

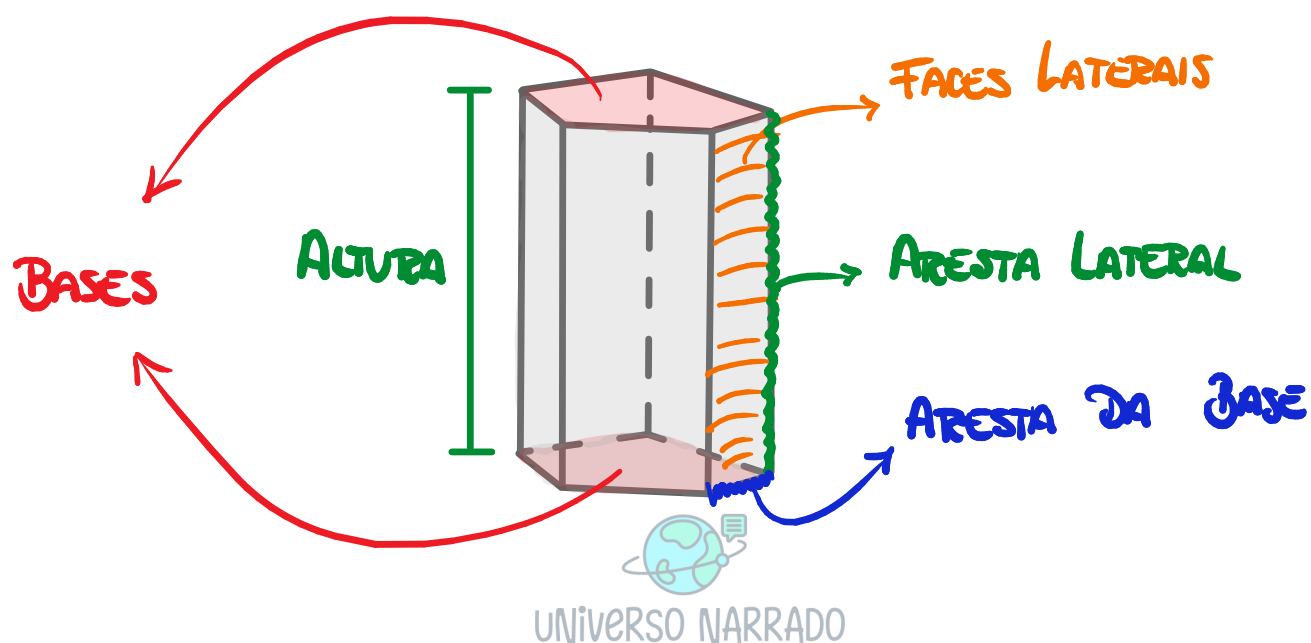
PRISMAS

DEFINIÇÃO

SÃO POLIEDROS QUE POSSUEM:

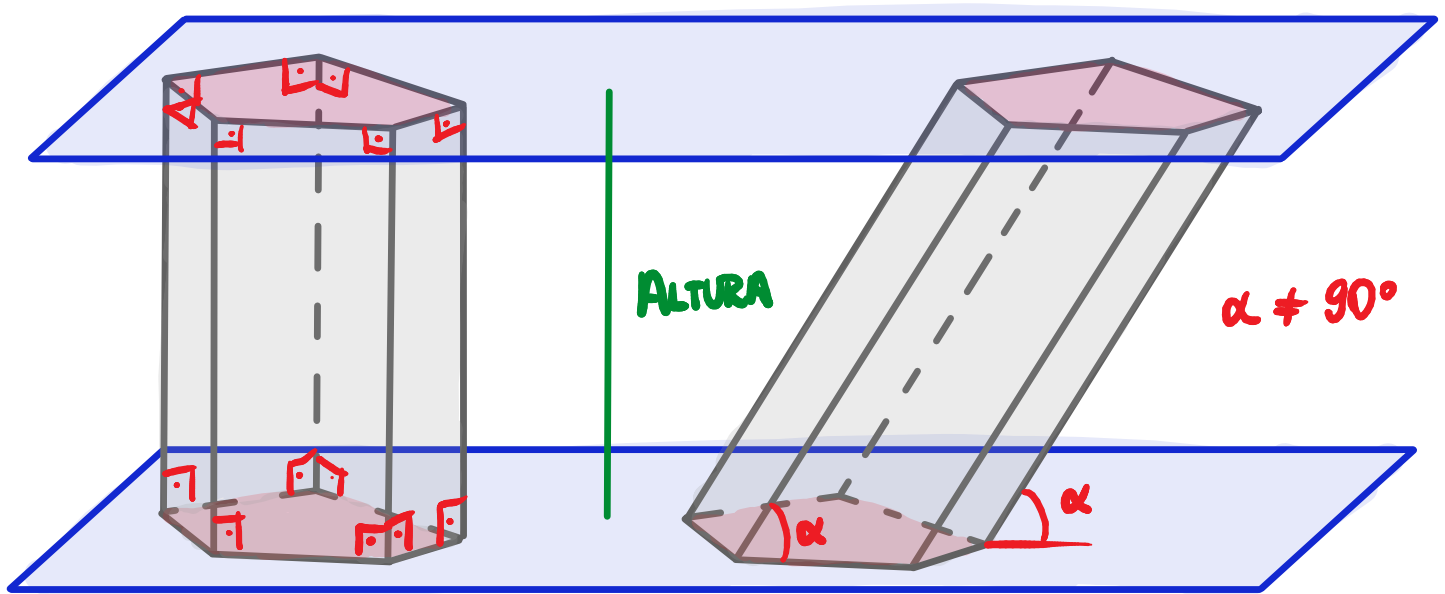
↳ DUAS FACES PARALELAS E CONGRUENTES.
(DENOMINADAS BASES)

↳ FACES LATERAIS SÃO PARALELOGRAMOS.



CLASSIFICAÇÃO DOS PRISMAS

→ QUANTO À INCLINAÇÃO DAS ARESTAS LATERAIS



PRISMA
RETO

PRISMA
OBLÍQUO

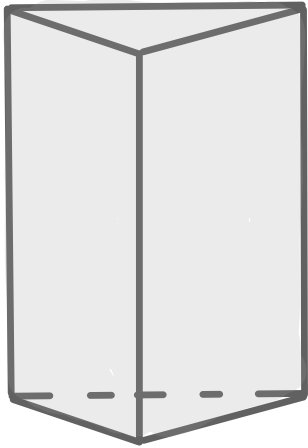
OBSERVATIONS

ALTURA : DISTÂNCIA ENTRE OS PLANOS DA BASE

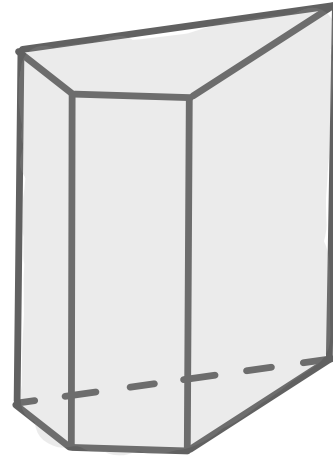
ARESTAS LATERAIS : PARALELAS



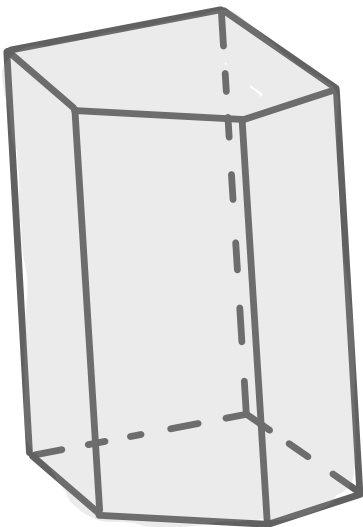
→ QUANTO AO POLÍGONO DA BASE



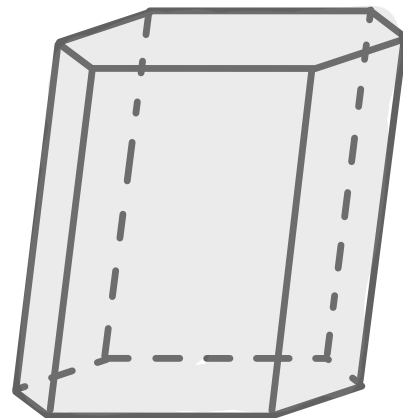
PRISMA
TRIANGULAR



PRISMA
QUADRILATERAL



PRISMA
PENTAGONAL

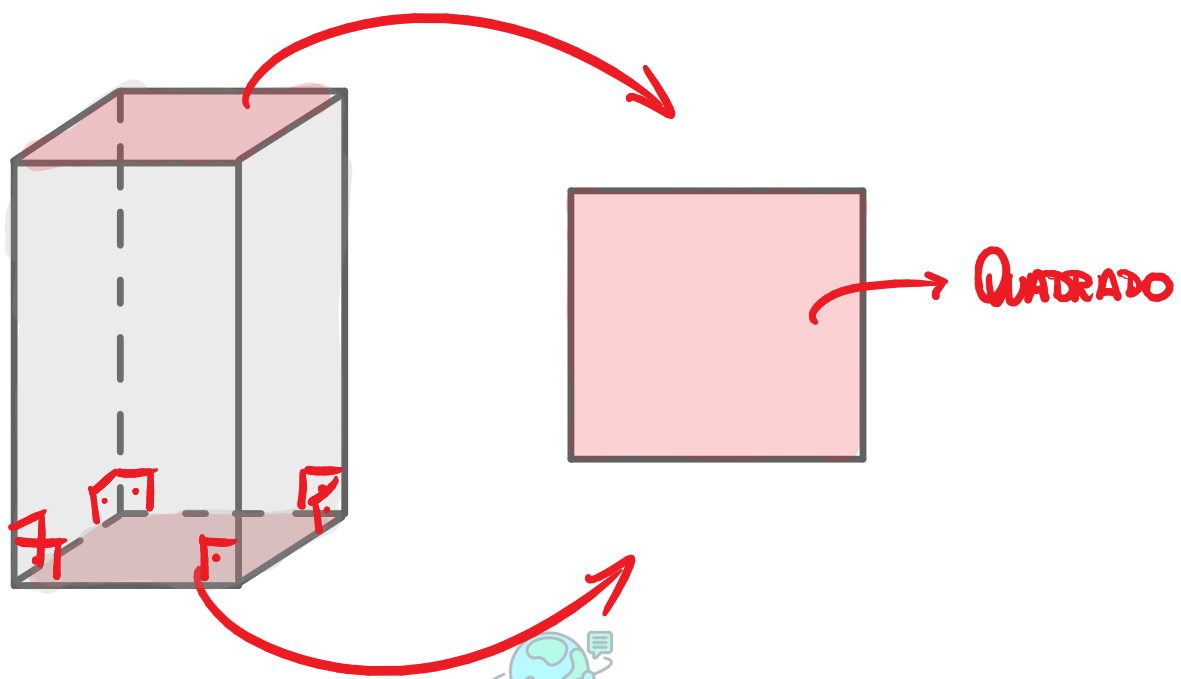
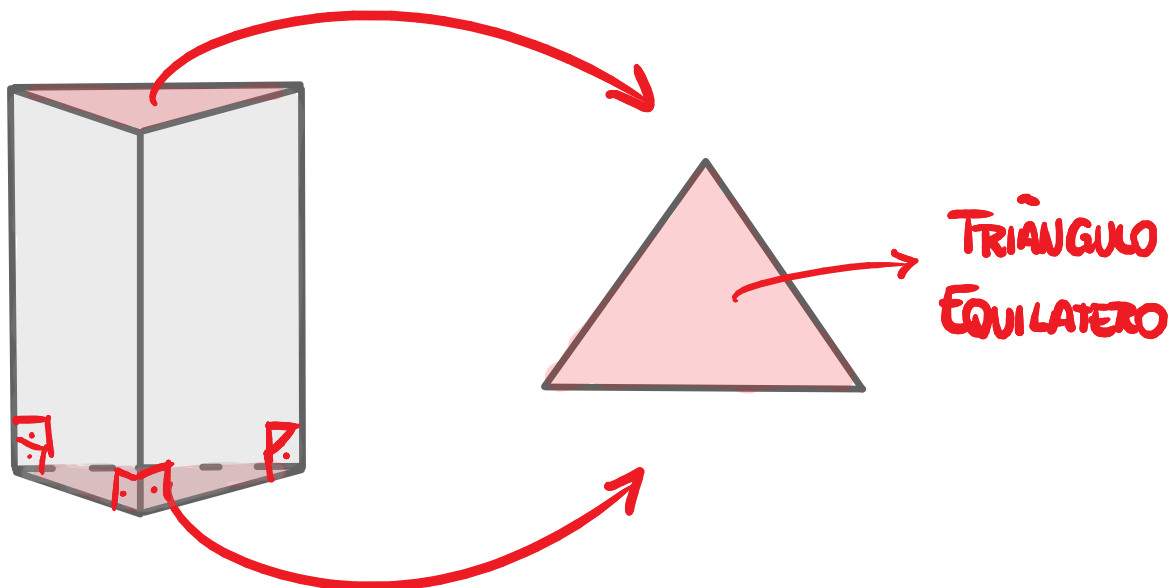


