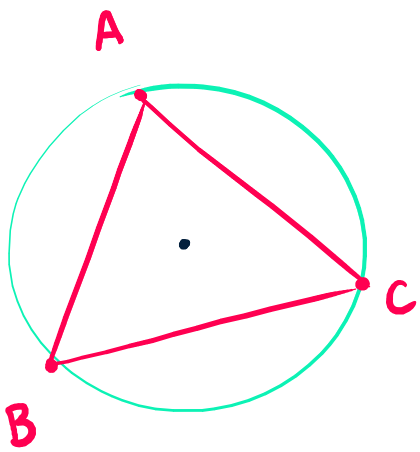
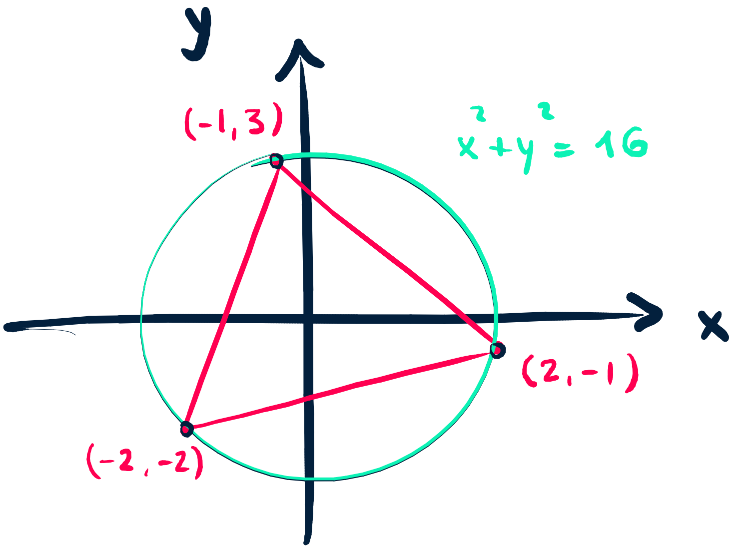
GEOMETRIA ANALÍTICA: INTRODUÇÃO





