UNIDADE IMAGINÁRIA

A EQUAÇÃO DO SEGUNDO GRAU

$$ax^2 + bx + c = 0$$

SOLUÇÃO:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

FÓRMULA DE BHÁSKARA

A EQUAÇÃO DO TERCEIRO GRAU

$$ax^3 + bx^2 + cx + d = 0$$

$$x^3 + p \cdot x + q = 0$$

SOLUÇÃO:

$$\boxed{x = \frac{-b}{3a} + \sqrt[3]{-\frac{q}{2} + \sqrt{q^2/4 + p^3/27}} + \sqrt[3]{-\frac{q}{2} - \sqrt{q^2/4 + p^3/27}}}$$

· FÓRMULA DE CARDANO TARTAGLIA ·

onde $p = \frac{-b^2}{3a^2} + \frac{c}{a}$ e $q = \frac{2b^3}{27a^3} - \frac{bc}{3a^2} + \frac{d}{a}$

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EXEMPLO

$$\boxed{x^3 = 15x + 4}$$

$$x = \frac{-b}{3a} + \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

$a = 1, \quad b = 0, \quad c = -15, \quad d = -4$

$$x = \sqrt[3]{\frac{4}{2} + \sqrt{\frac{2^2}{4} + \frac{15^3}{27}}} + \sqrt[3]{\frac{4}{2} - \sqrt{\frac{2^2}{4} - \frac{15^3}{27}}}$$

(. . .)

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$$

?

· Bombelli · Itália · séc. <u>XVI</u>

$$\sqrt[3]{2+\sqrt{-121}} = a + b\sqrt{-1}$$

$$\sqrt[3]{2+\sqrt{-121}} = a - b\sqrt{-1}$$

$$(\; . \; . \; . \;)$$

$a = 2$ e $b = 1$

$$\sqrt[3]{2+\sqrt{-121}} = 2 + b\sqrt{-1}$$

$$\sqrt[3]{2+\sqrt{-121}} = 2 - \sqrt{-1}$$

Logo: $x = \sqrt[3]{2+\sqrt{-121}} + \sqrt[3]{2-\sqrt{-121}}$

$$= 2 + \cancel{\sqrt{-1}} + 2 - \cancel{\sqrt{-1}}$$

$$= 4$$

A unidade imaginária

↳ A estrutura fundamental dos nºs complexos

$$\boxed{i^2 = -1 \quad \therefore \quad i = \sqrt{-1}}$$

- $\sqrt{-121} = \sqrt{121 \cdot (-1)} = \sqrt{121} \cdot \overbrace{\sqrt{-1}}^{i} = 11 \cdot i$

- $\sqrt{-25} = \sqrt{25 \cdot (-1)} = \sqrt{25} \cdot \overbrace{\sqrt{-1}}^{i} = 5i$

- $\sqrt{-407} = \sqrt{407 \cdot (-1)} = \sqrt{407} \cdot \overbrace{\sqrt{-1}}^{i} = \sqrt{407} \cdot i$

POTÉNCIAS DA UNIDADE IMAGINÁRIA

- $i^1 = \sqrt{-1}$
- $i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = \cancel{\sqrt{-1}^{\,2}} = -1$
- $i^3 = \overbrace{i \cdot i}^{-1} \cdot i = \overbrace{-1} \cdot i = -i$
- $i^4 = \overbrace{i^3}^{-i} \cdot i = -i^2 = -(-1) = 1$
- $i^5 = \underbrace{i^4}_{1} \cdot i = \underline{1} \cdot i = i$
- $i^6 = \underline{\underline{i^5}} \cdot i = \underline{\underline{i}} \cdot i = -1$
- $i^7 = \underbrace{i^6}_{-1} \cdot i = -i$
- $i^8 = \underbrace{i^7}_{-i} \cdot i = -i^2 = +1$

EXEMPLO

Calcule i^{2041}

R:

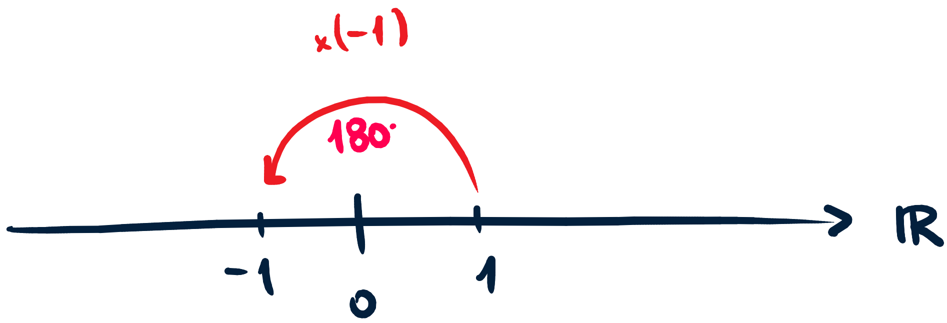
$$\begin{array}{r|l} 2041 & 4 \\ \cline{2-2} -20 & 510 \\ \hline 041 & \\ -40 & \\ \hline 01 & \end{array}$$

← RESTO

$$2041 = 510 \times 4 + 1$$

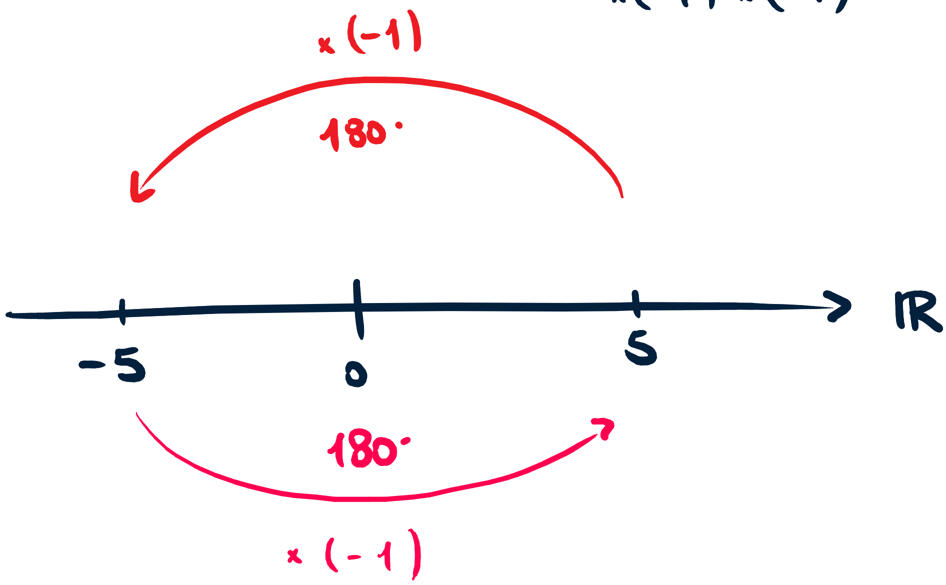
$$i^{2041} = i^{1} = i$$

O PLANO COMPLEXO