PRISMA
HEXAGONAL



PRISMA REGULAR

É UM PRISMA RETO CUA BASE É UM POLÍGONO REGULAR.

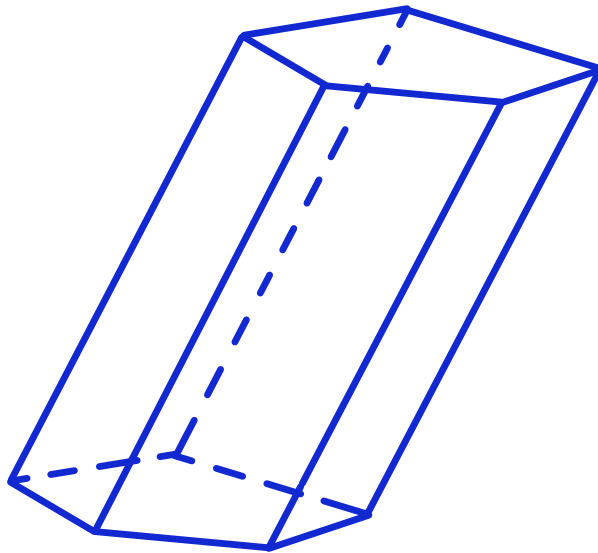


EXEMPLO

Um PRISMA POSSUI 7 FACES. CLASSIFIQUE ESSE PRISMA.

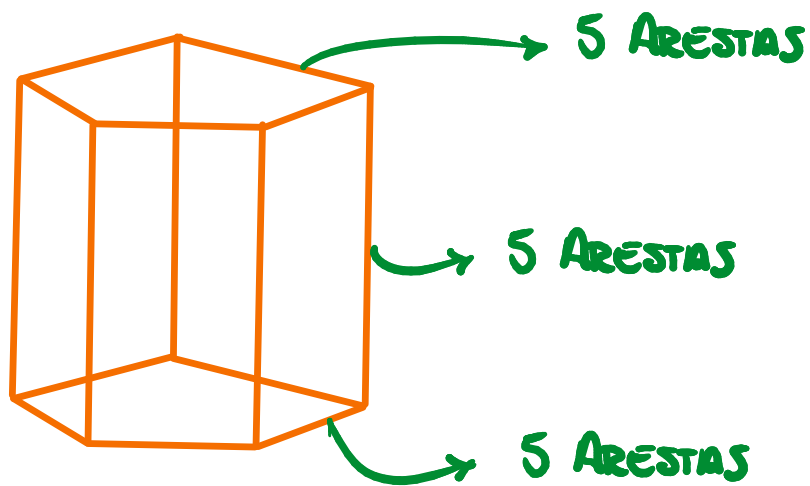
FACES DA BASE : 2

FACES LATERAIS : 5 → PRISMA PENTAGONAL



EXEMPLO

UM PRISMA TEM n FACES LATERAIS.
CALCULE O NÚMERO DE ARESTAS.

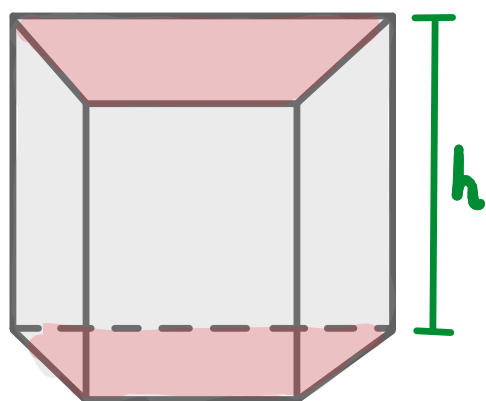


n BASE SUPERIOR	}	$3n$ ARESTAS
n BASE INFERIOR		
n LIGANDO AS BASES		

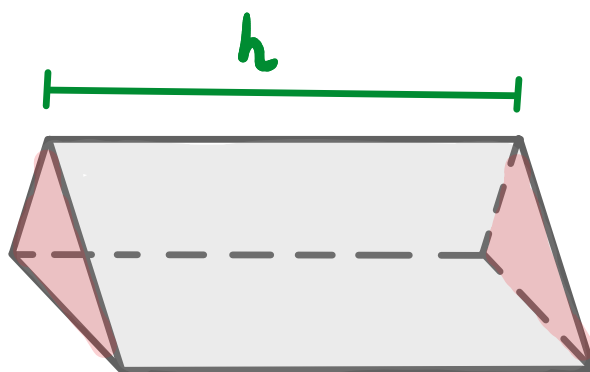


EXEMPLO

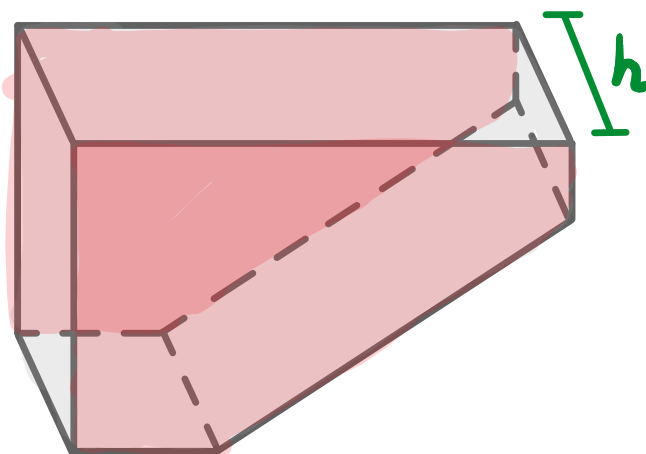
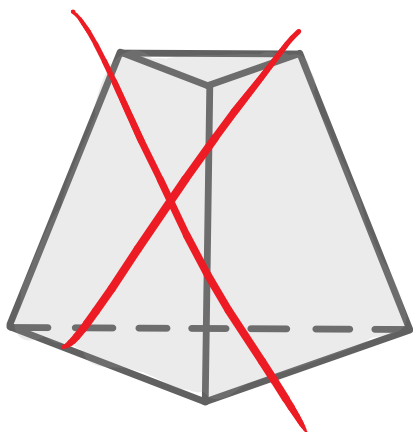
VERIFIQUE QUAIS SÓLIDOS ABAIXO SÃO PRISMAS,
CLASSIFIQUE-OS E IDENTIFIQUE SUAS ALTURAS.



PRISMA
QUADRILATERAL



PRISMA
TRIANGULAR



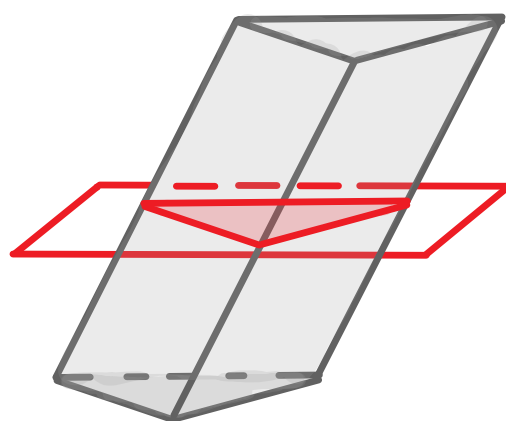
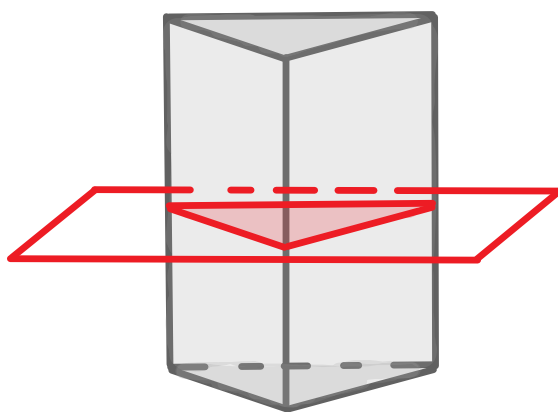
PRISMA
PENTAGONAL



SEÇÕES DE UM PRISMA

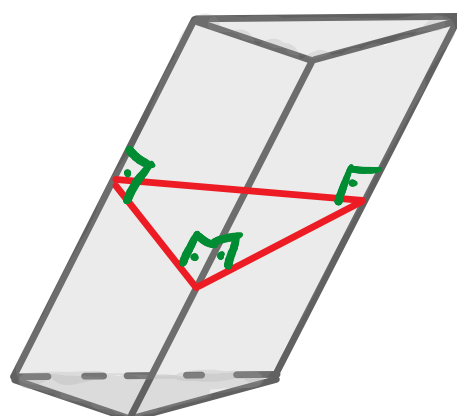
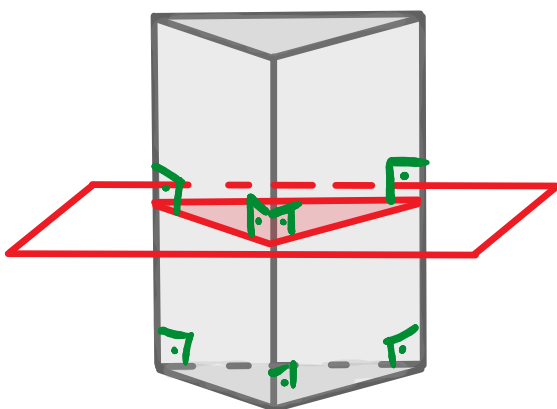
SEÇÃO TRANSVERSAL

SEÇÃO PARALELA ÀS BASES.