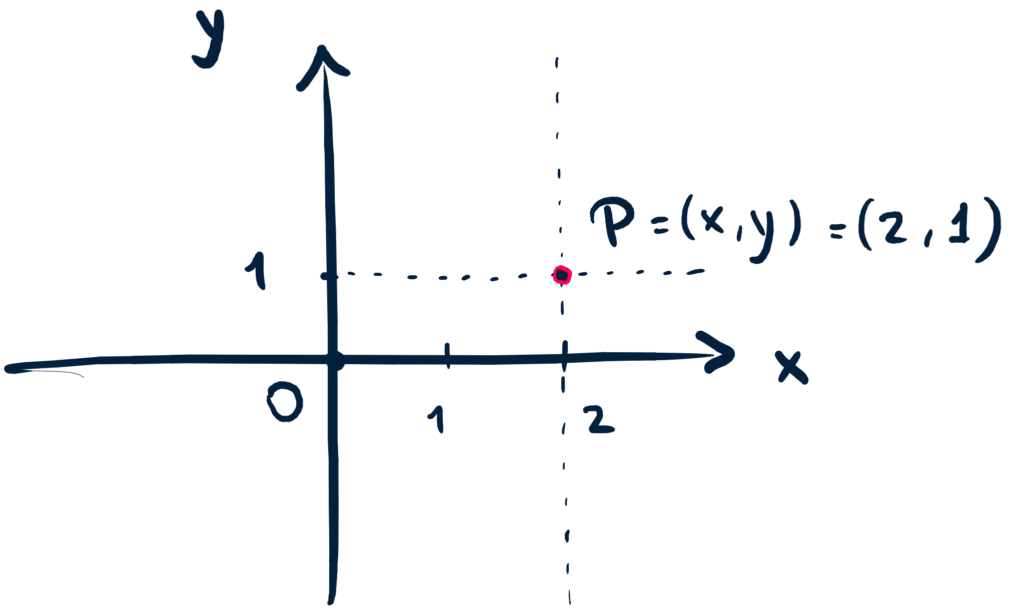
$\overline{AB} = 4$

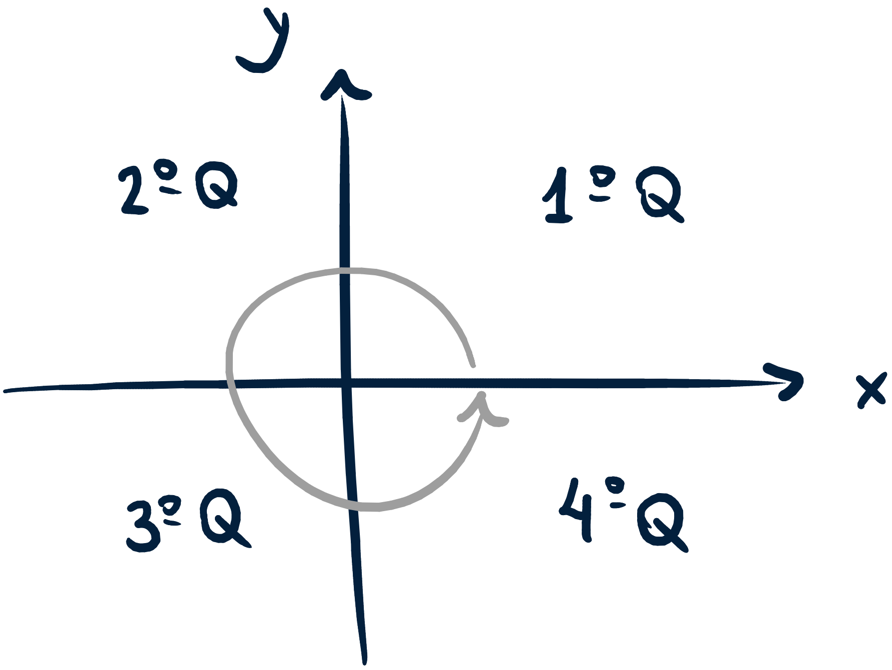
$\overline{BC} = 5$

$\overline{AC} = 4{,}5$

SISTEMA CARTESIANO

René Descartes





EXEMPLO

$A = (1, -1)$

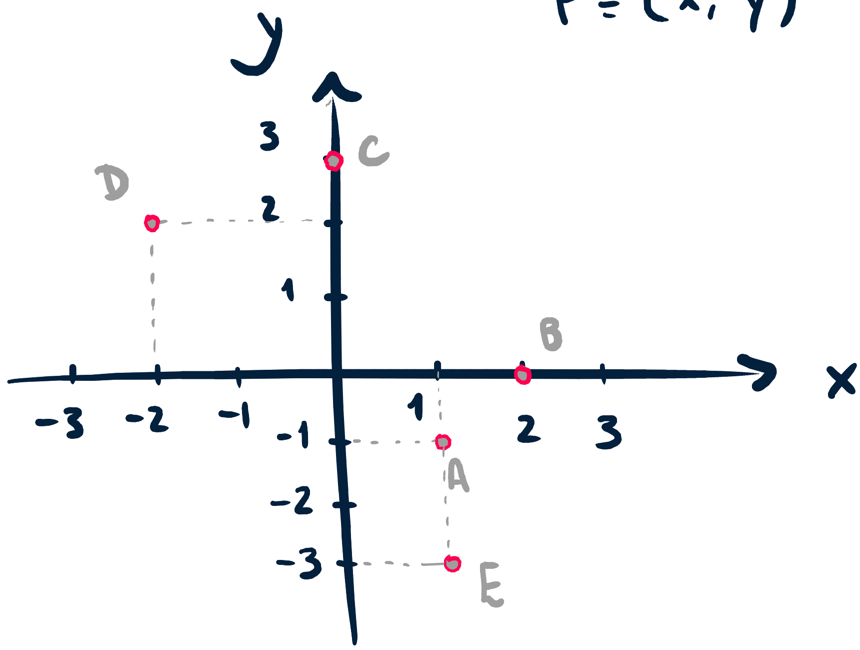
$B = (2, 0)$

