

# **GEOMETRIA DE POSIÇÃO**

**GEOMETRIA DE POSIÇÃO É A ÁREA DA MATEMÁTICA QUE ESTUDA AS POSIÇÕES RELATIVAS ENTRE FORMAS GEOMÉTRICAS NO ESPAÇO.**



# **CONCEITOS PRIMITIVOS**

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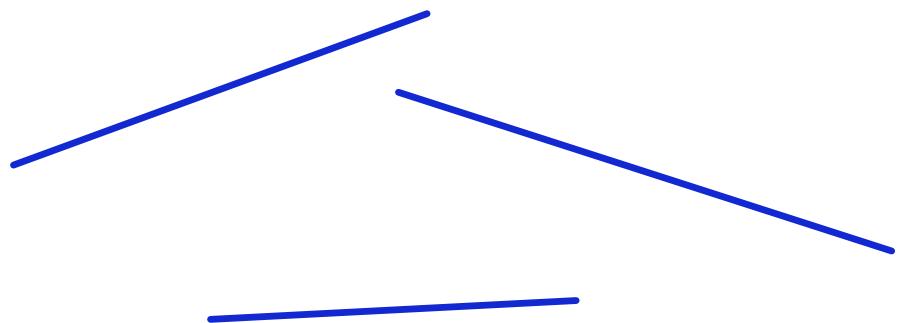
**PONTO**

. A

. C

. B

**RETA**



**PLANO**



# POSTULADOS - EXISTÊNCIA

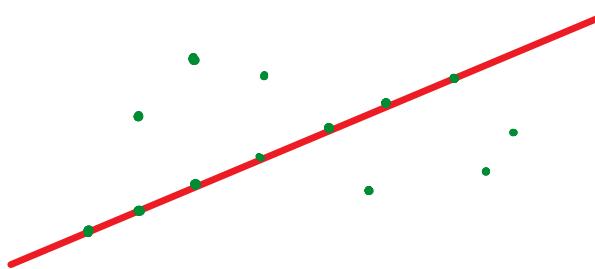
(I)

EXISTEM INFINITOS PONTOS.



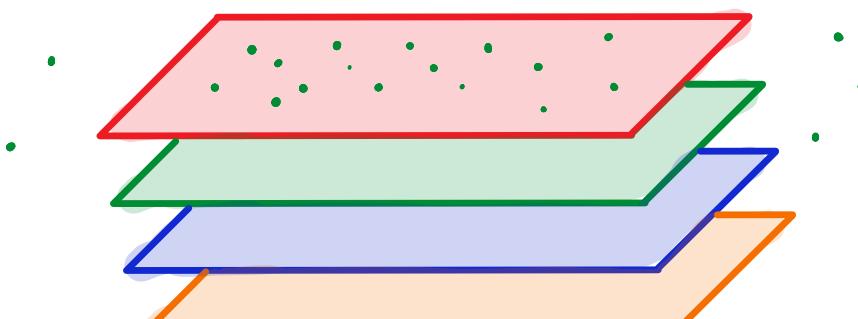
(II)

EXISTEM INFINITAS RETAS. EM CADA UMA, ASSIM COMO FORA DELA, EXISTEM INFINITOS PONTOS.



(III)

EXISTEM INFINITOS PLANOS. EM CADA UM, ASSIM COMO FORA DELE, EXISTEM INFINITOS PONTOS.

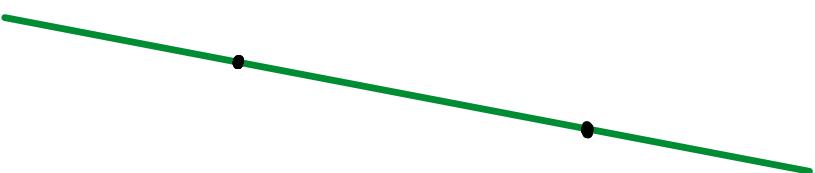


# POSTULADOS - DETERMINAÇÃO

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I

DOIS PONTOS DISTINTOS  
DETERMINAM UMA ÚNICA RETA.



II

TRÊS PONTOS NÃO COLINEARES  
DETERMINAM UM ÚNICO PLANO.



# **POSTULADOS - INCLUSÃO**

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(I)

**SE UMA RETA CONTÉM DOIS PONTOS DISTINTOS DE UM PLANO, ENTÃO A RETA ESTÁ CONTIDA NO PLANO.**



# POSTULADOS      SEPARAÇÃO

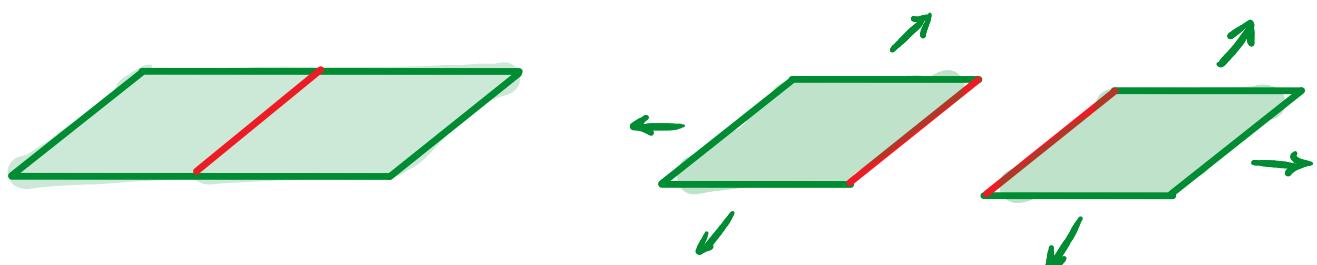
(I)

UM PONTO SEPARA UMA RETA EM DUAS SEMIRRETAS.



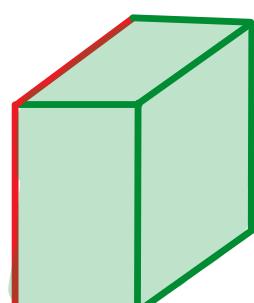
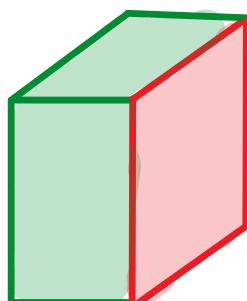
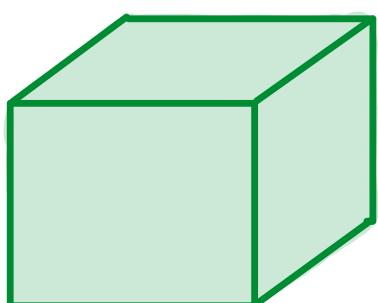
(II)

UMA RETA SEPARA UM PLANO EM DOIS SEMIPLANOS.



(III)

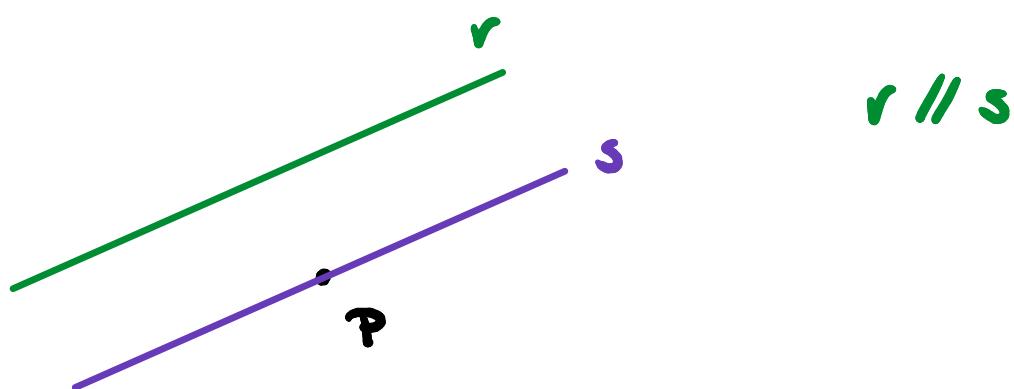
UM PLANO SEPARA UM ESPAÇO EM DOIS SEMIESPAÇOS.



# POSTULADO DAS PARALELAS

(I)

POR UM PONTO PASSA UMA ÚNICA RETA PARALELA A UMA RETA DADA.



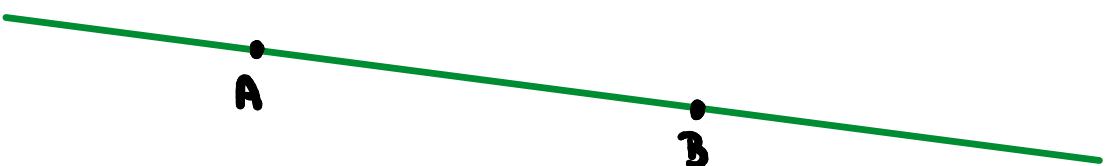
"QUINTO POSTULADO DE EUCLIDES"



# DETERMINAÇÃO DE RETA

(I)

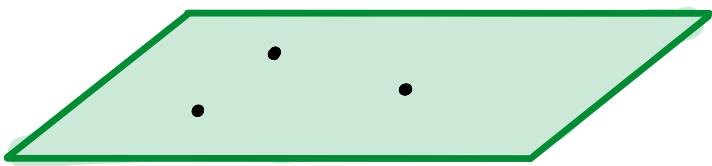
DOIS PONTOS.



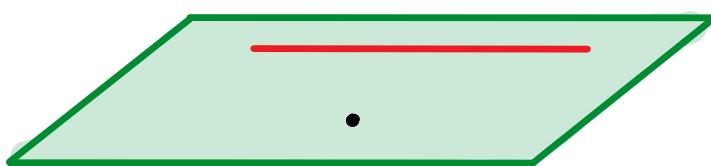
UNIVERSO NARRADO

# DETERMINAÇÃO DE PLANO

**I** TRÊS PONTOS NÃO COLINEARES.



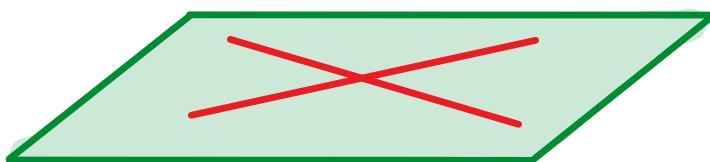
**II** RETA E UM PONTO FORA DELA.