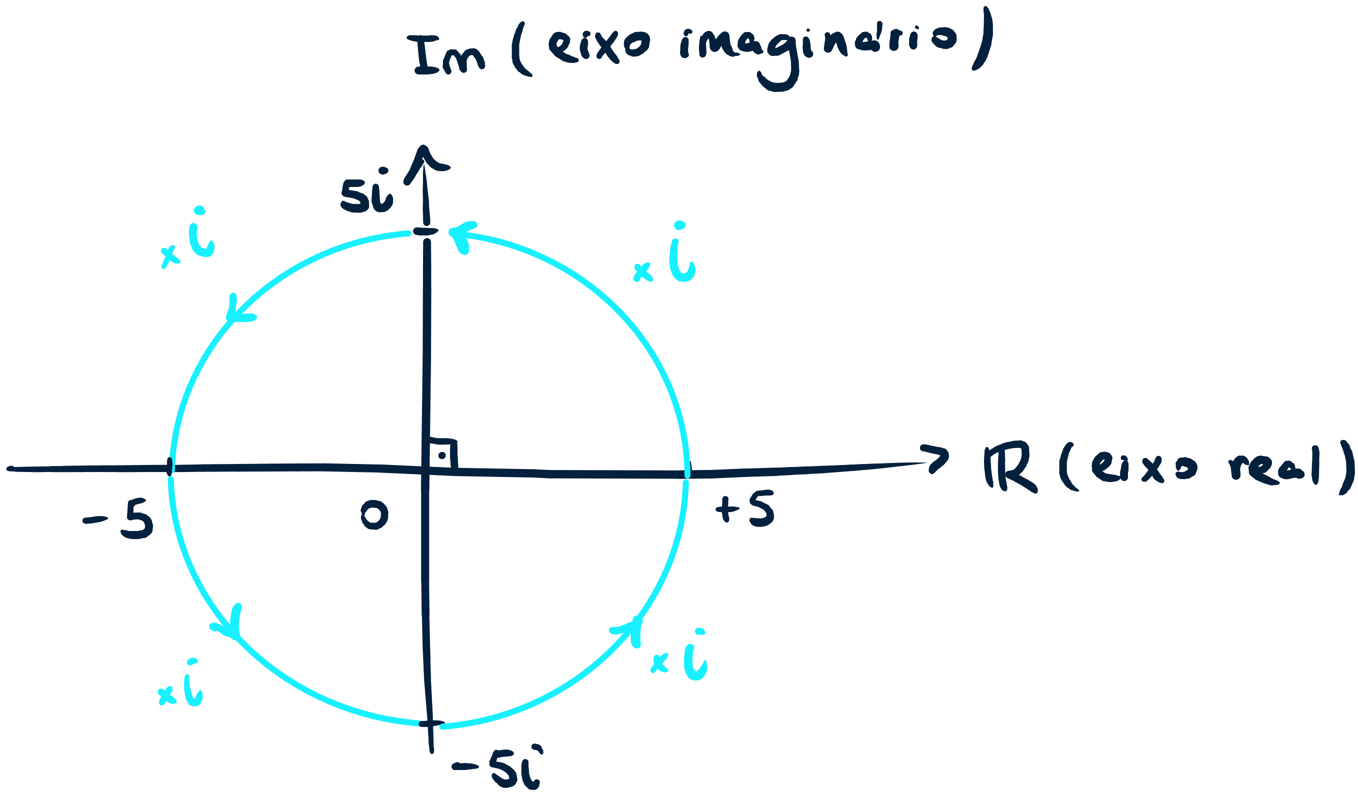
$$\times(-1) \longrightarrow \overset{\curvearrowleft}{180°}$$

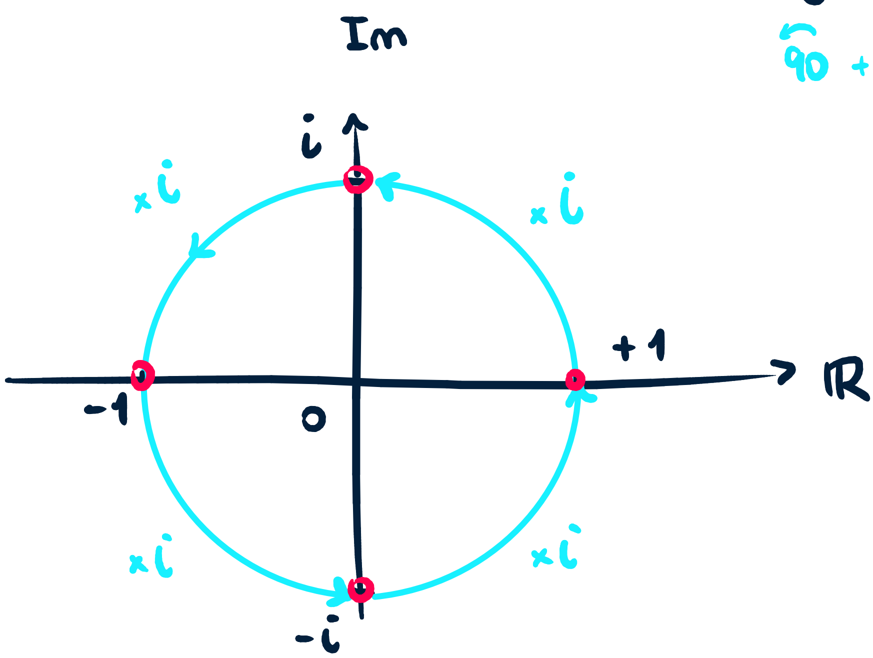
$$\times(-1)\times(-1) \longrightarrow \underbrace{\overset{\curvearrowleft}{180°} + \overset{\curvearrowleft}{180°}}_{360°}$$