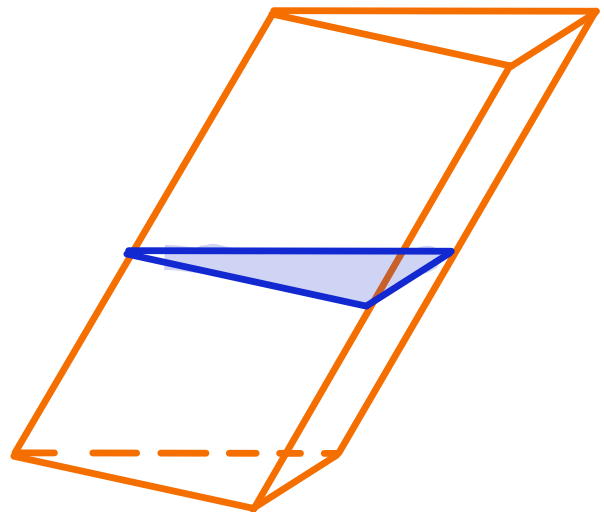
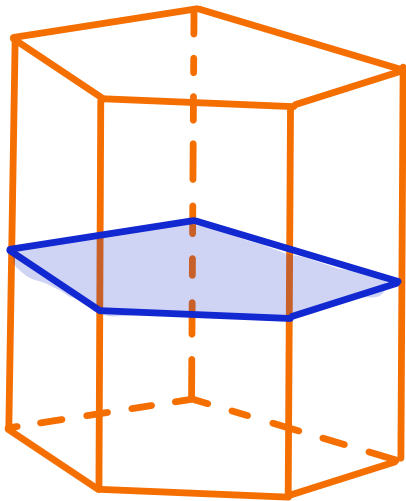
SEÇÃO RETA

SEÇÃO PERPENDICULAR ÀS ARESTAS LATERAIS



EXEMPLO

DESENHE UM PRISMA PENTAGONAL RETO E UM TRIANGULAR OBLÍQUO E FAÇA UMA SEÇÃO TRANSVERSAL EM CADA UM DELES.

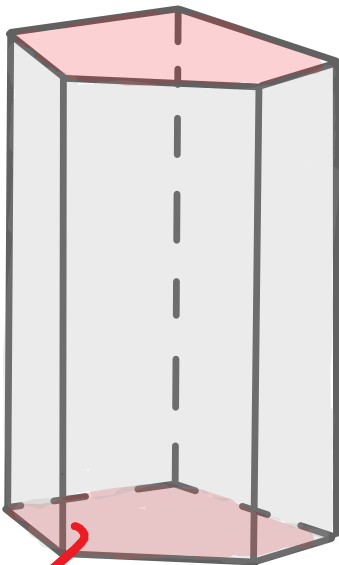


VOLUME

$$V = A_b \cdot h$$

A_b : ÁREA DA BASE

h : ALTURA



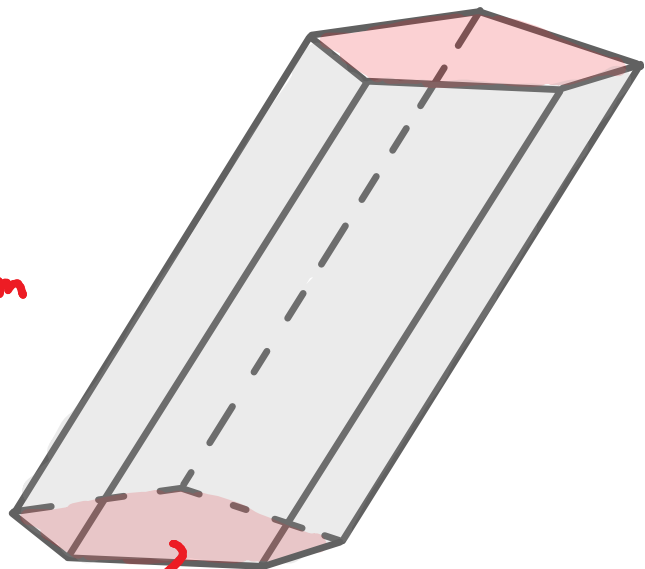
4cm

$$A_b = 12 \text{ cm}^2$$

$$V_1 = A_b \cdot h$$

$$V_1 = 12 \cdot 4$$

$$V_1 = 48 \text{ cm}^3$$



4cm

$$A_b = 12 \text{ cm}^2$$

$$V_2 = A_b \cdot h$$

$$V_2 = 12 \cdot 4$$

$$V_2 = 48 \text{ cm}^3$$



ÁREAS

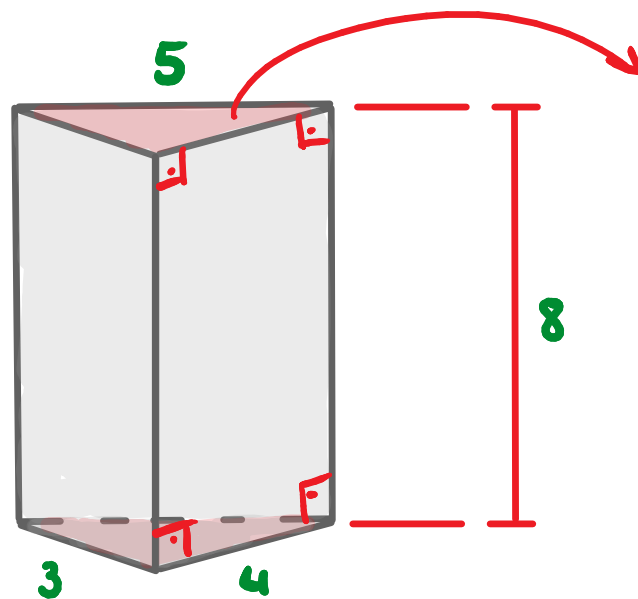
$$A_T = 2A_b + A_L$$

A_T : ÁREA TOTAL

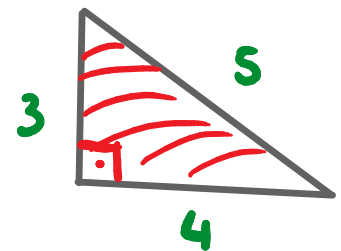
A_b : ÁREA DA BASE

A_L : ÁREA LATERAL

PRISMA
TRIANGULAR
RETO



BASE



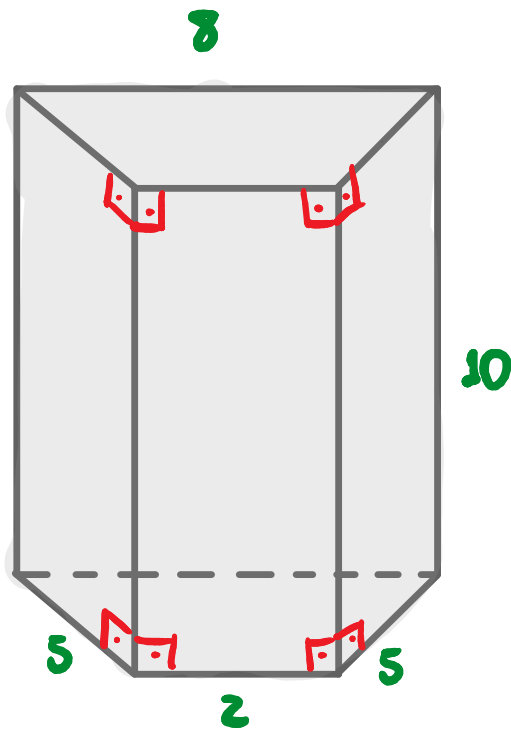
$$A_L = 4 \cdot 8 + 3 \cdot 8 + 5 \cdot 8 = 8(4+3+5) = 96 \quad \left. \begin{array}{l} A_T = 96 + 2 \cdot 6 \\ A_T = 108 \end{array} \right\}$$

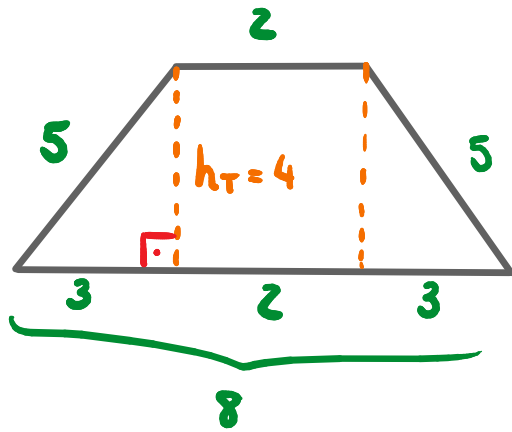
$$A_b = \frac{3 \cdot 4}{2} = 6$$



EXEMPLO

CALCULE O VOLUME E A ÁREA TOTAL DO PRISMA TRAPEZOIDAL ABAIXO.





$$A_b = \frac{(B + b) h_T}{2}$$

$$A_b = \frac{(8 + 2) \cancel{4}^2}{\cancel{2}} = 20$$

$$V = A_b \cdot h \rightarrow V = 20 \cdot 10 \rightarrow V = 200$$

$$A_T = 2 \cdot A_b + A_L$$

$$A_L = 2 \cdot 10 + 5 \cdot 10 + 8 \cdot 10 + 5 \cdot 10$$

$$A_L = 10(2 + 5 + 8 + 5) = 200$$

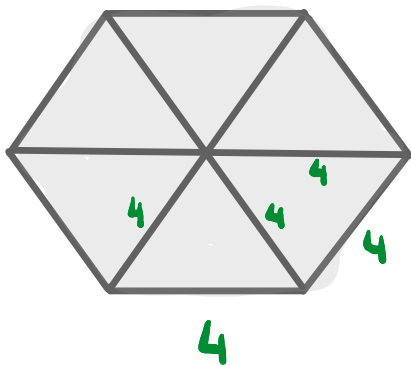
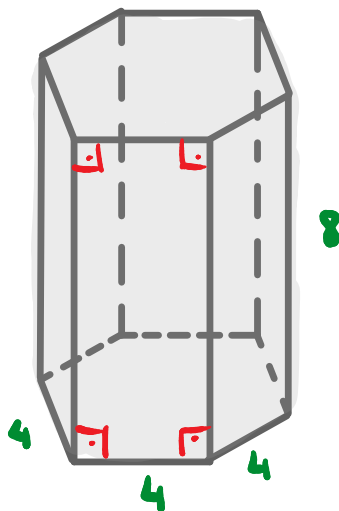
$$A_T = 2 \cdot 20 + 200$$

$$A_T = 240$$



EXEMPLO

CALCULE O VOLUME E A ÁREA TOTAL DE UM PRISMA HEXAGONAL REGULAR CUJO LADO DA BASE É 4 E POSSUI ALTURA 8.