**III** DUAS RETAS PARALELAS NÃO COINCIDENTES.



**IV** DUAS RETAS CONCORRENTES.



# POSIÇÃO RELATIVA - RETAS

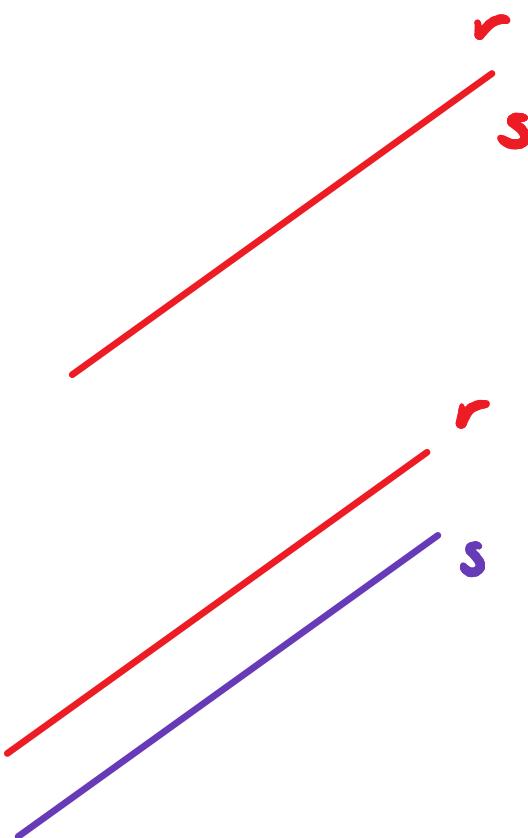
(I)

**RETAS PARALELAS**

$$r \parallel s$$

**PARALELAS  
COINCIDENTES**

**PARALELAS  
DISTINTAS**

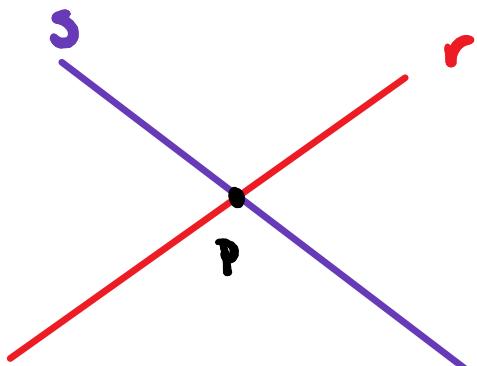


**RETAS PARALELAS SÃO COPLANARES**



II

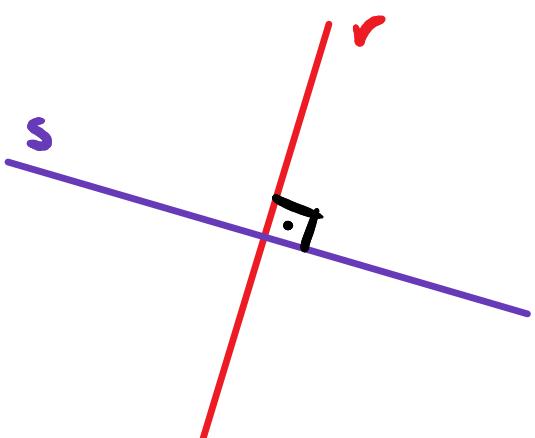
## RETAS CONCORRENTES



$$r \cap s = P$$

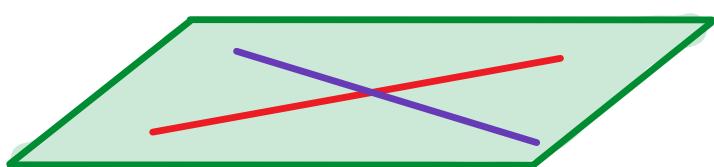
CASO PARTICULAR:

### RETAS PERPENDICULARES



$$r \perp s$$

RETAS CONCORRENTES SÃO COPLANARES

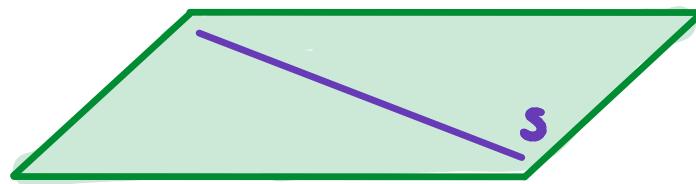
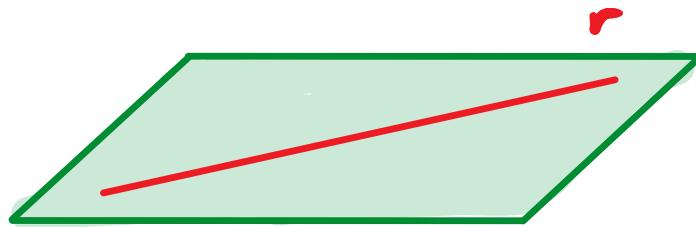


Universo Narrado

III

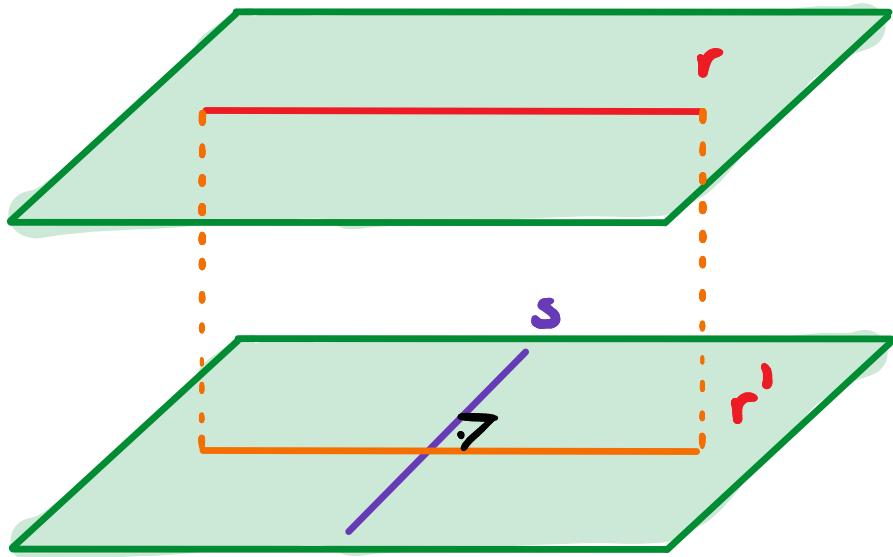
## RETAS REVERSAS

SÃO RETAS NÃO COPLANARES.



CASO PARTICULAR: RETAS ORTOGONIAIS

A PROJEÇÃO DE UMA É PERPENDICULAR À OUTRA.

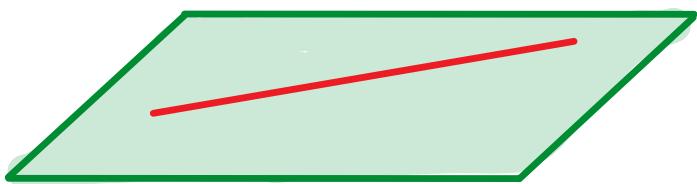


# POSIÇÃO RELATIVA

## RETA E PLANO

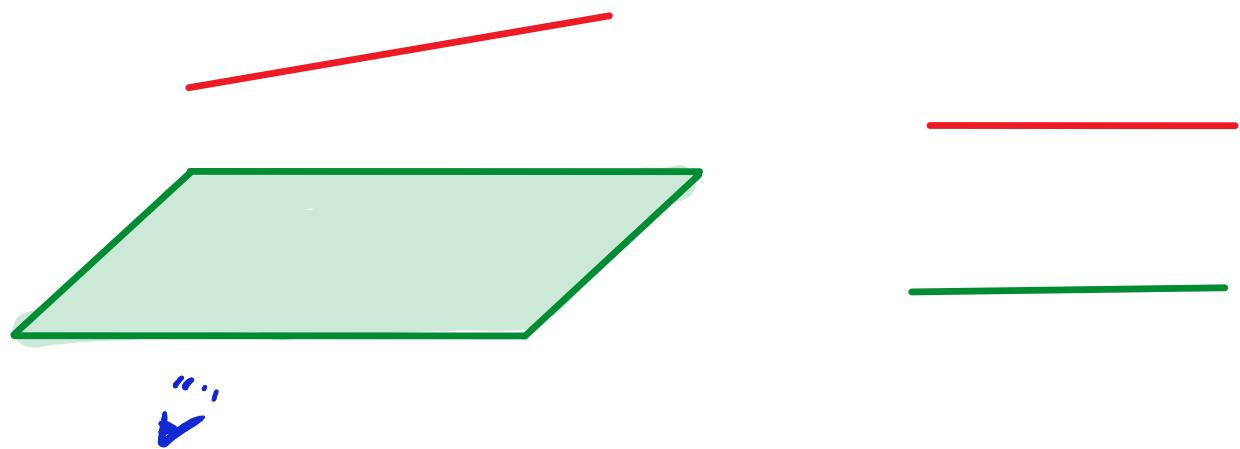
(I)

RETA CONTIDA NO PLANO



(II)

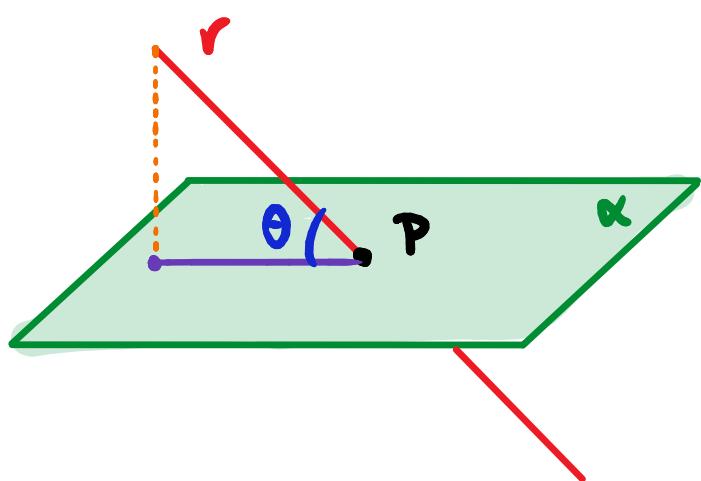
RETA PARALELA AO PLANO



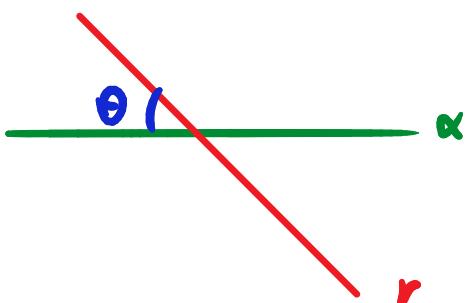
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III