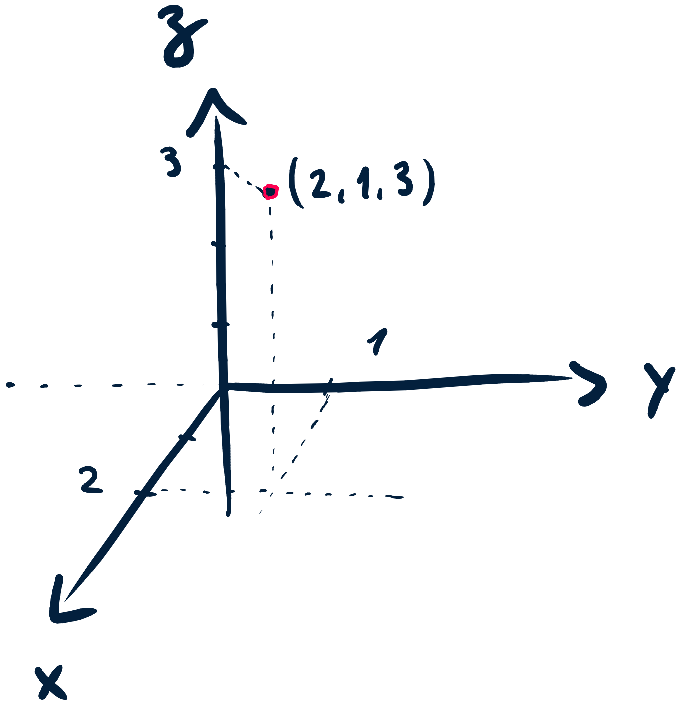
$C = (0, 3)$

$D = (-2, 2)$

$E = (1, -3)$

$P = (x, y)$





$P = (x, y, z) = (2, 1, 3)$

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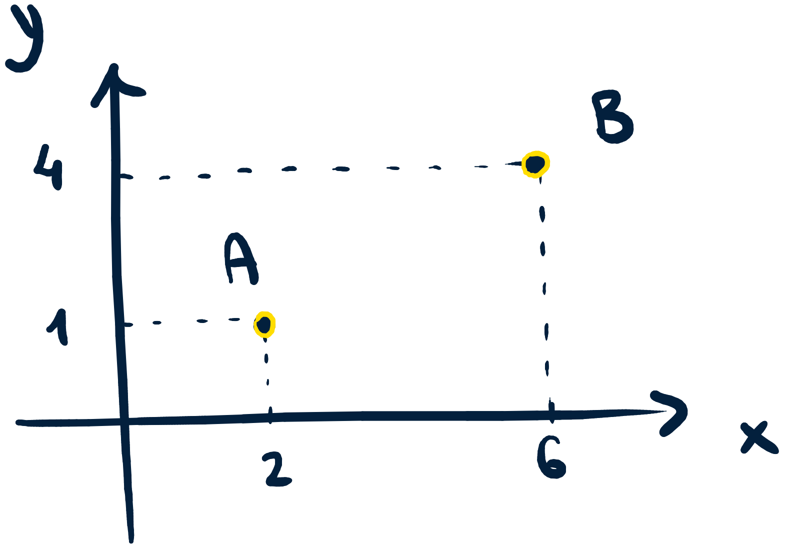
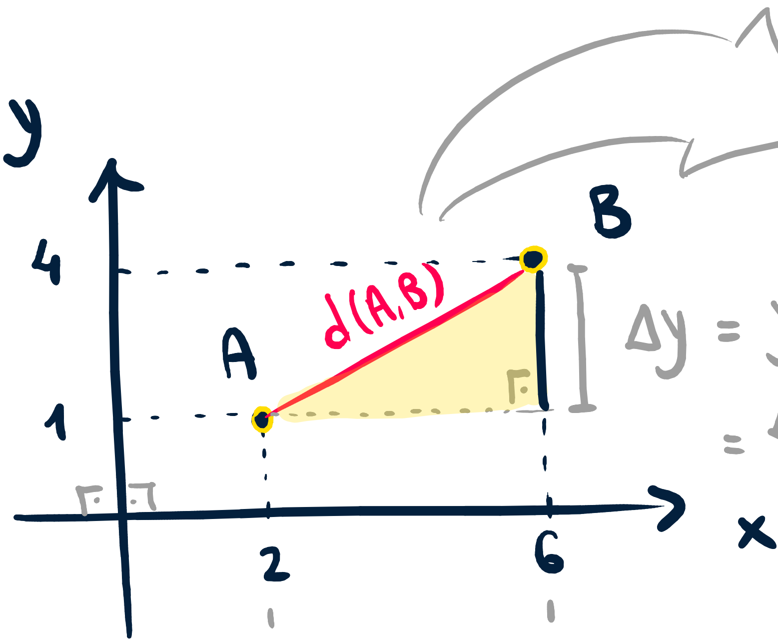
· DISTÂNCIA ENTRE PONTOS ·