$$\begin{aligned} A_b &= 6 \cdot A_{\Delta EQ} \\ &= 6 \cdot \cancel{4} \cdot \sqrt{3} \\ &\quad \quad \quad \cancel{4} \end{aligned}$$

$$A_b = 24\sqrt{3}$$

$$V = A_b \cdot h \rightarrow V = 24\sqrt{3} \cdot 8 \rightarrow V = 192\sqrt{3}$$



$$A_T = 2A_b + A_L$$

$$\begin{aligned} A_L &= \text{perímetro base} \cdot h \\ &= 4 \cdot 6 \cdot 8 \end{aligned}$$

$$A_L = 192$$

$$A_T = 2 \cdot 24\sqrt{3} + 192$$

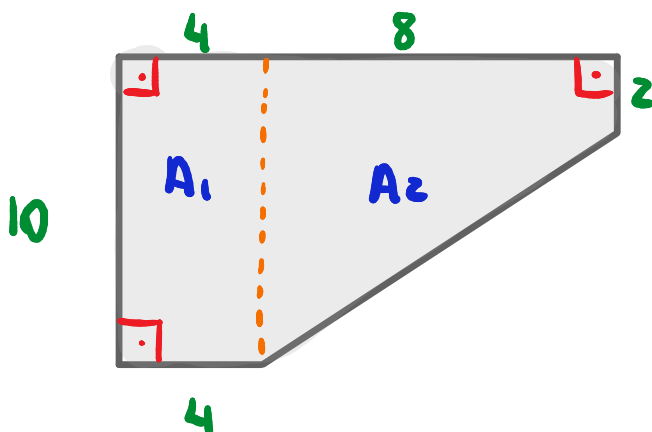
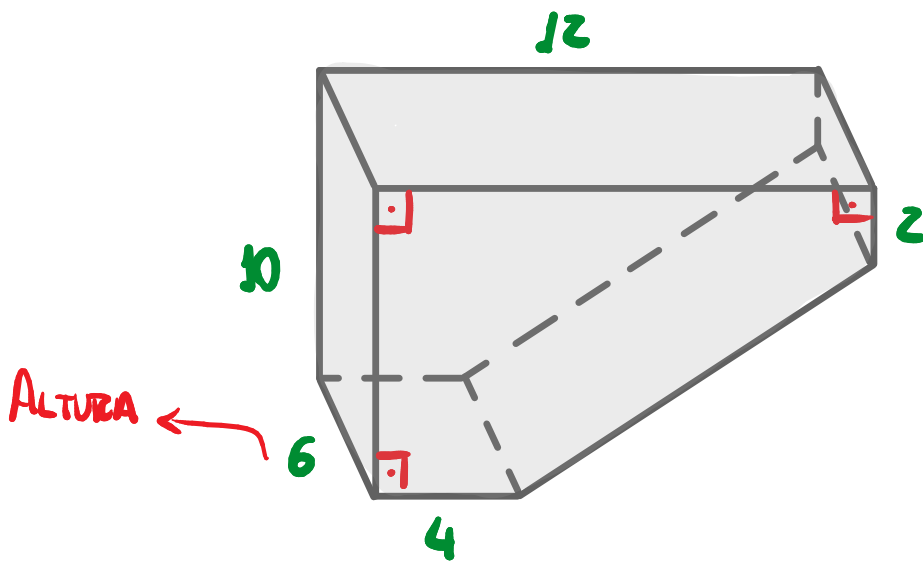
$$A_T = 48\sqrt{3} + 192$$

$$A_T = 48(\sqrt{3} + 4)$$



EXEMPLO

CALCULE O VOLUME DO SÓLIDO.



$$A_1 = 4 \cdot 10 = 40$$

$$A_2 = \frac{(10 + 2) \cdot 8}{2} = 48$$

$$A_b = A_1 + A_2$$

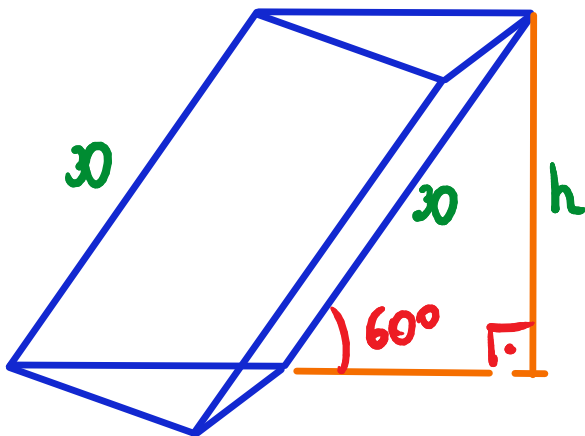
$$A_b = 88$$

$$V = A_b \cdot h \rightarrow V = 88 \cdot 6 \rightarrow V = 528$$



EXEMPLO

CALCULE O VOLUME DE UM PRISMA CUJA ÁREA DA BASE É 20 E CUJAS ARESTAS LATERAIS MEDEM 30 E FORMAM 60° COM O PLANO DA BASE.



$$\sin 60^\circ = \frac{h}{30}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{\cancel{30}_{15}}$$

$$\underline{h = 15\sqrt{3}}$$

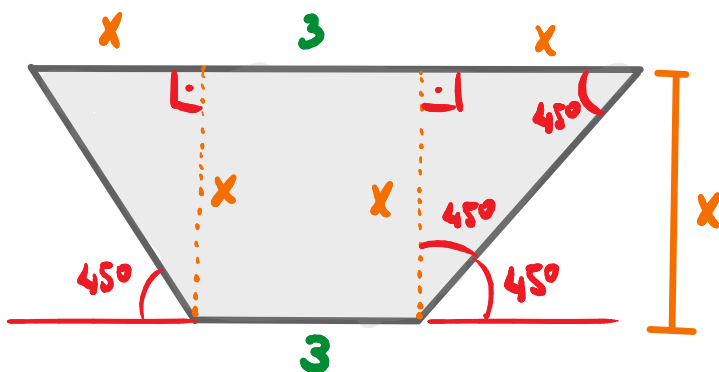
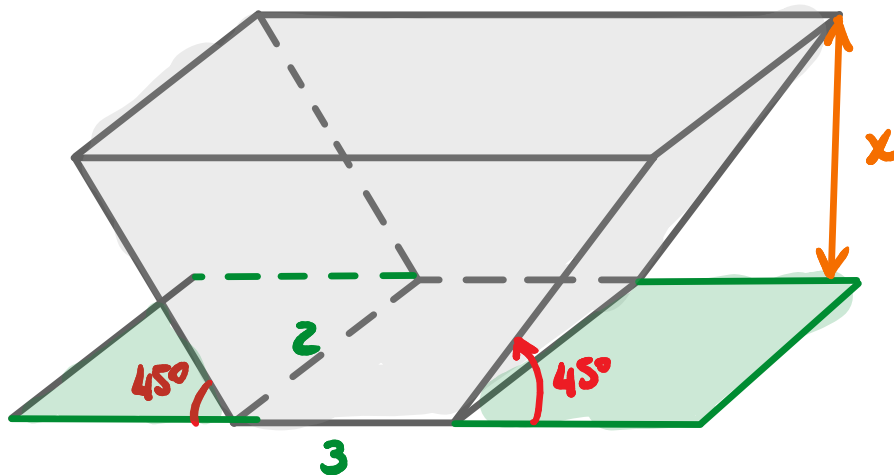
$$V = A_b \cdot h \rightarrow V = 20 \cdot 15\sqrt{3}$$

$$\boxed{V = 300\sqrt{3}}$$



EXEMPLO

CALCULE x DE MODO QUE O VOLUME SEJA 8.



$$A_b = \frac{(B + b)h}{2}$$

$$A_b = \frac{(2x + 3 + 3)x}{2}$$

$$A_b = \frac{2x(x + 3)}{2}$$

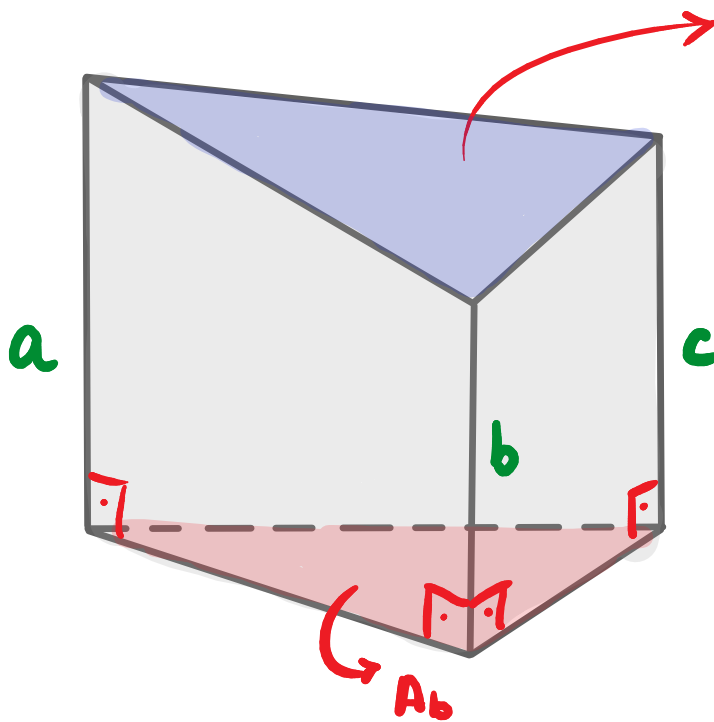
$$A_b = x^2 + 3x$$

$$V = A_b \cdot h \rightarrow \frac{4}{8} = (x^2 + 3x) \cdot \frac{1}{2} \rightarrow \underline{x^2 + 3x - 4 = 0}$$

$$\begin{cases} S = -3 \\ P = -4 \end{cases} \left\{ \begin{array}{l} x_1 = 1 \\ x_2 = -4 \end{array} \right.$$