## RETA CONCORRENTE AO PLANO

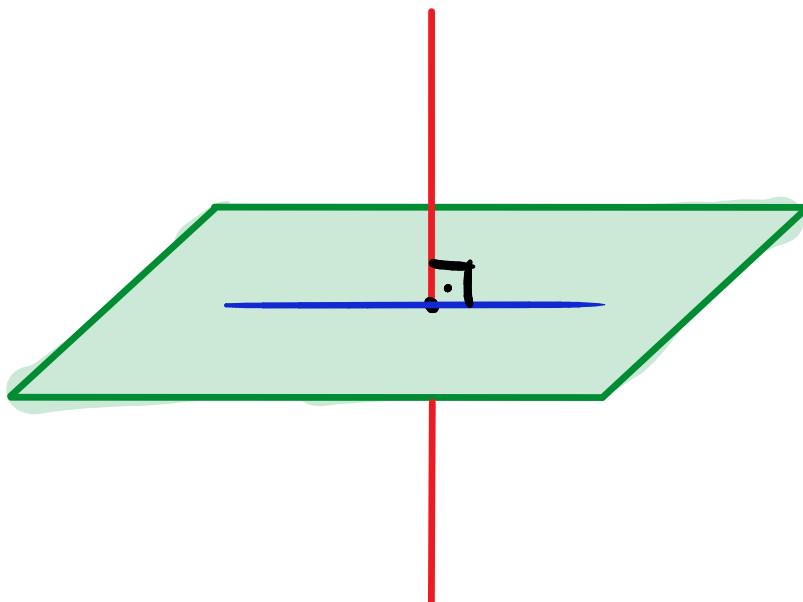


$$r \cap \alpha = P$$



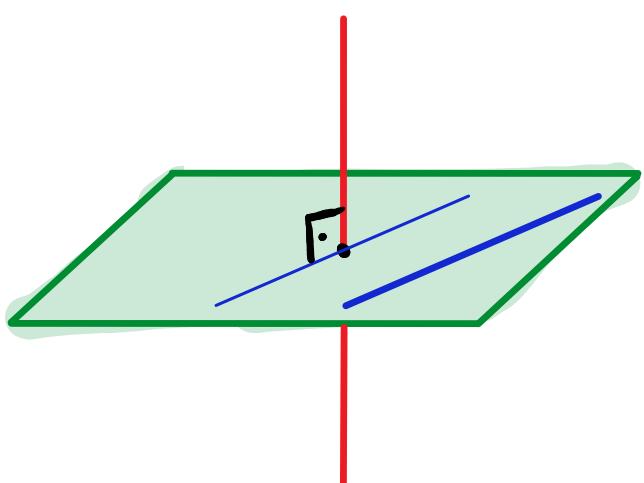
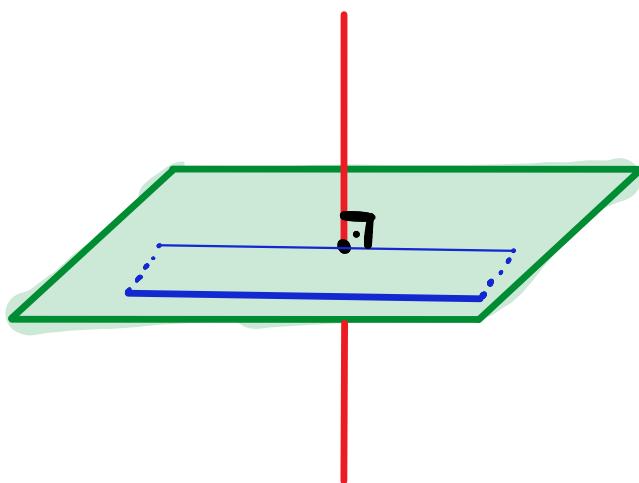
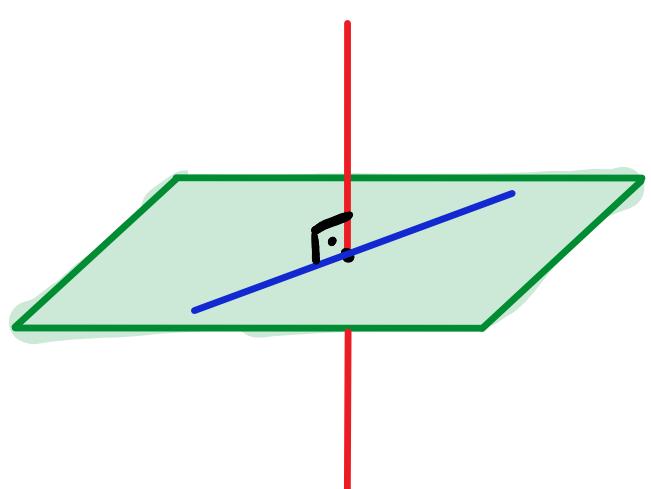
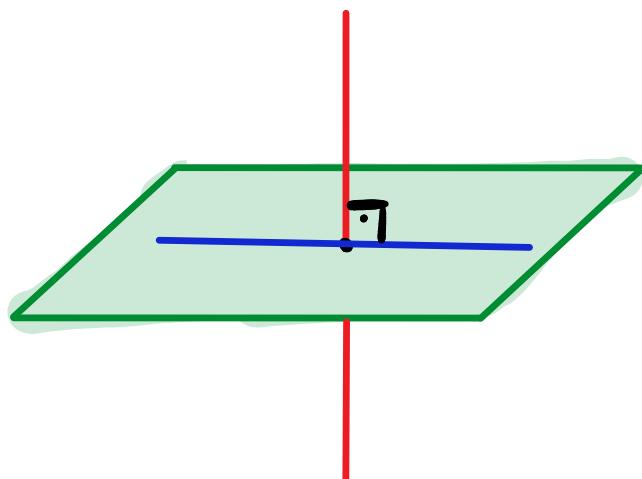
CASO PARTICULAR:

## RETA PERPENDICULAR AO PLANO



## TEOREMA

SE UMA RETA É PERPENDICULAR A UM PLANO, ELA É PERPENDICULAR OU ORTOGONAL A QUALQUER RETA CONTIDA NESSE PLANO



# POSIÇÃO RELATIVA - PLANOS

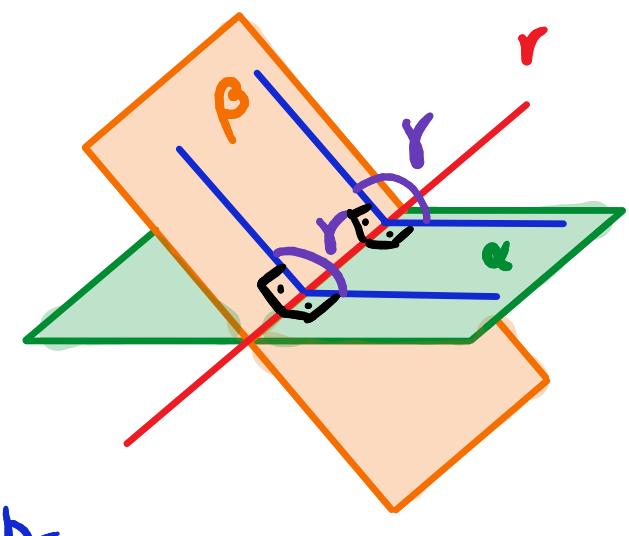
## I PLANOS PARALELOS



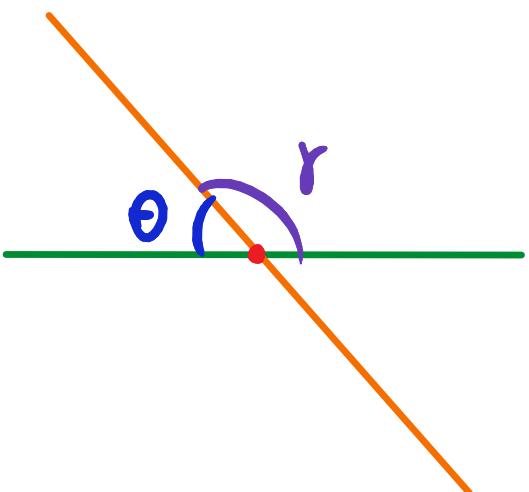
## II PLANOS COINCIDENTES



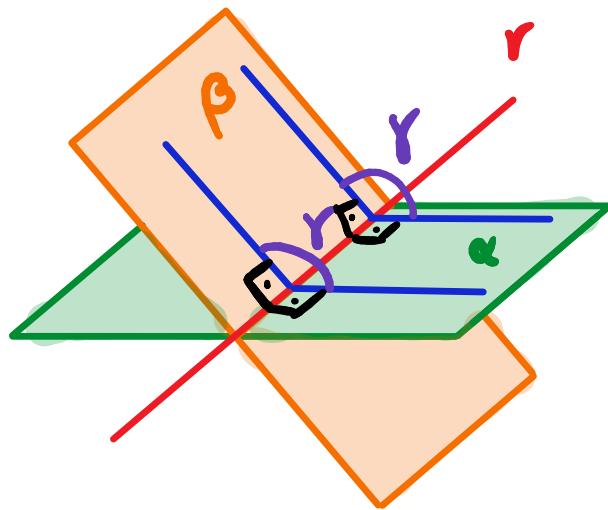
## III PLANOS SECANTES



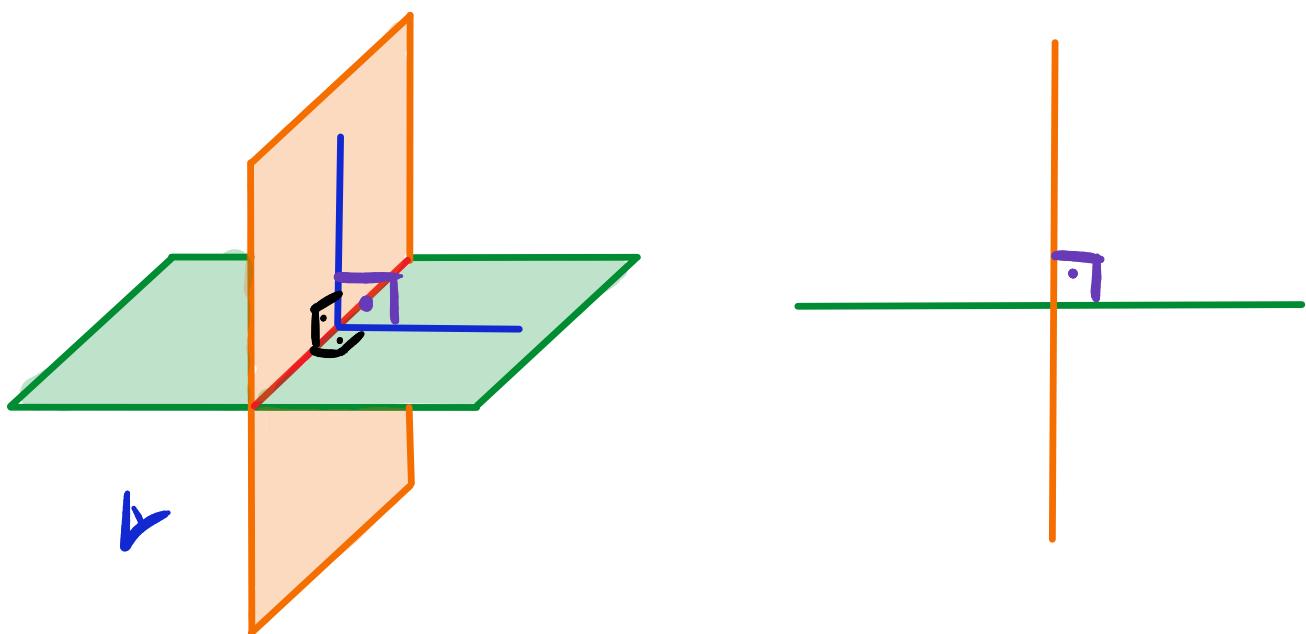
$$\alpha \cap \beta = r$$



# ÂNGULO ENTRE PLANOS



## CASO PARTICULAR: PLANOS PERPENDICULARES

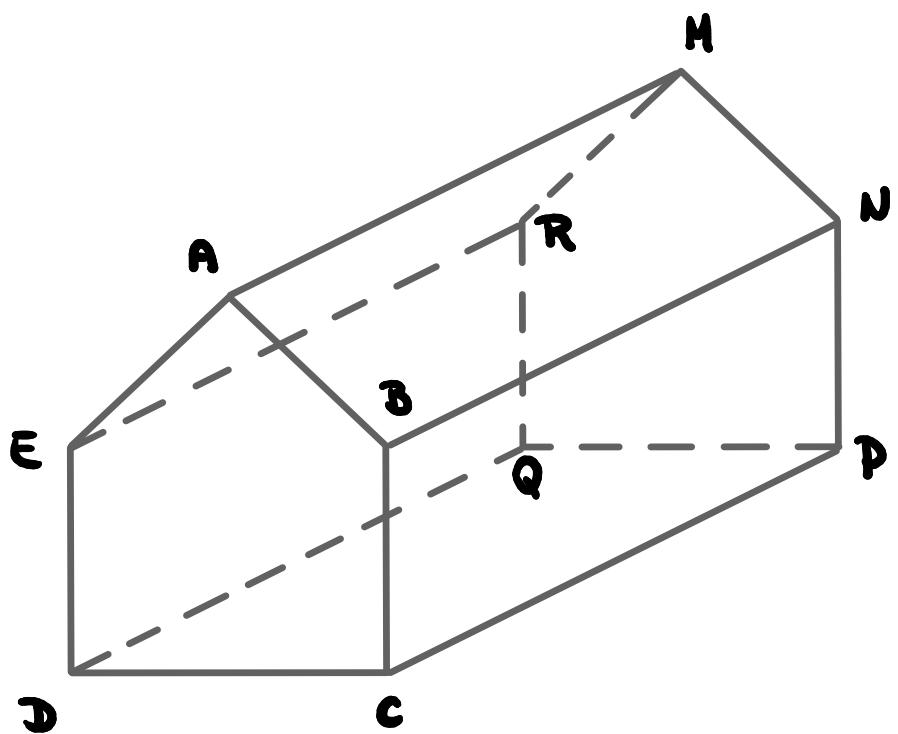


## **EXEMPLO**

CONSIDERANDO A FIGURA, DETERMINE A POSIÇÃO RELATIVA ENTRE AS RETAS:

- a. AM E ER **PARALELAS**
- b. BN E DQ **PARALELAS**
- c. AE E MN **REVERSAS**
- d. AM E CD **REVERSAS / ORTOGONALIS**
- e. AE E ED **Concorrentes**
- f. MR E PQ **Concorrentes**



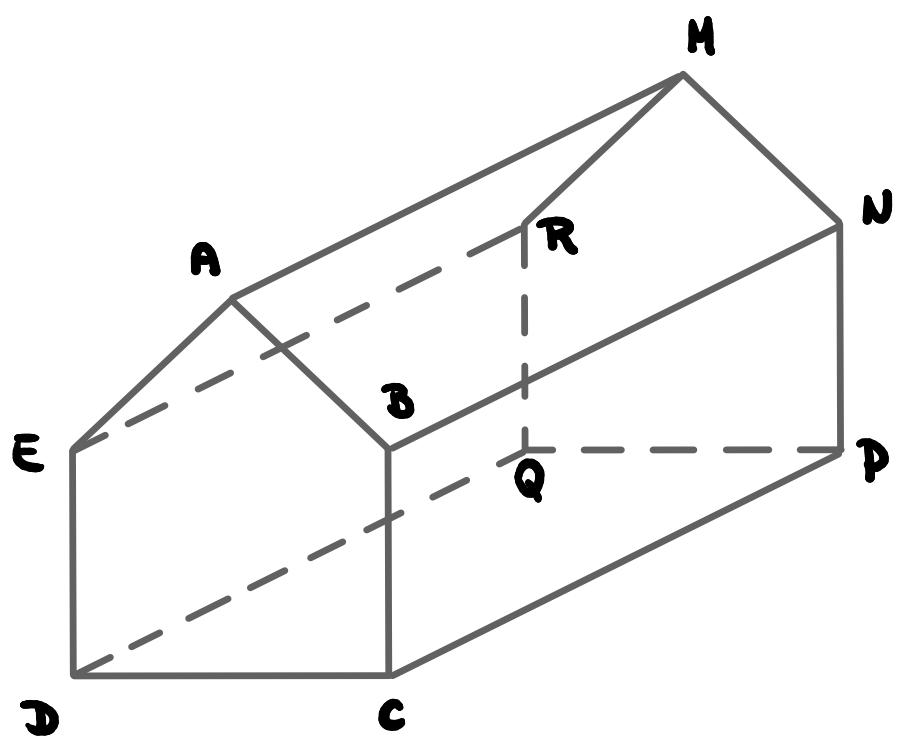


## EXEMPLO

CONSIDERANDO A FIGURA, DETERMINE A POSIÇÃO RELATIVA ENTRE OS PLANOS:

- a. BCD E MNR **PARALELOS**
- b. ABC E BDE **COINCIDENTES**
- c. AMR E DEQ **SECANTES**
- d. AMR E PQD **SECANTES**
- e. PQD E EQD **SECANTES / PERPENDICULARES**





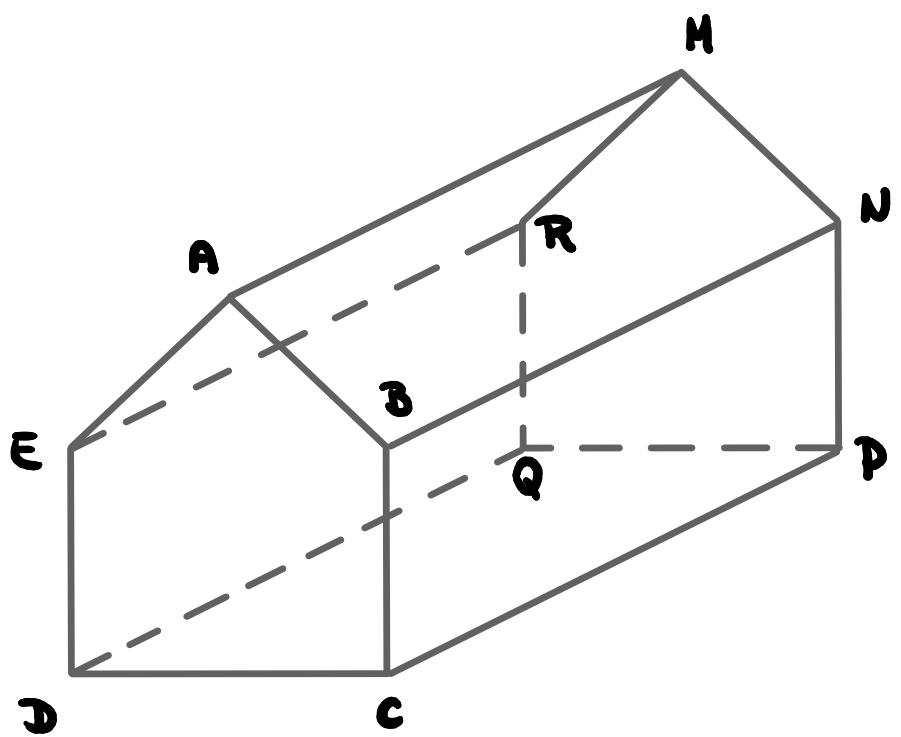
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## EXEMPLO

CONSIDERANDO A FIGURA, DETERMINE A POSICAO RELATIVA ENTRE AS RETAS E PLANOS ABAIXO:

- a. PQD E ER **PARALELOS**
- b. EDQ E AM **PARALELOS**
- c. MNR E AE **PARALELOS**
- d. AMR E ED **SECANTES**
- e. CDP E RQ **SECANTES / PERPENDICULARES**