$A = (2, 1)$

$B = (6, 4)$



$\Delta y = y_B - y_A$

$= 4 - 1 = 3$

$\Delta x = x_B - x_A = 6 - 2 = 4$

Pitágoras:

$d^2(A,B) = \Delta x^2 + \Delta y^2$

$= 4^2 + 3^2$

$= 25$

$\boxed{d(A,B) = 5}$

$$\boxed{d(A,B) = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}}$$

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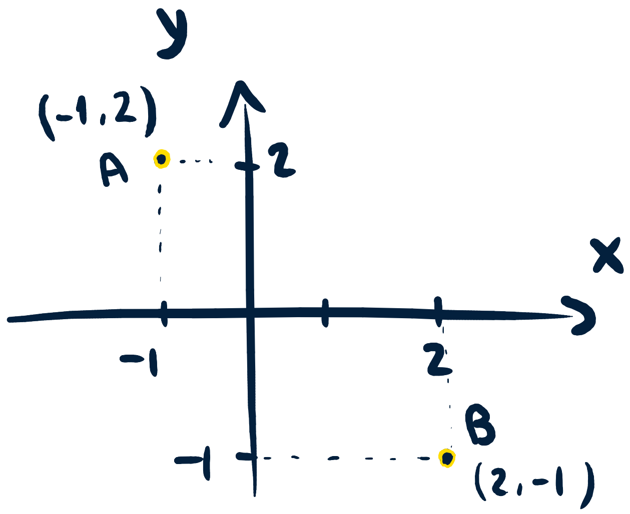
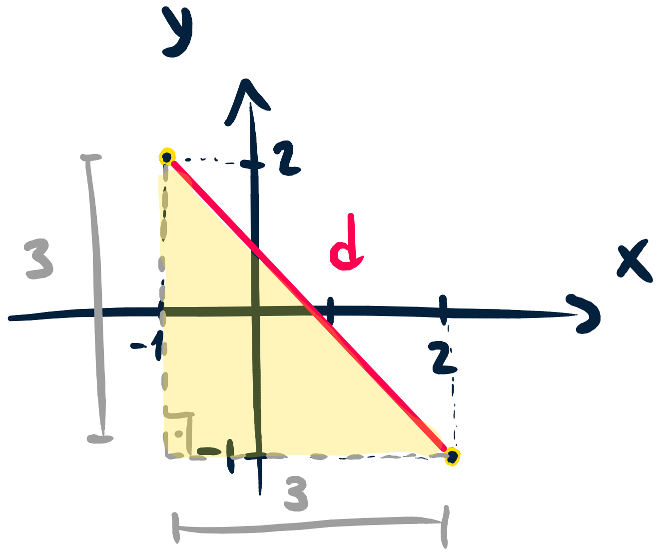
EXEMPLO

Calcule a distância entre os pontos

$A = (-1, 2)$ e $B = (2, -1)$.