TRONCO DE PRISMA TRIANGULAR



FACE NÃO
PARALELA À BASE

ALTURA MÉDIA : h_m

$$h_m = \frac{a + b + c}{3}$$

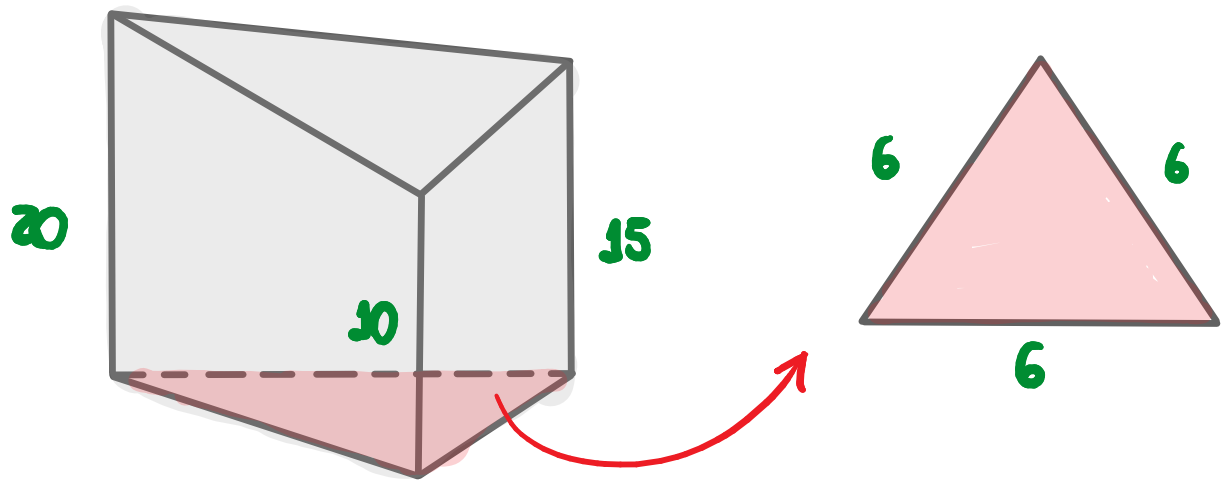
VOLUME : $V_T = A_b \cdot h_m$

OBSERVATION

FACES LATERAIS: TRAPEZÍOS



UNIVERSO NARRADO



$$h_n = \frac{20 + 10 + 15}{3} \rightarrow \underline{h_n = 15}$$

$$A_b = \frac{6^2 \sqrt{3}}{4} \rightarrow \underline{A_b = 9\sqrt{3}}$$

$$V = A_b \cdot h_n$$

$$V = 9\sqrt{3} \cdot 15$$

$$\boxed{V = 135\sqrt{3}}$$

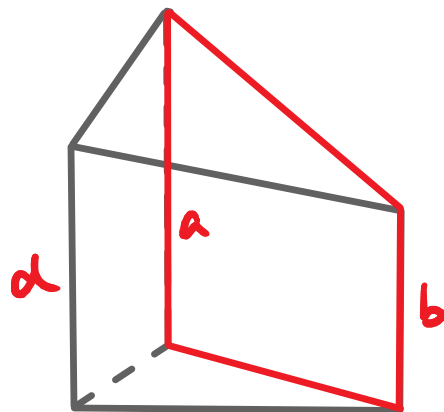
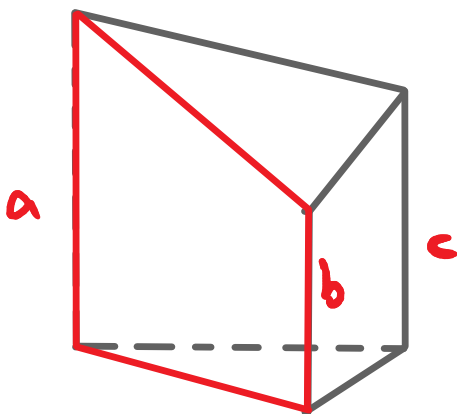
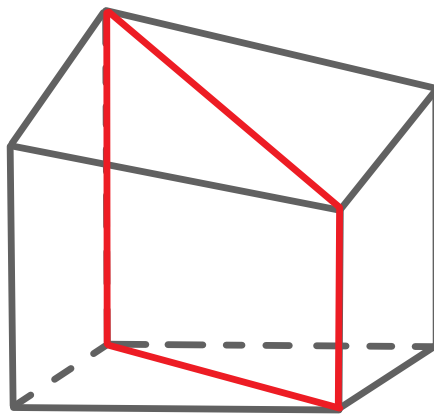


TRONCO DE PRISMA GERAIS

E SE O TRONCO NÃO FOR TRIANGULAR ?

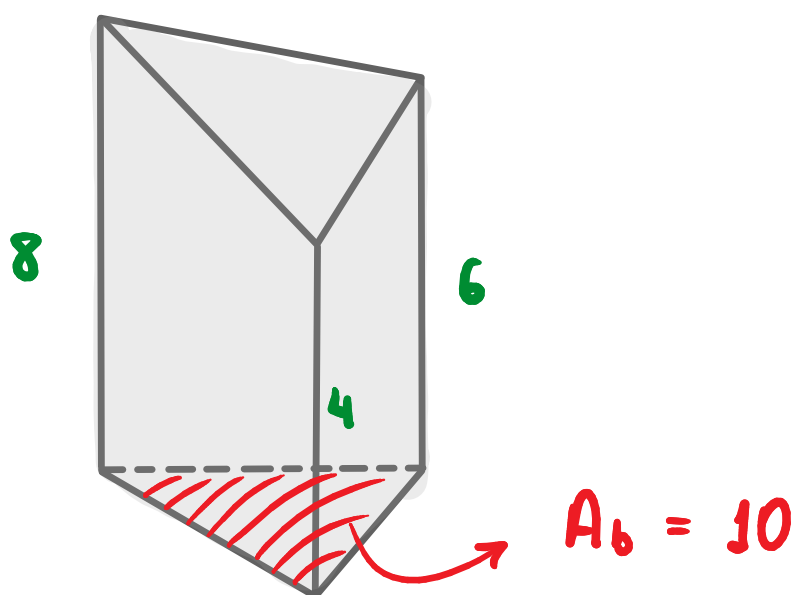


SUBDIVIDIR EM TRONCOS TRIANGULAR



EXEMPLO

CALCULE O VOLUME DO TRONCO DE PRISMA TRIANGULAR ABAIXO.



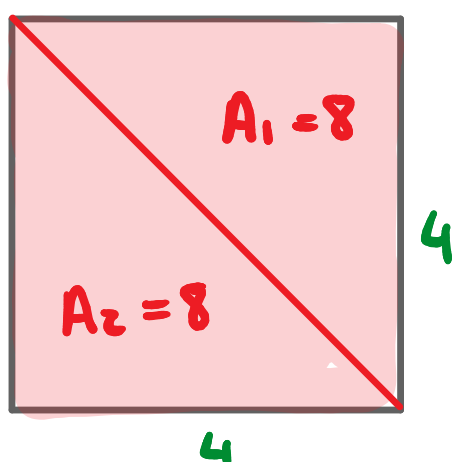
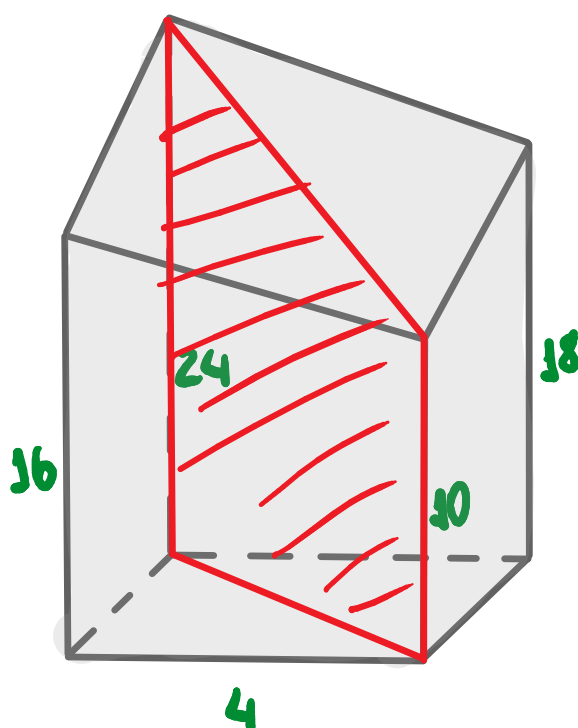
$$h_m = \frac{8 + 6 + 4}{3} \rightarrow \underline{h_m = 6}$$

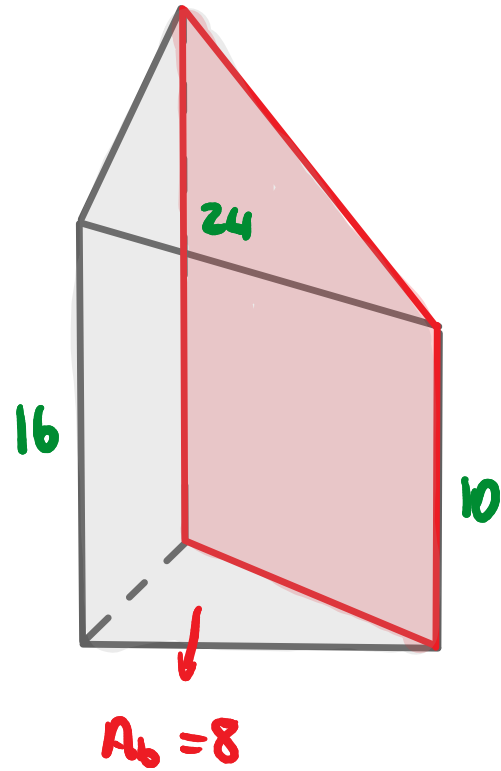
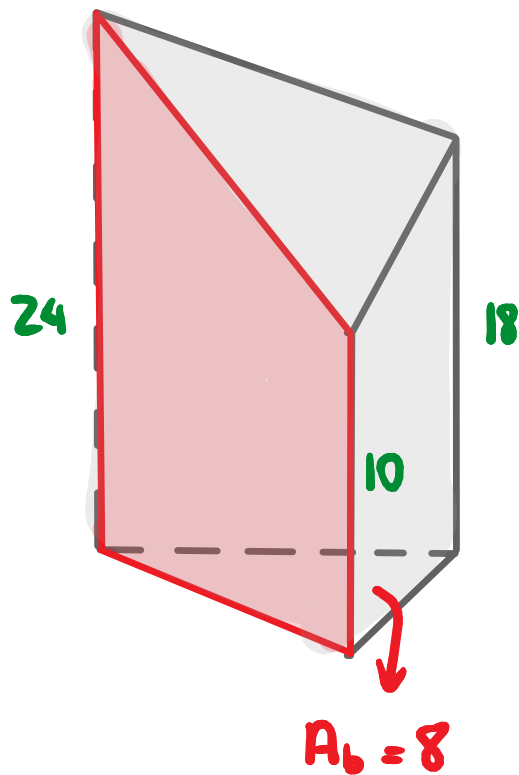
$$V = A_b \cdot h_m \rightarrow V = 10 \cdot 6 \rightarrow \boxed{V = 60}$$



EXEMPLO

CALCULE O VOLUME DO TRONCO DE PRISMA DE BASE QUADRADA ABAIXO.





$$h_{m1} = \frac{24 + 18 + 10}{3}$$

$$h_{m1} = \frac{52}{3}$$

$$V_1 = 8 \cdot \frac{52}{3} = \frac{416}{3}$$

$$h_{m2} = \frac{24 + 16 + 10}{3}$$

$$h_{m2} = \frac{50}{3}$$

$$V_2 = \frac{8 \cdot 50}{3} = \frac{400}{3}$$

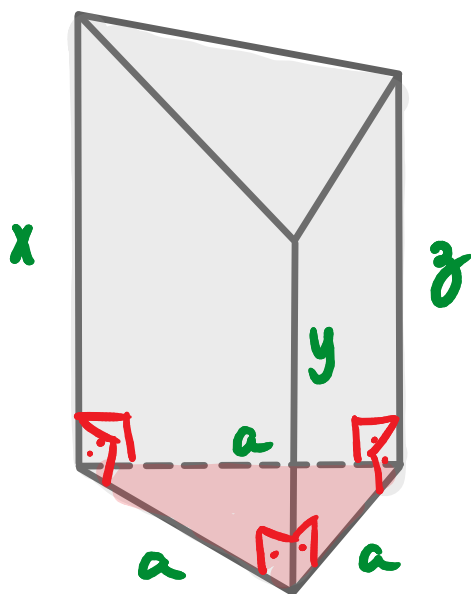
$$V_T = V_1 + V_2 \rightarrow V_T = \frac{416}{3} + \frac{400}{3} = \frac{816}{3}$$

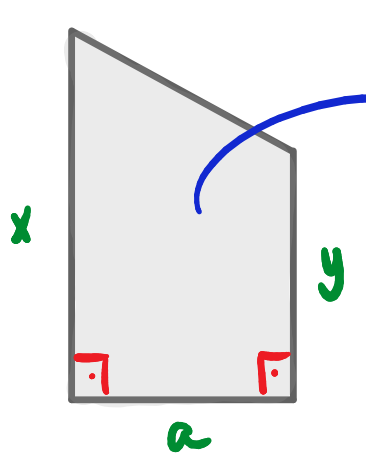
$$V_T = 272$$



EXEMPLO

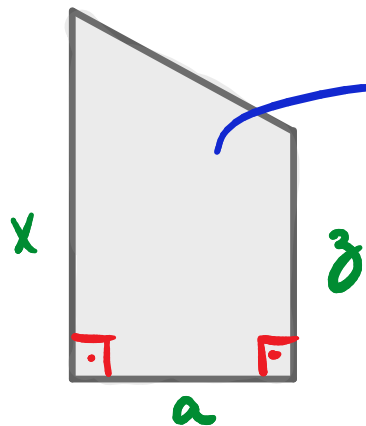
UM PRISMA TRIANGULAR REGULAR CUJA ARESTA DA BASE MEDE a É INTERSECTADO POR UM PLANO OBTENDO UM TRONCO DE ARESTAS LATERAIS IGUAIS A x , y E z . CALCULE A ÁREA LATERAL.