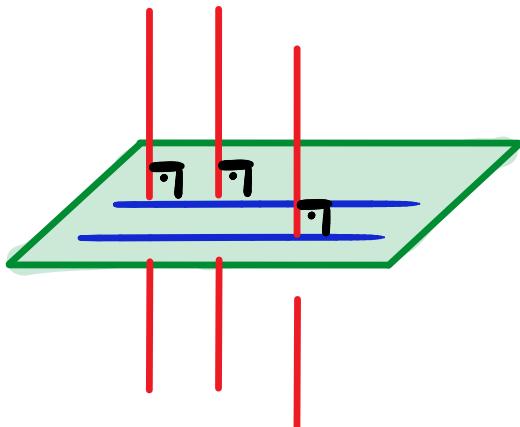




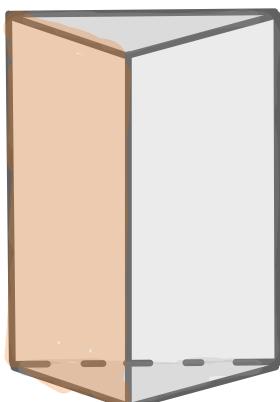
## EXEMPLO

JULGUE AS AFIRMATIVAS A SEGUIR:

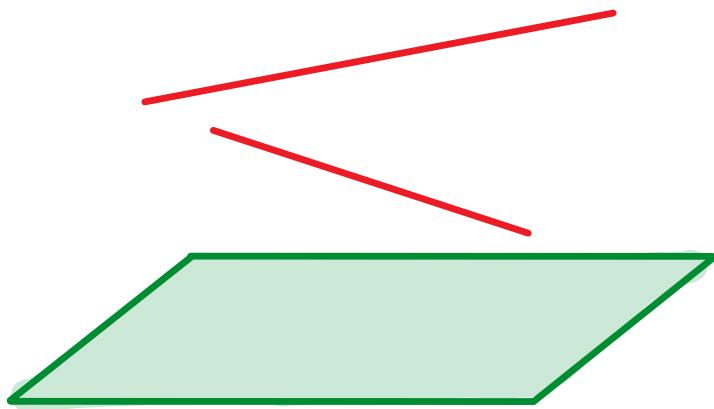
( ✓ ) DUAS RETAS PERPENDICULARES A UM PLANO DADO SÃO PARALELAS ENTRE SI.



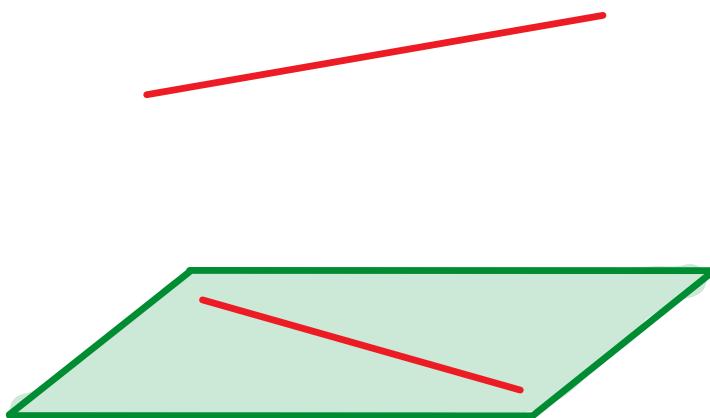
( F ) SE UM PLANO INTERSECTA OUTROS DOIS PLANOS, ENTÃO ESSES DOIS PLANOS SÃO PARALELOS.



**( F ) DUAS RETAS PARALELAS A UM PLANO  
DADO SÃO PARALELAS ENTRE SI.**



**( F ) SE UMA RETA É PARALELA A UM PLANO  
ENTÃO ELA É PARALELA A QUALQUER RETA  
DESSE PLANO.**



## EXEMPLO

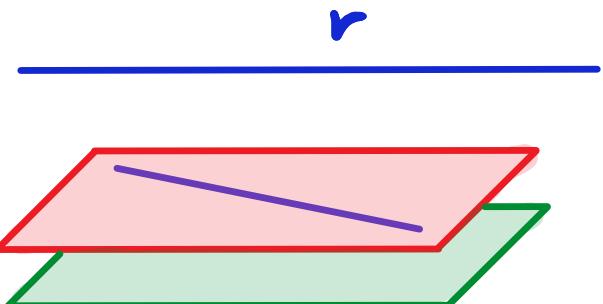
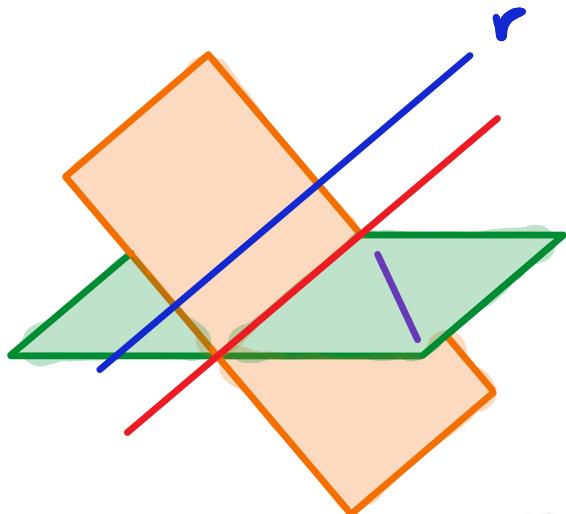
SE UMA RETA  $r$  É PARALELA A DOIS PLANOS NÃO COINCIDENTES,  $\alpha$  e  $\beta$ , JULGUE AS ALTERNATIVAS:

(F)  $\alpha$  e  $\beta$  SÃO PERPENDICULARES.

(F)  $\alpha$  e  $\beta$  SÃO PARALELOS.

(F) QUALQUER RETA CONTIDA EM  $\beta$  É PARALELA A  $r$ .

(F) EXISTEM RETAS CONTIDAS EM  $\beta$  QUE SÃO PERPENDICULARES A  $r$ .



## EXEMPLO

TRÊS RETAS DISTINTAS  $r$ ,  $s$  E  $t$  SÃO TAIS QUE  $s$  E  $t$  SÃO PERPENDICULARES A  $r$ .

Sobre a posição relativa das retas  $s$  e  $t$ , julgue as afirmativas:

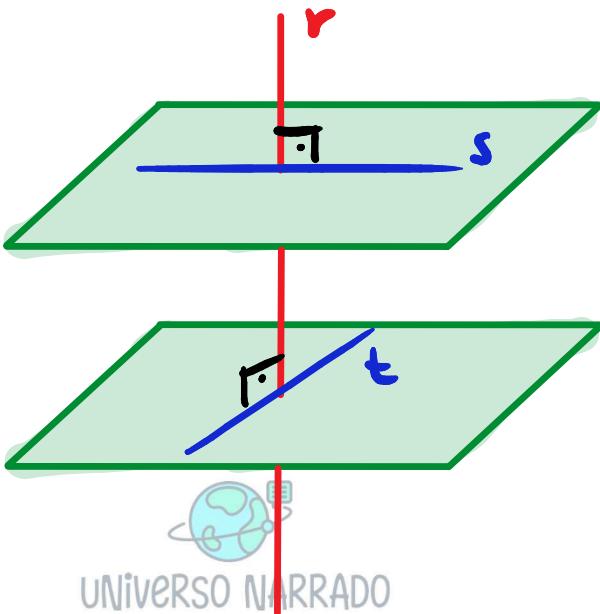
(F) ELAS SÃO PARALELAS.

(F) ELAS SÃO PARALELAS OU CONCORRENTES.

(F) ELAS SÃO PARALELAS OU REVERSAS.

(F) ELAS SÃO CONCORRENTES OU REVERSAS.

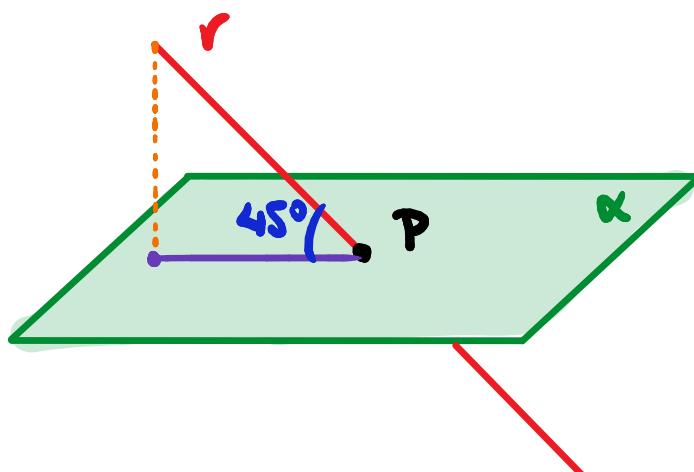
(✓) ELAS SÃO PARALELAS, CONCORRENTES OU REVERSAS.



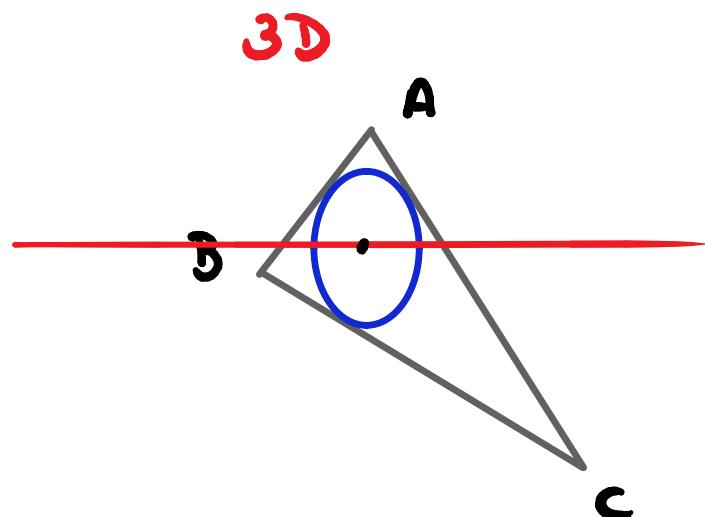
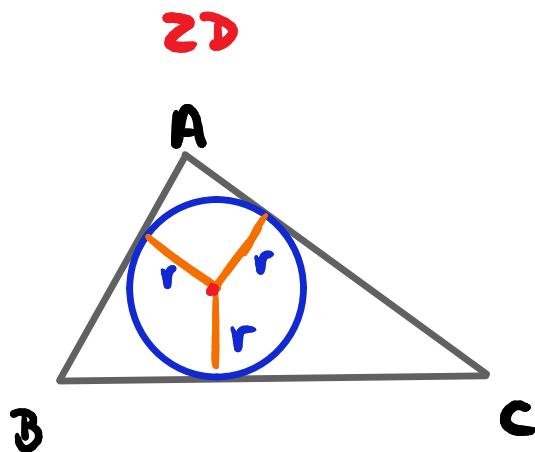
## EXEMPLO

JULGUE AS AFIRMATIVAS:

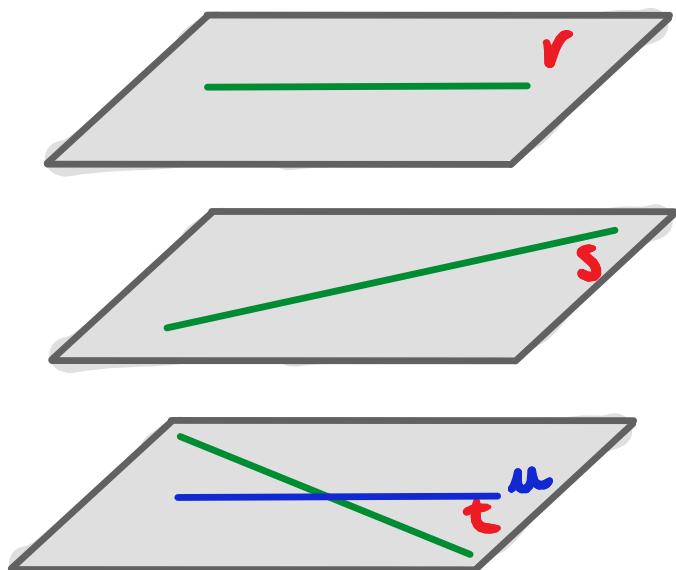
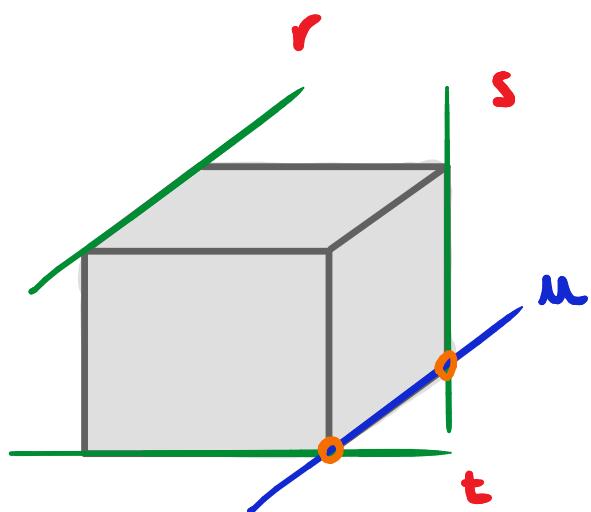
(F) SE A MEDIDA DO ÂNGULO AGUDO ENTRE UMA RETA  $r$  E UM PLANO  $\alpha$  É  $45^\circ$ , ENTÃO EXISTE UMA RETA  $s$  CONTIDA EM  $\alpha$  TAL QUE A MEDIDA DO ÂNGULO ENTRE  $r$  E  $s$  É  $30^\circ$ .



**(F)** SEJAM A, B E C TRÊS PONTOS NÃO COLINEARES. ENTÃO EXISTE APENAS UM PONTO EQUIDISTANTE ÀS RETAS AB, BC E AC.



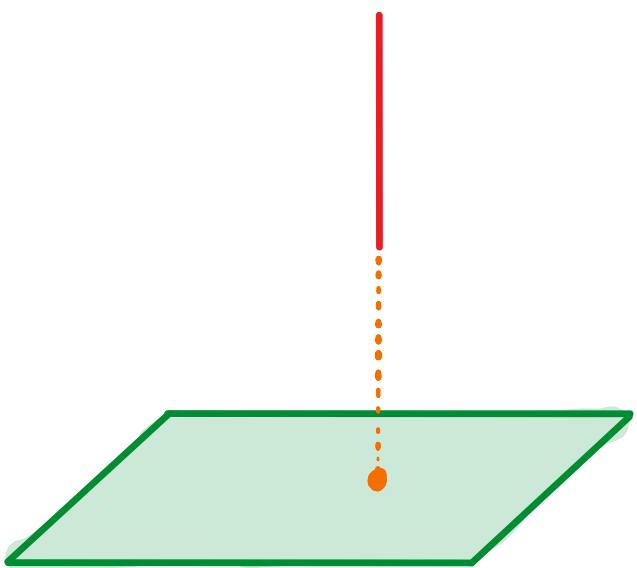
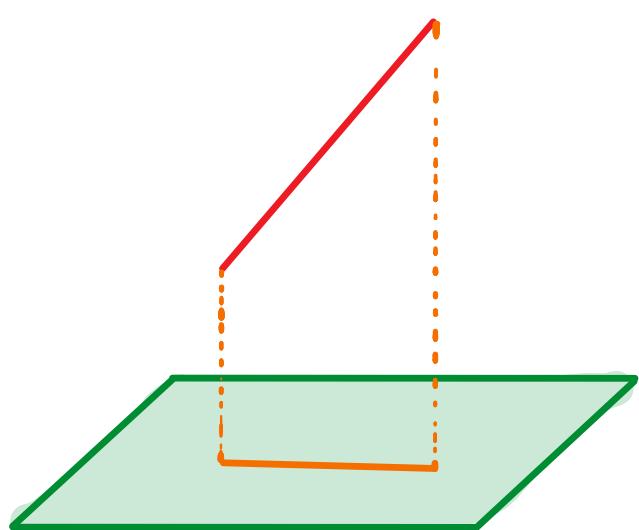
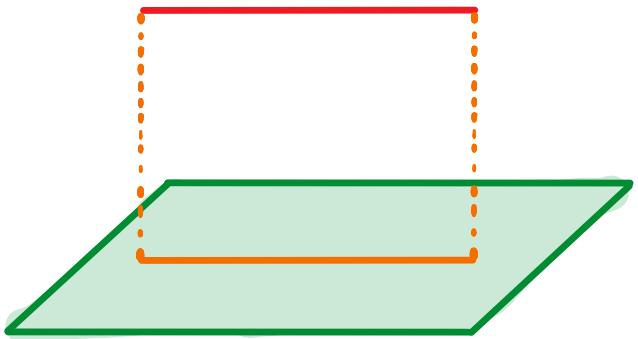
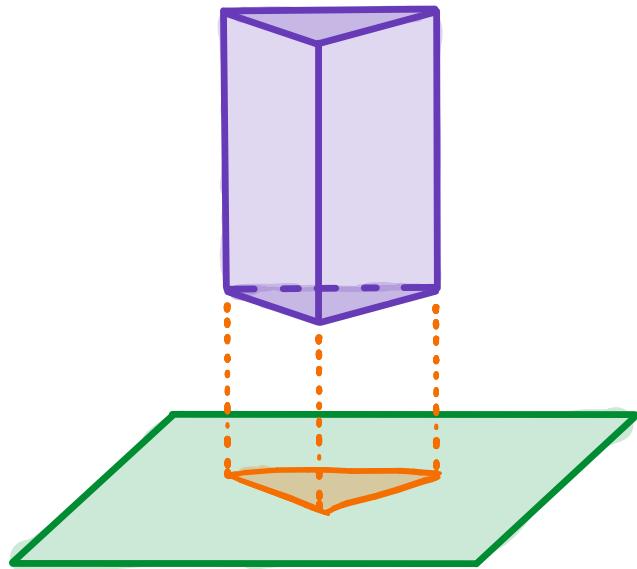
**(F)** PARA QUAISQUER RETAS  $r$ ,  $s$  E  $t$  REVERSAS DUAS A DUAS, EXISTE UMA RETA  $u$  PARALELA A  $r$  E CONCORRENTE COM  $s$  E COM  $t$ .



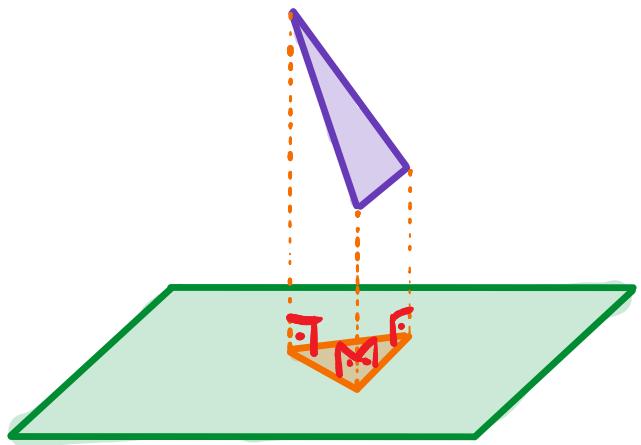
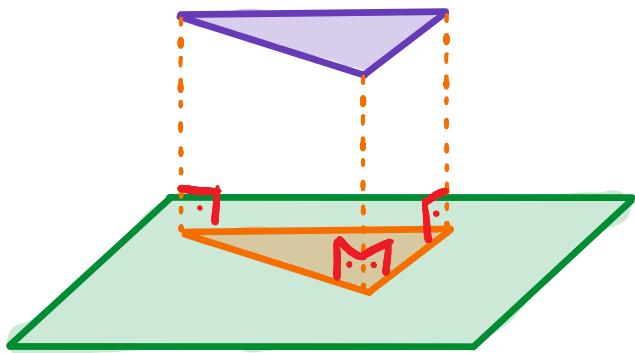
# PROJEÇÕES

PROJEÇÃO DE OBJETOS SOBRE SUPERFÍCIES.