R :





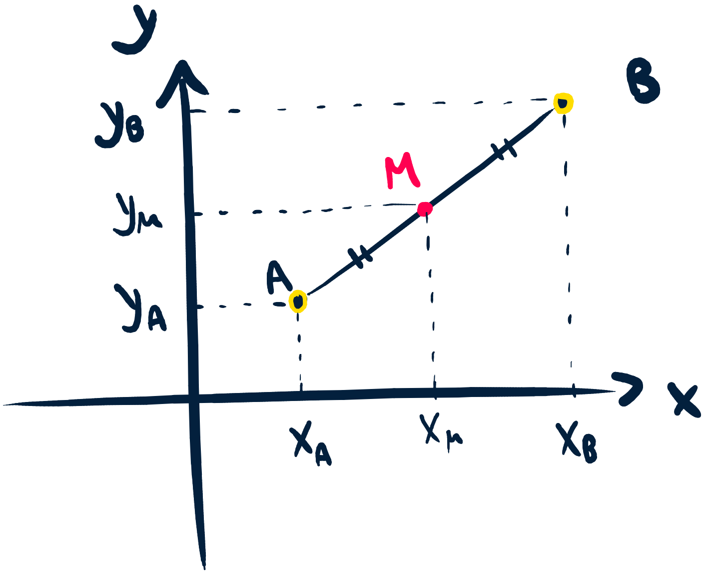
• $d^2 = 3^2 + 3^2 = 2 \cdot 3^2 \therefore \underline{d = 3\sqrt{2}}$

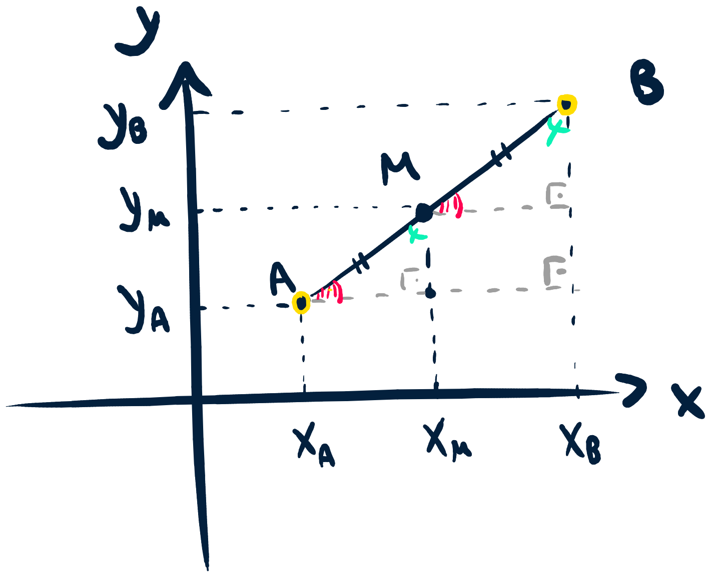
• $d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(-1-2)^2 + (2-(-1))^2}$

$= \sqrt{(-3)^2 + 3^2}$

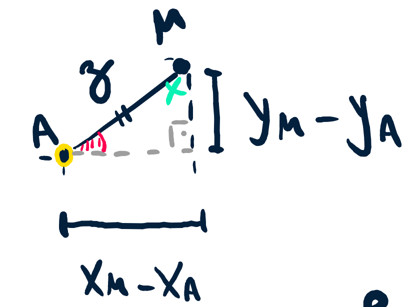
$= \sqrt{9 \cdot 2} = \underline{3\sqrt{2}}$

· PONTO MÉDIO ·



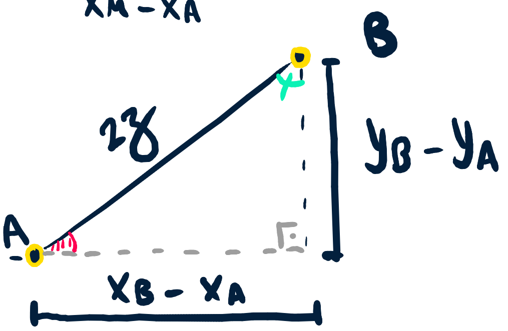


Semelhança:



(i) $$\frac{x_M - x_A}{x_B - x_A} = \frac{\cancel{\gamma}}{2\cancel{\gamma}} \therefore$$

$$x_M = x_A + \frac{x_B - x_A}{2} = \frac{x_A + x_B}{2}$$



(ii) $$\frac{y_M - y_A}{y_B - y_A} = \frac{1}{2} \therefore y_M = \frac{y_A + y_B}{2}$$

$$\boxed{M = (x_M, y_M) = \left(\frac{x_A + x_B}{2} ; \frac{y_A + y_B}{2}\right)}$$

EXEMPLO

Quais são as coordenadas do ponto médio do segmento $\overline{AB}$ onde $A = (-1, 3)$ e $B = (2, 4)$?