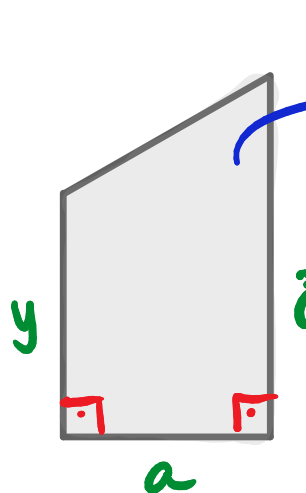
$$A_1 = \frac{(x + y)a}{2}$$

$$A_1 = \frac{ax + ay}{2}$$



$$A_2 = \frac{(x + z)a}{2}$$

$$A_2 = \frac{ax + az}{2}$$



$$A_3 = \frac{(y + z)a}{2}$$

$$A_3 = \frac{ay + az}{2}$$

$$A_L = A_1 + A_2 + A_3$$

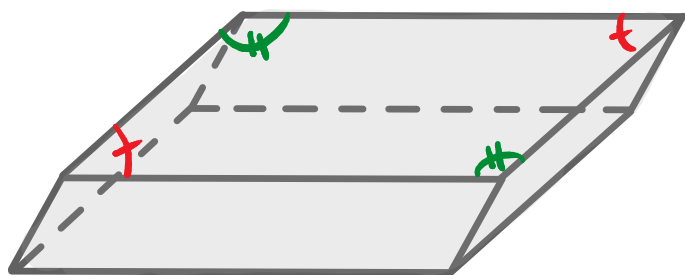
$$A_L = \frac{ax + ay}{2} + \frac{ax + az}{2} + \frac{ay + az}{2}$$

$$A_L = ax + ay + az \rightarrow A_L = a(x + y + z)$$

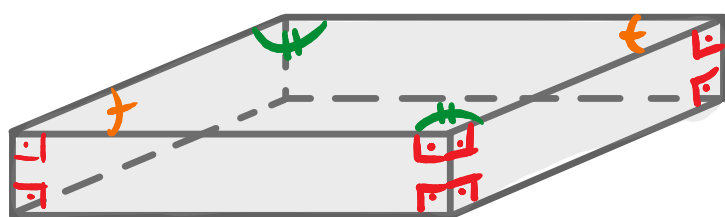
PRISMAS IMPORTANTES

PARALELEPÍPEDOS

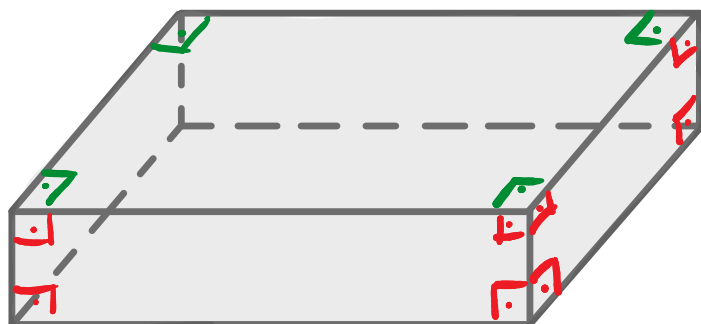
PRISMAS CUJAS FACES SÃO PARALELOGRAMOS.



PARALELEPÍPEDO



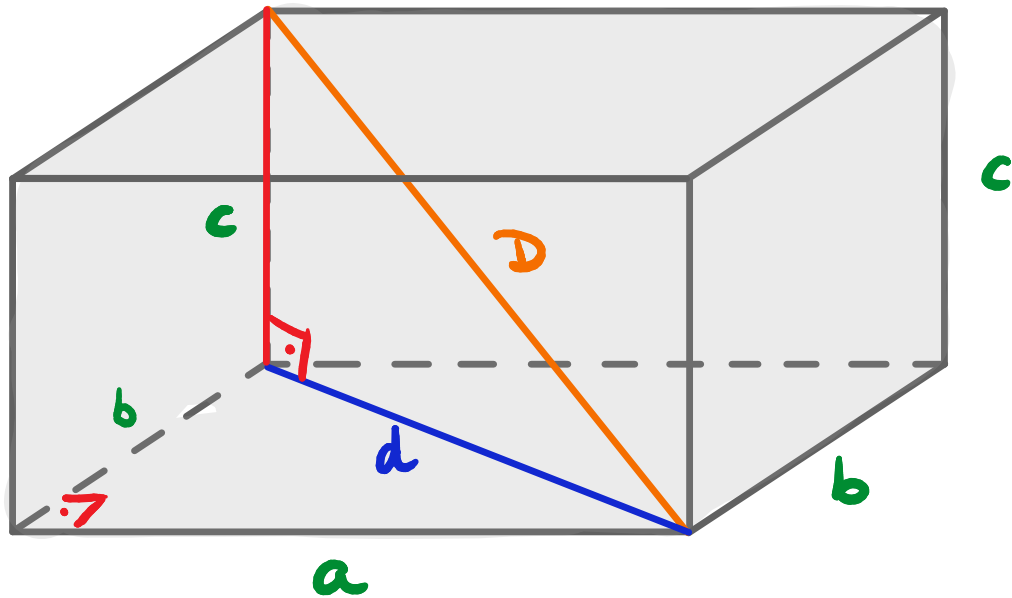
PARALELEPÍPEDO
RETO



PARALELEPÍPEDO
RETO RETO RETÂNGULO



PARALELEPÍPEDO RETO RETÂNGULO



Volume: $V = A_b \cdot h \rightarrow V = abc$

ÁREA
TOTAL : $2ab + 2bc + 2ac \rightarrow$

$$A_T = 2(ab + ac + bc)$$

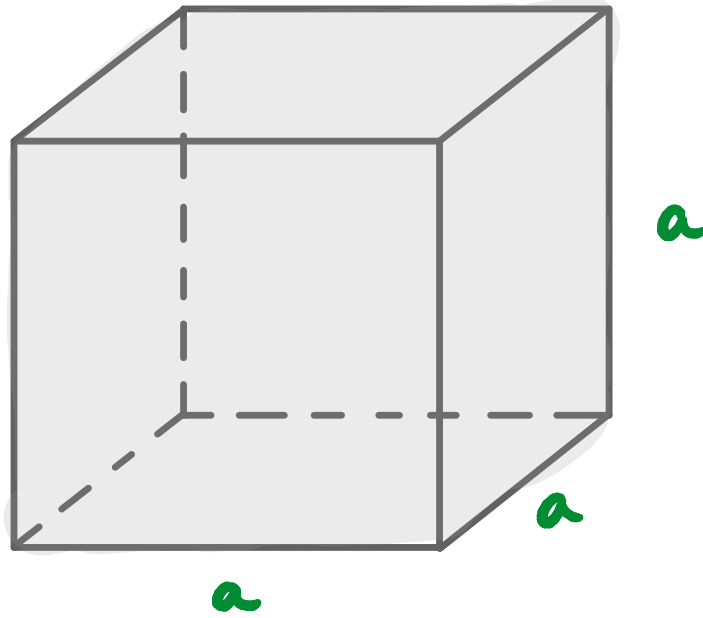
DIAGONAL :

$$d^2 = a^2 + b^2$$
$$D^2 = d^2 + c^2 \rightarrow$$

$$D = \sqrt{a^2 + b^2 + c^2}$$



CUBO



$$V = a^3$$

$$A_T = 6a^2$$

$$D = \sqrt{a^2 + a^2 + a^2}$$

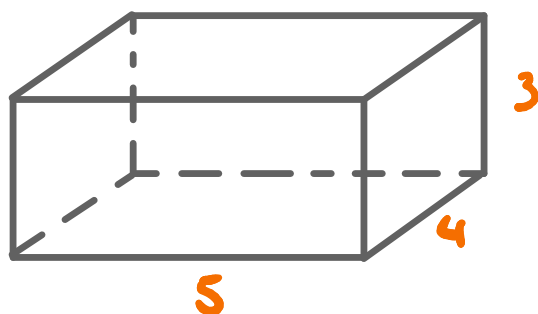
$$D = \sqrt{3a^2}$$

$$D = a\sqrt{3}$$



EXEMPLO

CALCULE A ÁREA TOTAL E O VOLUME DE UM PARALELEPÍPEDO RETO RETÂNGULO CUJAS ARESTAS MEDEM 3, 4 E 5. CALCULE TAMBÉM O COMPRIMENTO DA DIAGONAL.



$$V = 3 \cdot 4 \cdot 5 \rightarrow V = 60$$

$$A_T = 2(3 \cdot 4 + 3 \cdot 5 + 4 \cdot 5) \rightarrow A_T = 94$$

$$D = \sqrt{3^2 + 4^2 + 5^2} \rightarrow D = \sqrt{50}$$

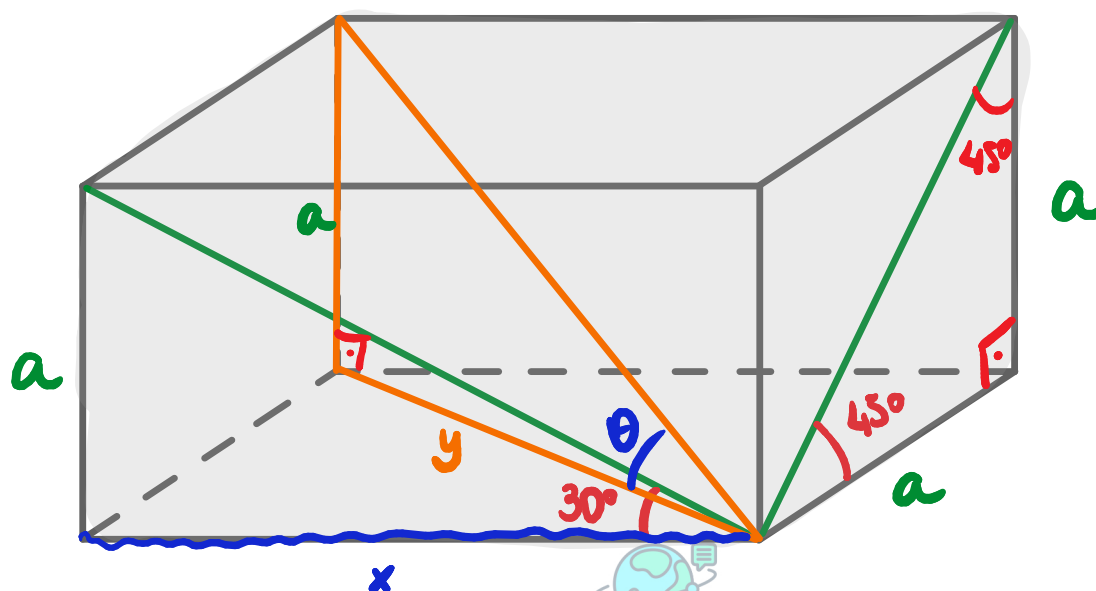
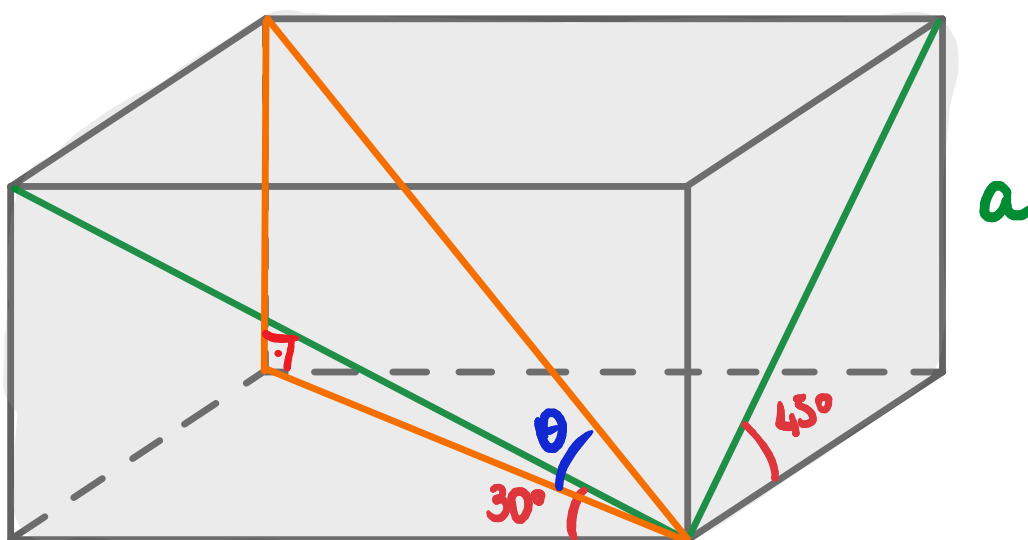
$$D = 5\sqrt{2}$$