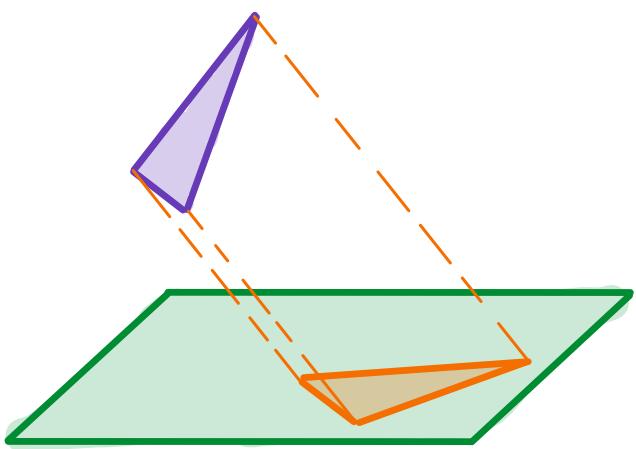
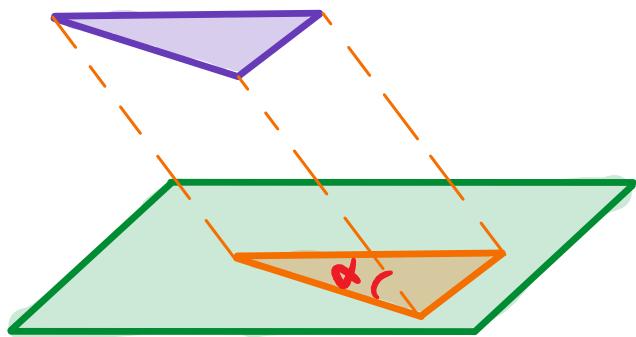
"SOMBRA DO OBJETO"



# PROJEÇÃO ORTOGONAL



# PROJEÇÃO OBLÍQUA

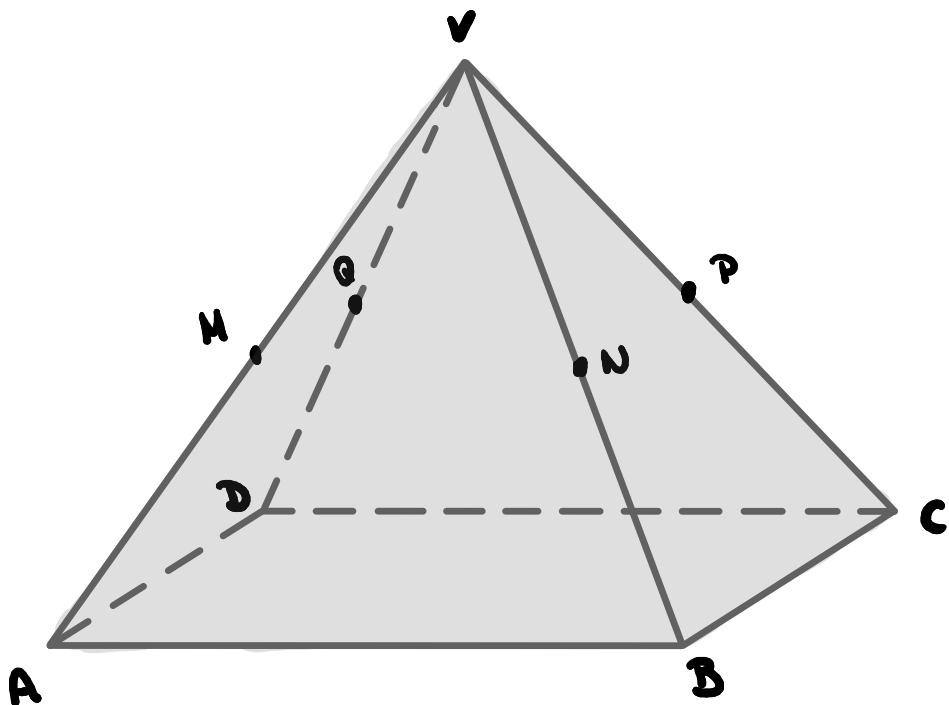


## EXEMPLO

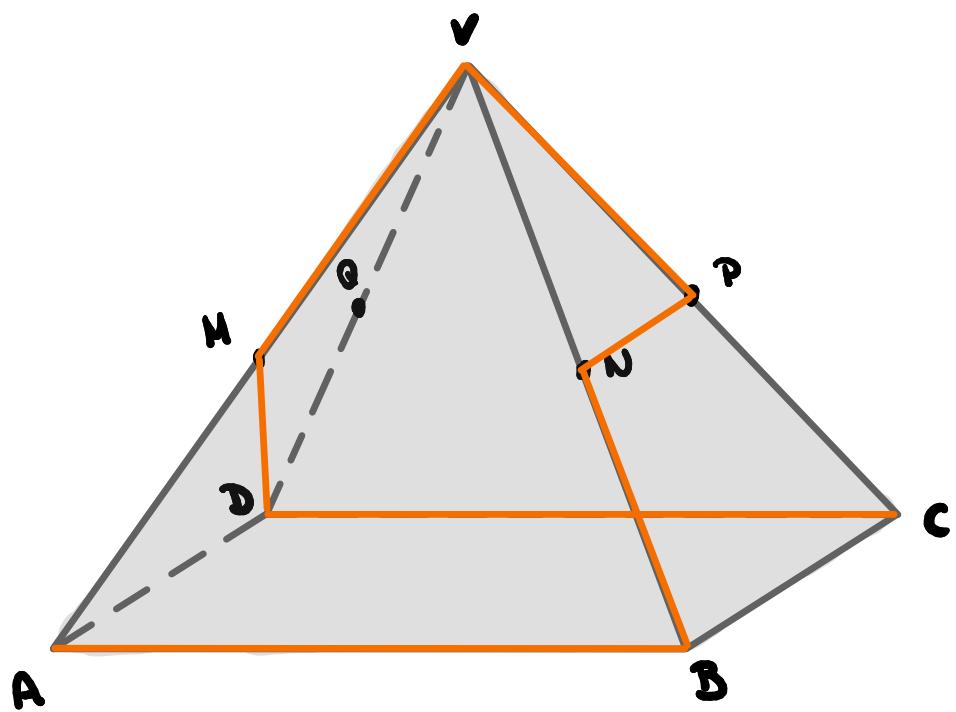
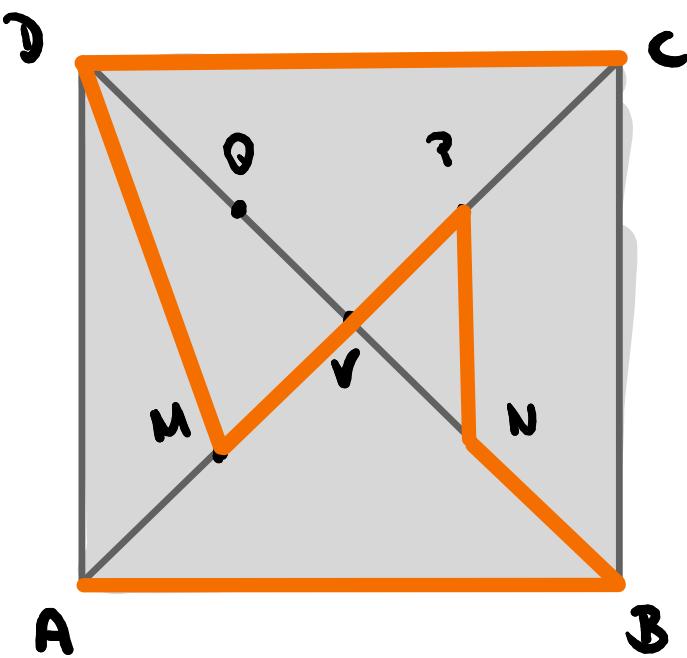
UMA FORMIGA DESLOCA SE SOBRE A SUPERFÍCIE DA PIRÂMIDE, SEMPRE EM LINHA RETA, COMEÇANDO NO PONTO A.

ELA PASSA SEQUENCIALMENTE PELOS PONTOS:  
A, B, N, P, V, M, D, C.

DESCREVA A TRAJETOÓRIA DA PROJEÇÃO ORTOGONAL DA FORMIGA SOBRE O PLANO DA BASE DA PIRÂMIDE.



# VISTA SUPERIOR



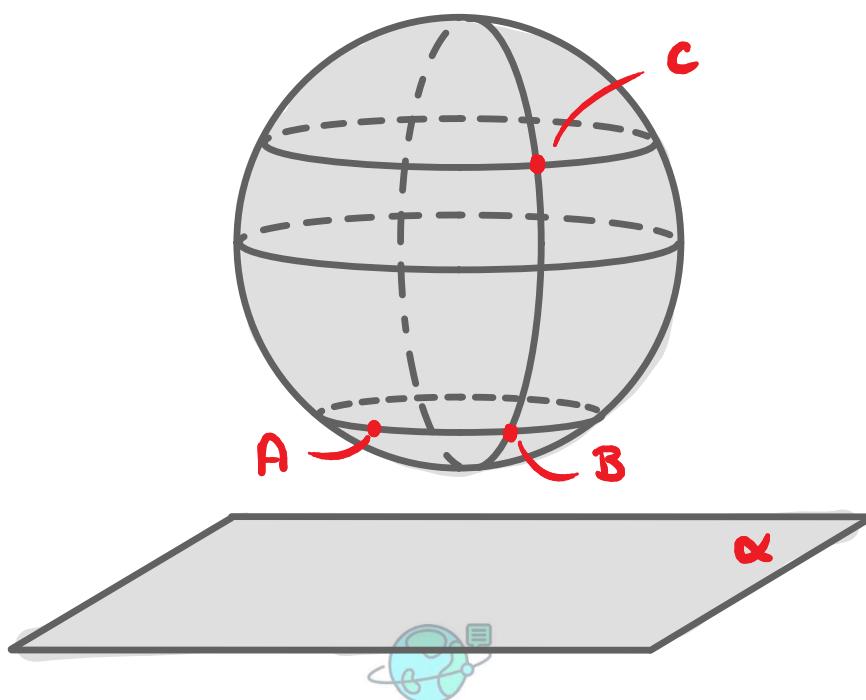
## EXEMPLO

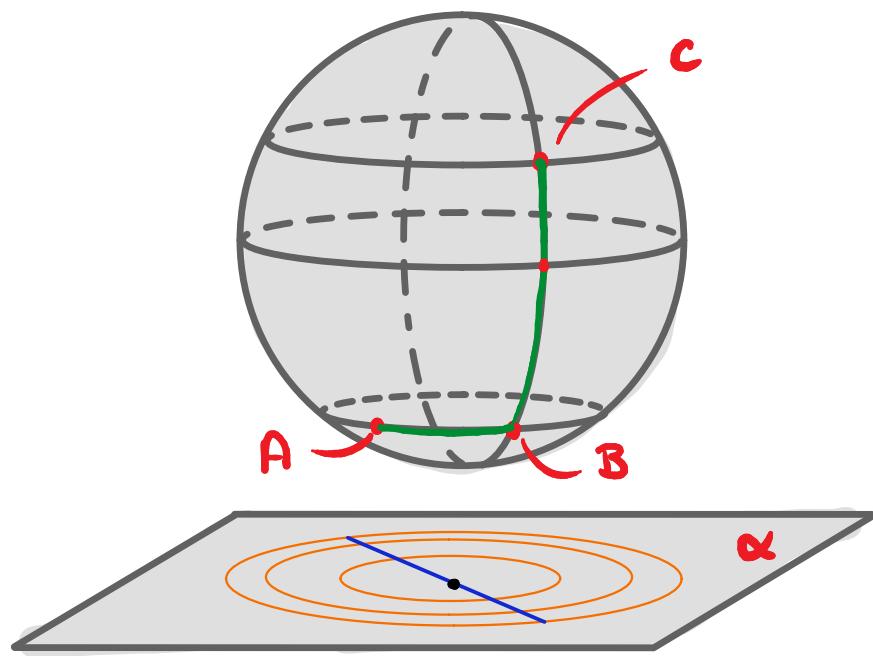
CONSIDERE O GLOBO REPRESENTANDO O PLANETA TERRA ABAIXO.