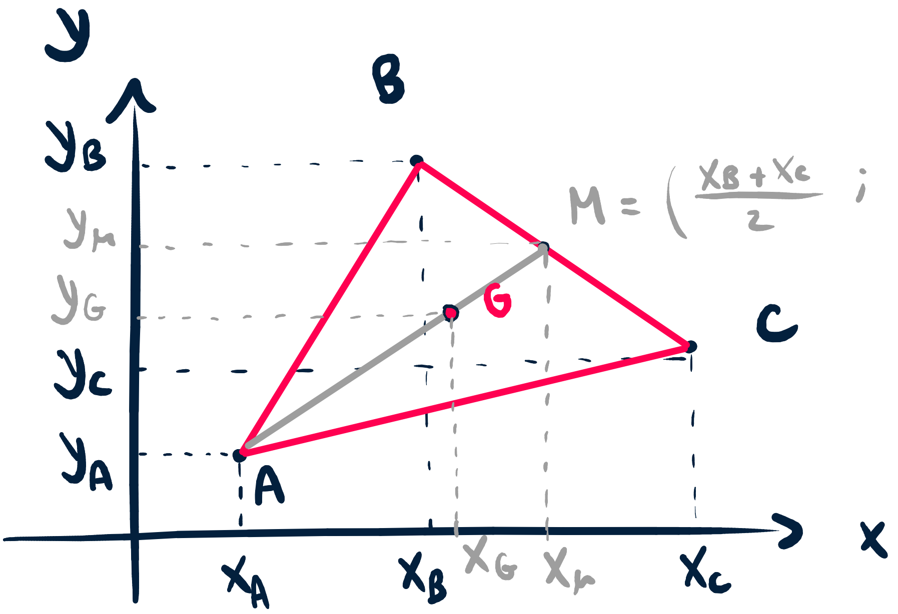
R:

$$M = (x_M, y_M) = \left(\frac{x_A + x_B}{2} ; \frac{y_A + y_B}{2} \right)$$

$$= \left(\frac{-1 + 2}{2} ; \frac{3 + 4}{2} \right)$$

$$= (0,5 ; 3,5)$$

• COORDENADAS DO BARICENTRO •



$$M = \left(\frac{x_B + x_C}{2} \; ; \; \frac{y_B + y_C}{2} \right)$$

$$\frac{\overline{AG}}{\overline{GM}} = 2 = \frac{(x_G - x_A)}{(x_M - x_G)} \quad \therefore \quad 2x_M - 2x_G = x_G - x_A$$

$$2x_M + x_A = 3x_G \quad \therefore \quad \cancel{2} \cdot \left(\frac{x_B + x_C}{\cancel{2}} \right) + x_A = 3x_G$$

$$x_G = \frac{x_A + x_B + x_C}{3}$$

$$\boxed{G = \left(\frac{x_A + x_B + x_C}{3} \; ; \; \frac{y_A + y_B + y_C}{3} \right)}$$

EXEMPLO

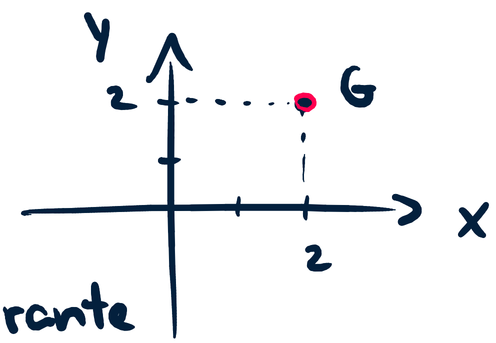
Dado o ΔABC de vértices $A = (-1, 2)$, $B = (1, 0)$ e $C = (6, 4)$, em qual quadrante se encontra seu baricentro?