$$\begin{array}{r|l} 50 & 2 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$



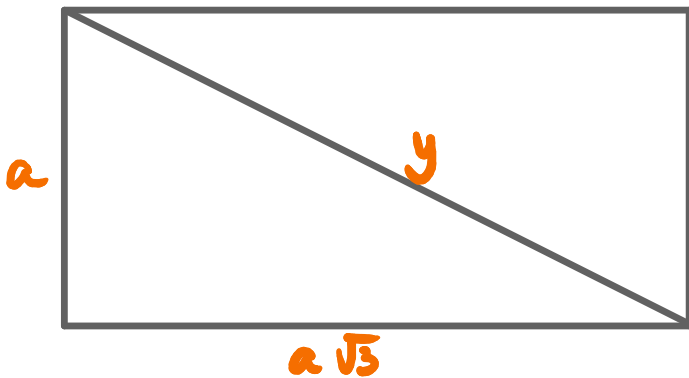
EXEMPLO

CALCULE A TANGENTE DO ÂNGULO QUE A
DIAGONAL DO PARALELEPÍPEDO RETO RETÂNGULO
FORMA COM A BASE.



$$\operatorname{tg} 30^\circ = \frac{a}{x} \rightarrow \frac{\sqrt{3}}{3} = \frac{a}{x}$$

$$x = \frac{3a}{\sqrt{3}} \rightarrow \boxed{x = a\sqrt{3}}$$



$$y^2 = a^2 + (a\sqrt{3})^2$$

$$y^2 = 4a^2$$

$$\boxed{y = 2a}$$

$$\operatorname{tg} \theta = \frac{a}{2a} \rightarrow \boxed{\operatorname{tg} \theta = \frac{1}{2}}$$

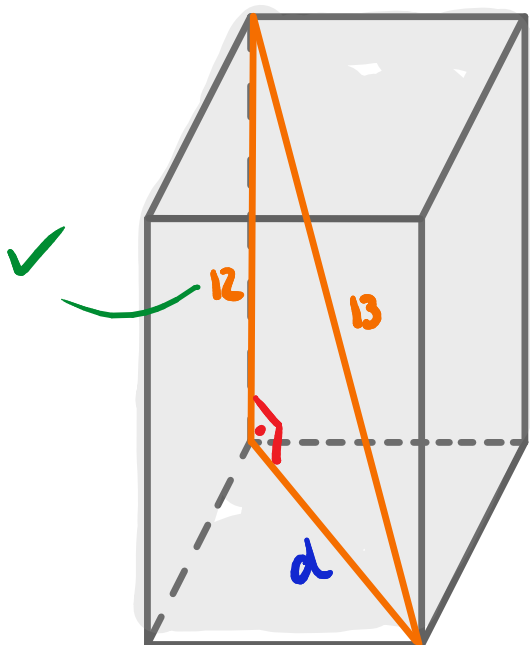


EXEMPLO

SEJA UM PARALELEPÍPEDO RETO RETÂNGULO:

- PERÍMETRO DA BASE : 14
- DIAGONAL : 13
- ALTURA : 12

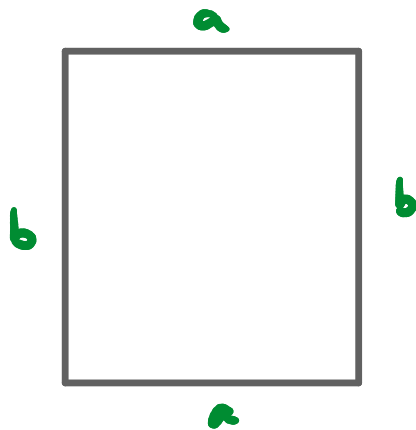
CALCULE O VOLUME.



$$\begin{aligned}13^2 &= 12^2 + d^2 \\169 &= 144 + d^2 \\25 &= d^2 \\d &= 5\end{aligned}$$

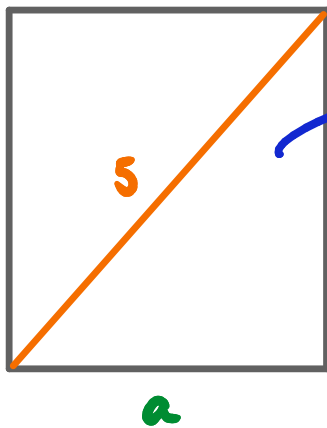


BASE



$$2a + 2b = 14$$

$$\begin{aligned} &\underline{a + b = 7} \\ &\rightarrow b = 7 - a \end{aligned}$$



$$\begin{aligned} &\rightarrow a^2 + (7-a)^2 = 5^2 \\ &a^2 + 49 - 14a + a^2 = 25 \\ &2a^2 - 14a + 24 = 0 \end{aligned}$$

$$a^2 - 7a + 12 = 0$$

$$\Delta = (-7)^2 - 4 \cdot 1 \cdot 12 \rightarrow \Delta = 1$$

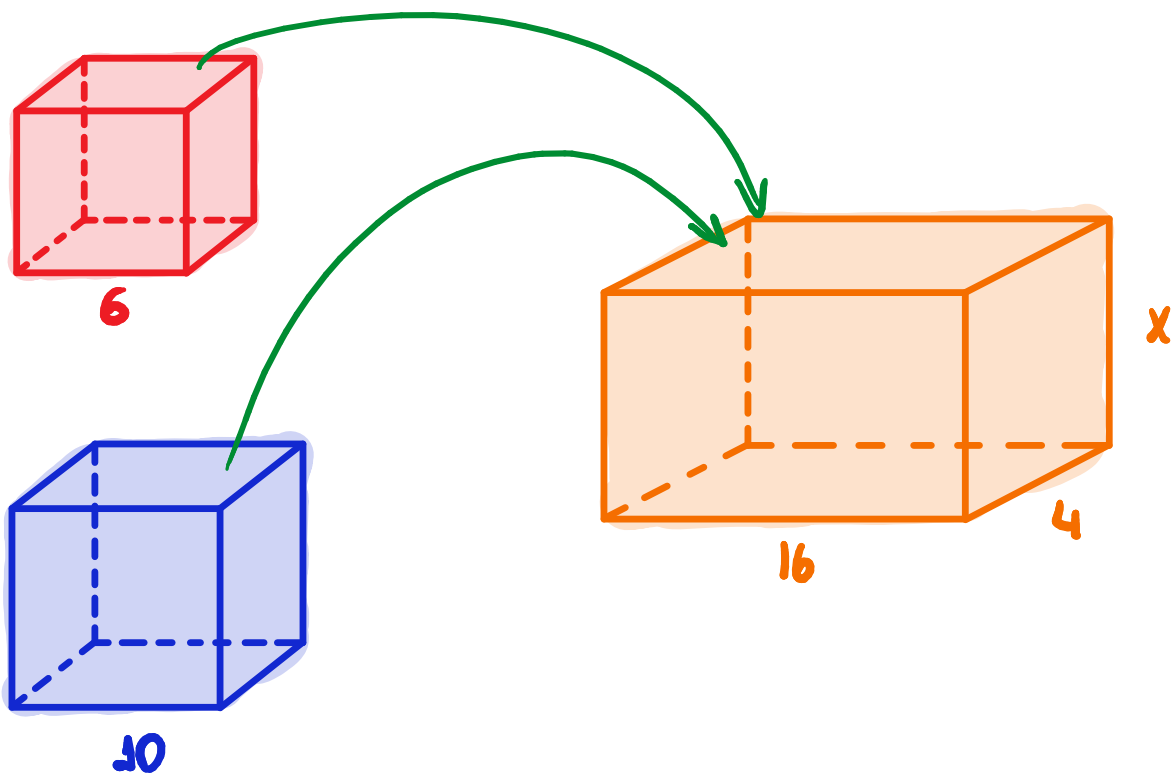
$$a = \frac{7 \pm \sqrt{1}}{2} \rightarrow \begin{array}{ll} a = 4 & ; \quad a = 3 \\ b = 3 & \quad b = 4 \end{array}$$

$$V = a \cdot b \cdot c \rightarrow V = 3 \cdot 4 \cdot 12 \rightarrow V = 144$$



EXEMPLO

DOIS CUBOS COM ARESTAS 6 E 10 SÃO FUNDIDOS E DEPOIS MOLDADOS COM UM PARALELEPÍPEDO RETO RETÂNGULO DE ARESTAS 4, 16 E x . CALCULE x .