$$\underset{\overset{\curvearrowleft}{90}}{i} \cdot \underset{+\ \overset{\curvearrowleft}{90}}{i} = \underbrace{-1}_{\overset{\curvearrowleft}{180°}}$$

$$5 \cdot \underset{\overset{\curvearrowleft}{90°}}{i} \cdot \underset{\overset{\curvearrowleft}{90°}}{i} = 5 \cdot \underbrace{(-1)}_{\overset{\curvearrowleft}{180°}} = -5$$





$$\underset{90}{i} \cdot \underset{90}{i} = \underset{180}{-1}$$

$90 + 90 \longrightarrow 180$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

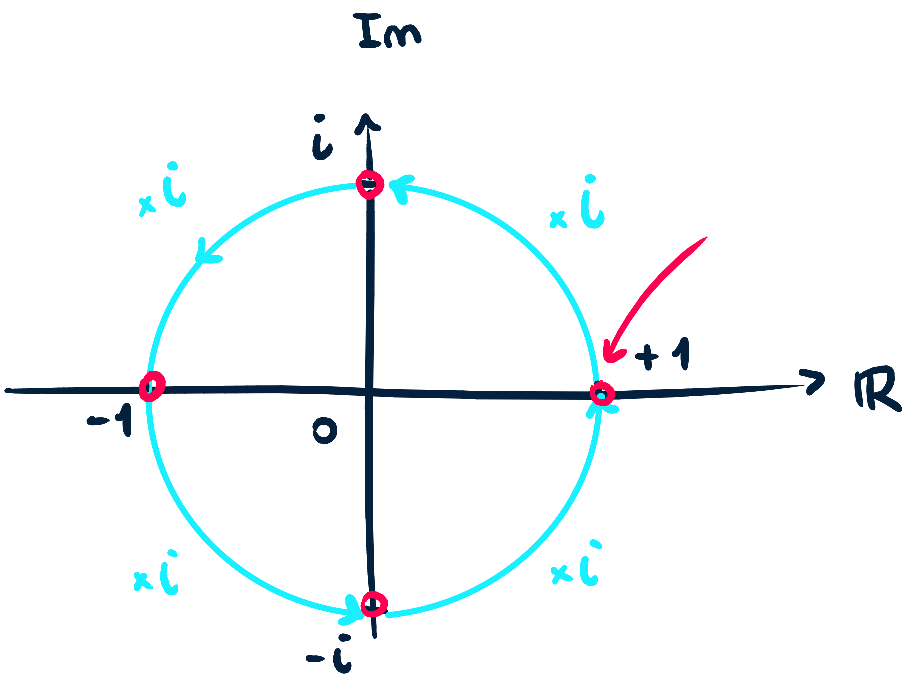
Multiplicar por $\underline{i}$ gera uma rotação de 90° no sentido A.H. no plano complexo.

· EXEMPLO 01 ·

Quanto vale i^{21} ?

R :

$$21 = 5 \times \underbrace{\overset{i^4}{4}}_{360^\circ} + \underbrace{\overset{\text{RESTO}\; i}{1}}_{90^\circ}$$