R:

$$G = \left(\frac{x_A + x_B + x_C}{3} \; ; \; \frac{y_A + y_B + y_C}{3} \right)$$

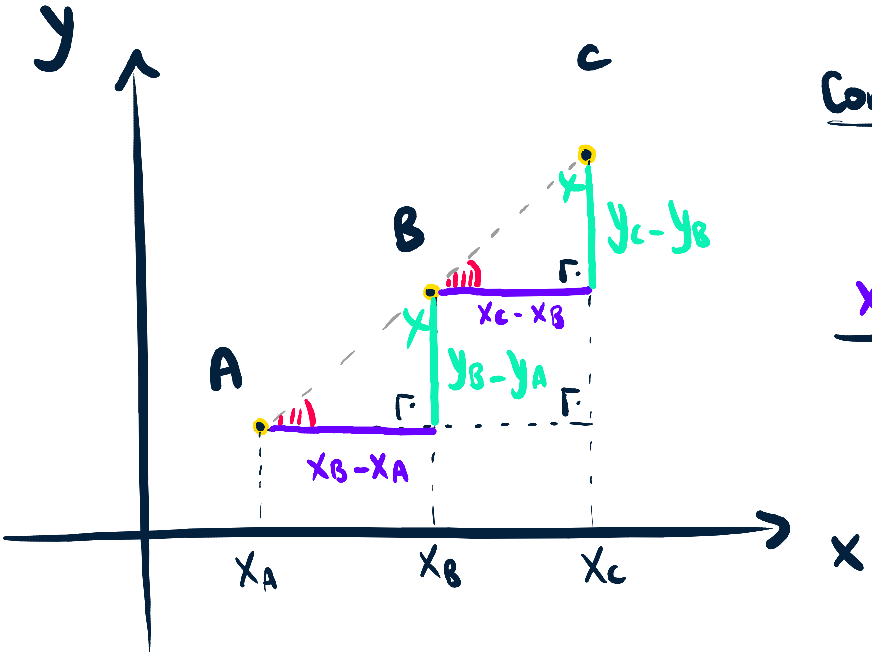
$$= \left(\frac{-1 + 1 + 6}{3} \; ; \; \frac{2 + 0 + 4}{3} \right)$$

$$= (2 \, , \, 2)$$



1º quadrante

· COLINEARIDADE DE PONTOS ·



Condição de alinhamento:
(semelhança)

$$\frac{x_C - x_B}{x_B - x_A} = \frac{y_C - y_B}{y_B - y_A}$$

$$(x_C - x_B)\cdot(y_B - y_A) = {}_B - x_A)\cdot(y_C - y_B)$$

$$x_C \cdot y_B - x_C \cdot y_A - \cancel{x_B \cdot y_B} + x_B \cdot y_A = x_B \cdot y_C - \cancel{x_B \cdot y_B} - x_A \cdot y_C + x_A \cdot y_B$$

$$\boxed{x_B \cdot y_A + x_C \cdot y_B + x_A \cdot y_C - x_C \cdot y_A - x_B y_C - x_A \cdot y_B = 0}$$

Se A, B e C são colineares, então:

$$\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = 0$$

$$\left|\begin{array}{ccc|cc} x_A & y_A & 1 & x_A & y_A \\ x_B & y_B & 1 & x_B & y_B \\ x_C & y_C & 1 & x_C & y_C \end{array}\right. = 0$$

$$x_A \cdot y_B + x_C \cdot y_A + x_B \cdot y_C - x_C \cdot y_B - x_A \cdot y_C - x_B \cdot y_A = 0$$

· EXEMPLO ·

Determine α p/ que A, B e C sejam colineares, dado $A = (1,0)$, $B(2,1)$ e $C = (\alpha, 2)$.

R :

$$\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = 0$$

$$\left|\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 & 1 \\ \alpha & 2 & 1 & \alpha & 2 \end{array}\right| = 0$$

$$1 + 0 + 4 - \alpha - 2 - 0 = 0 \therefore \boxed{\alpha = 3}$$