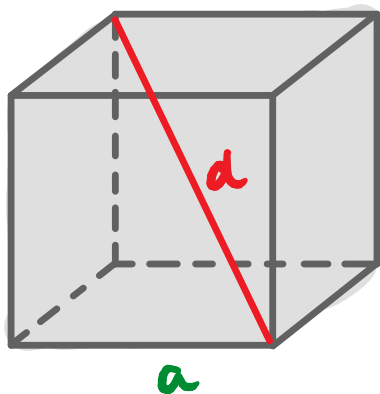
$$16 \cdot 4 \cdot x = 6^3 + 10^3 \rightarrow 64x = 1216$$

$$x = \frac{1216}{64} \rightarrow \boxed{x = 19}$$



EXEMPLO

CALCULE A ÁREA E O VOLUME DE UM CUBO DE DIAGONAL d .



$$d = a\sqrt{3}$$

$$a = \frac{d}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$a = \frac{d\sqrt{3}}{3}$$

$$V = a^3 \rightarrow V = \left(\frac{d\sqrt{3}}{3}\right)^3 \rightarrow V = \frac{d^3 \cdot \cancel{\sqrt{3}}}{3 \cdot 3 \cdot \cancel{\sqrt{3}}}$$

$$V = \frac{d^3 \sqrt{3}}{9}$$

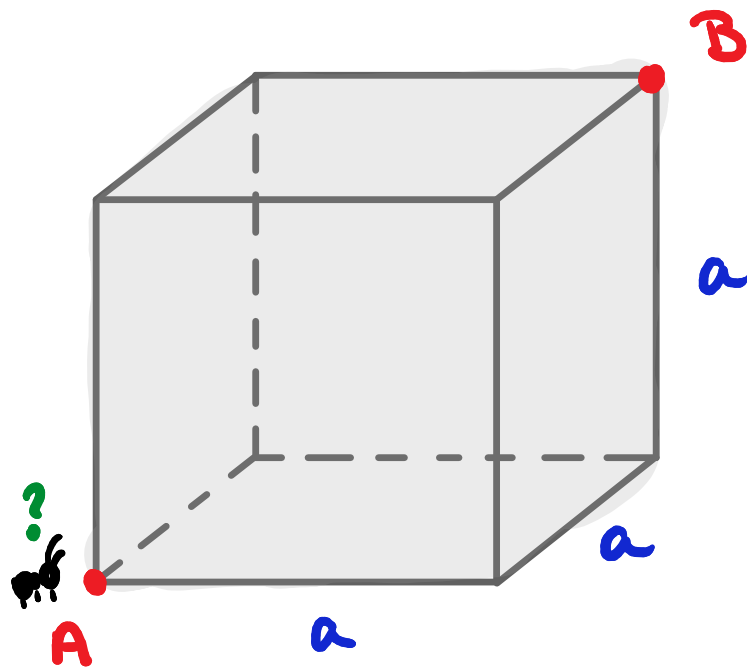
$$A = 6 \cdot a^2 \rightarrow A = 6 \cdot \left(\frac{d\sqrt{3}}{3}\right)^2 \rightarrow A = \frac{\overset{2}{\cancel{6}} \cdot d^2 \cdot \cancel{\sqrt{3}}}{\cancel{\sqrt{3}} \cdot \cancel{\sqrt{3}}}$$

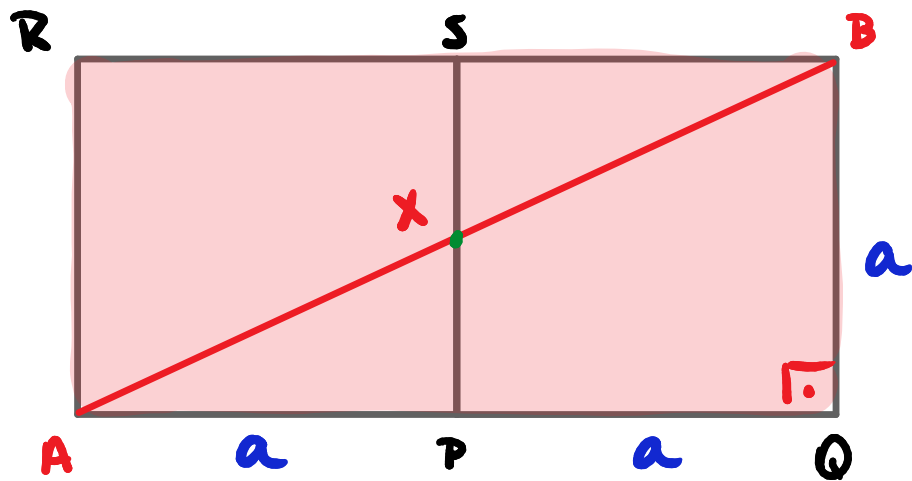
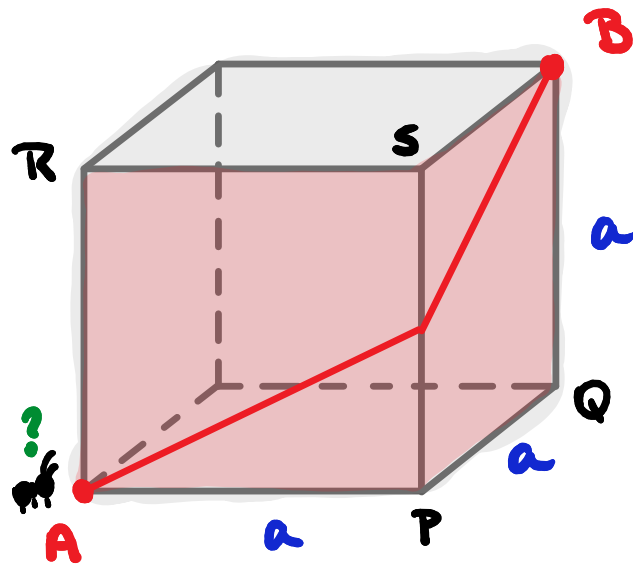
$$A = 2d^2$$



EXEMPLO

QUAL A MENOR DISTÂNCIA A FORMIGA DEVE ANDAR PARA IR DO VÉRTICE "A" AO VÉRTICE B DO CUBO ANDANDO PELA SUPERFÍCIE.





$$x^2 = a^2 + (2a)^2$$

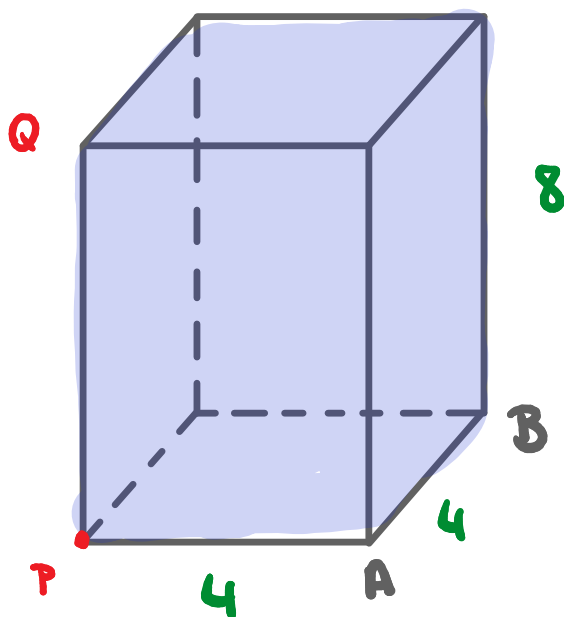
$$x^2 = 5a^2$$

$$x = a\sqrt{5}$$



EXEMPLO

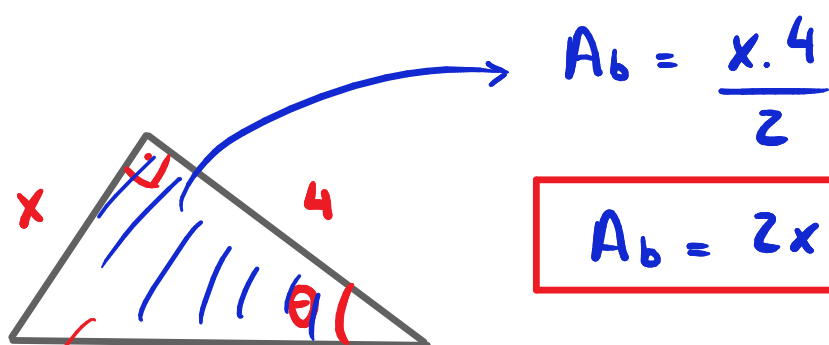
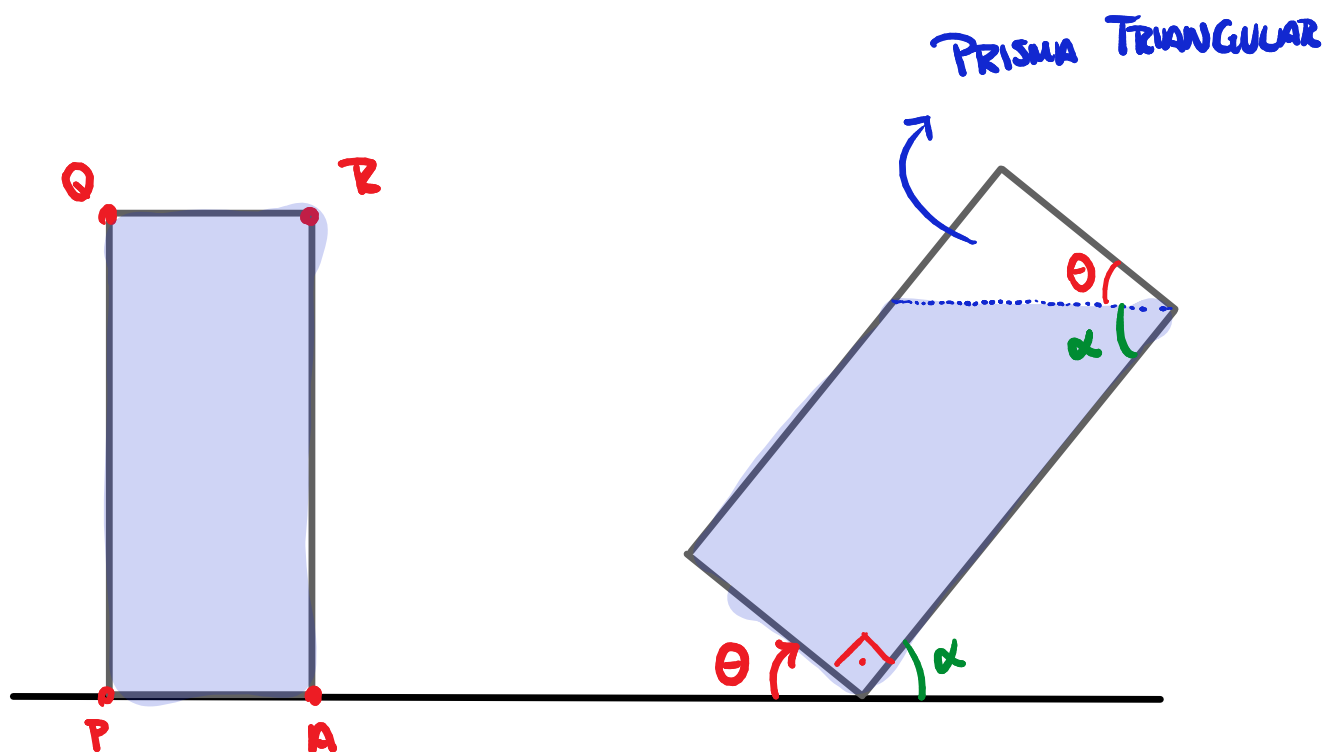
AO INCLINAR SOBRE A ARESTA $\overline{AB}$, DERRAMA-SE $\frac{1}{4}$ DO LÍQUIDO CONTIDO. CALCULE O ÂNGULO DE INCLINAÇÃO.



$$V_{\text{TOTAL}} = 4 \cdot 4 \cdot 8 \rightarrow \underline{V_{\text{TOTAL}} = 128}$$

$$V_{\text{DETRAMADO}} = \frac{1}{4} \cdot 128 \rightarrow \underline{V_{\text{DETRAMADO}} = 32}$$





$$A_b = 2x$$

$$h_{\text{PRISMA}} = 4$$

$$V_{\text{PRISMA}} = 2x \cdot 4 = 8x$$

$$8x = 32$$

$$\underline{x = 4}$$

$$\text{tg} \theta = \frac{x}{4} = \frac{4}{4} = 1 \rightarrow \theta = 45^\circ$$