$$i^{21} = i$$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

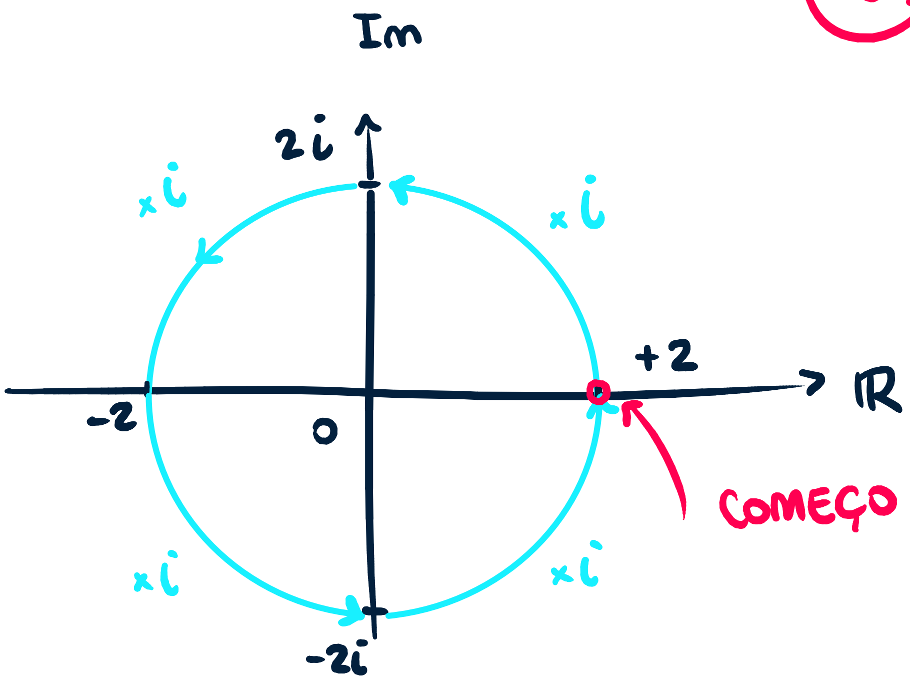
EXEMPLO 02.

Quanto vale $2 \cdot i^{302}$?

R:

$$302 = \underline{\underline{4}} \cdot 75 + 2$$

360 90 + 90



$$i^{302} = i^{2} = -1$$

1) $i^1 = i$

2) $i^2 = -1$

3) $i^3 = -i$

4) $i^4 = 1$

$$2 \cdot i^{302} = 2 \cdot (-1) = -2$$

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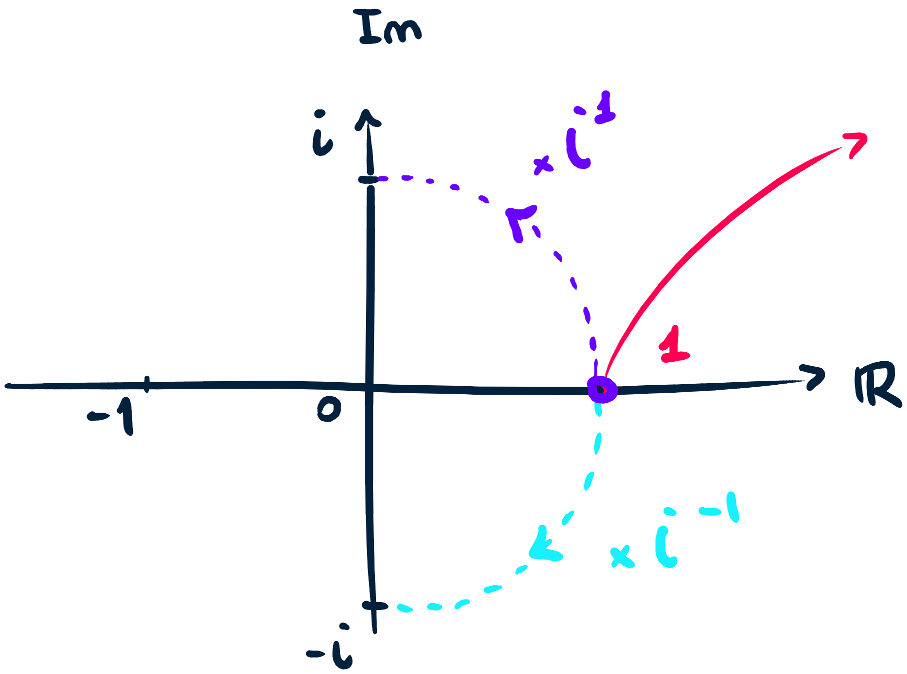
• EXEMPLO 03 •

Quanto vale i^{-1} ?

R :



$1 \times i^{1} = i$

$1 \times i^{-1} = -i$

$$i^{-1} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} \times \frac{(-1)}{(-1)} = \frac{-i}{1} = -i$$