OS PONTOS A E B ESTÃO SOBRE UM MESMO PARALELO E OS PONTOS B E C ESTÃO SOBRE UM MESMO MERIDIANO. SEJA  $\alpha$  UM PLANO PARALELO À LINHA DO EQUADOR, ABAIXO DO GLOBO.

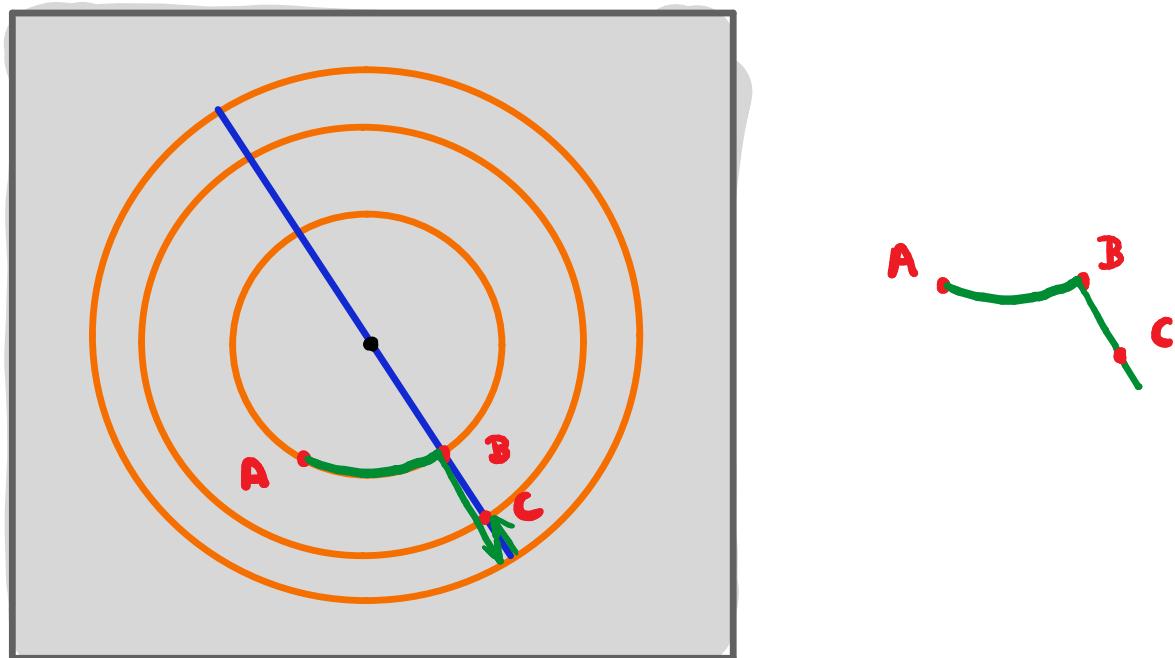
UMA PESSOA SE DESLOCA DE A PARA B SOBRE O MESMO PARALELO E DE B PARA C SOBRE O MESMO MERIDIANO.

DESCREVA A TRAJETÓRIA DA PROJEÇÃO ORTOGONAL DESSA PESSOA SOBRE O PLANO  $\alpha$ .





VISTA SUPERIOR

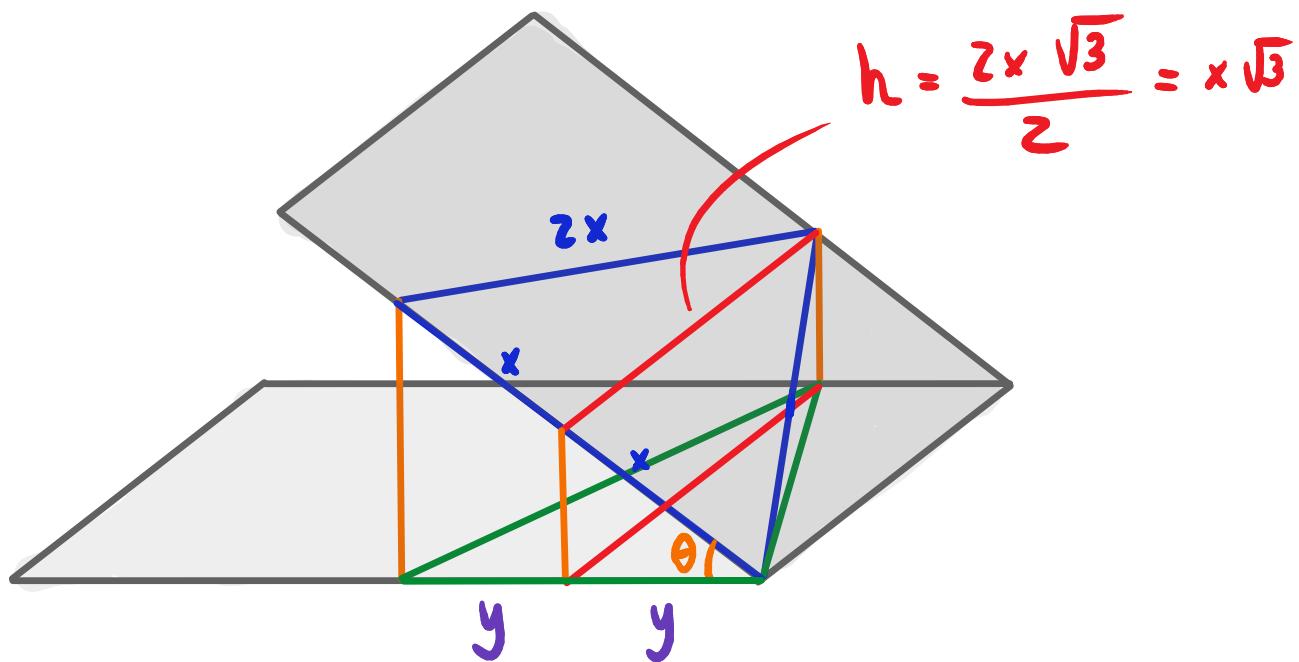


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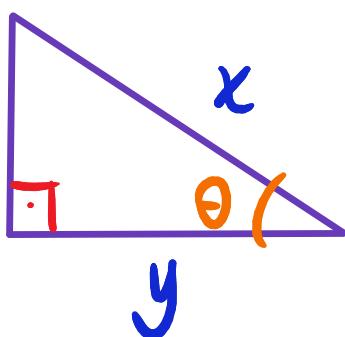
## EXEMPLO

A PROJEÇÃO DE UM TRIÂNGULO EQUILÁTERO É UM TRIÂNGULO ISÓCELES COM ÂNGULO DISTINTO IGUAL A  $30^\circ$ . CALCULE O COSENHO DO ÂNGULO FORMADO ENTRE O PLANO DO TRIÂNGULO ORIGINAL E O PLANO DA SUA PROJEÇÃO.



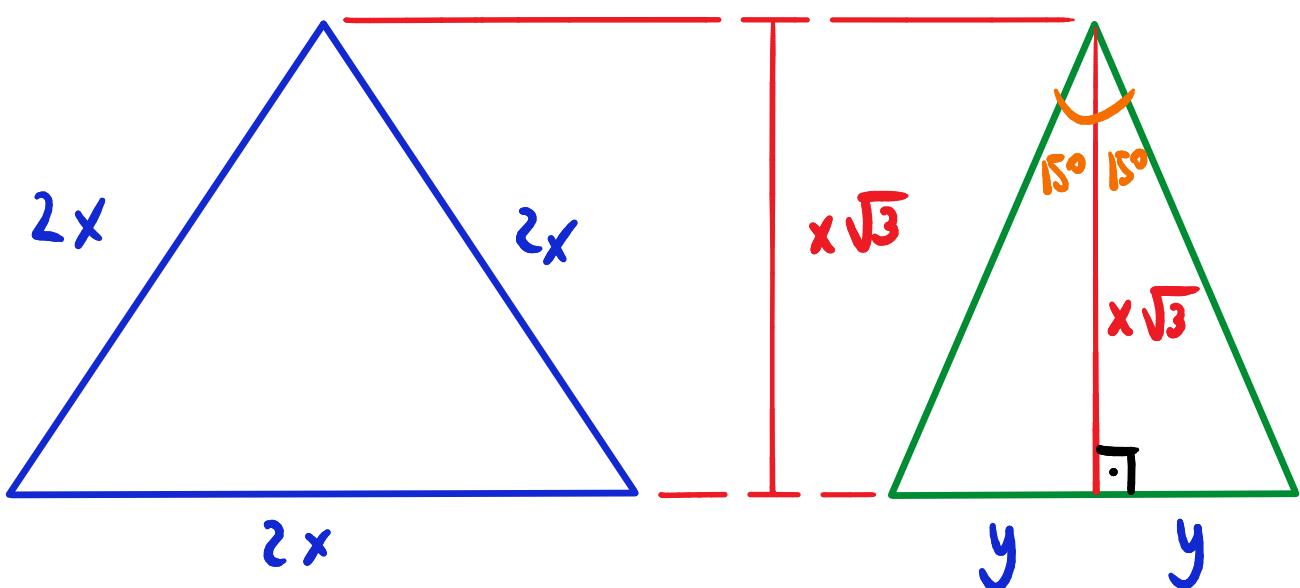


$$\cos \theta = ?$$



$$\cos \theta = \frac{y}{x}$$





$$\operatorname{tg} 15^\circ = \frac{y}{x\sqrt{3}} \rightarrow \frac{y}{x} = \underline{\sqrt{3} \cdot \operatorname{tg} 15^\circ}$$

$$\operatorname{tg} 15^\circ = \operatorname{tg}(45^\circ - 30^\circ) = \frac{\operatorname{tg} 45^\circ - \operatorname{tg} 30^\circ}{1 + \operatorname{tg} 45^\circ \cdot \operatorname{tg} 30^\circ}$$

$$\operatorname{tg} 15^\circ = \underline{2 - \sqrt{3}}$$

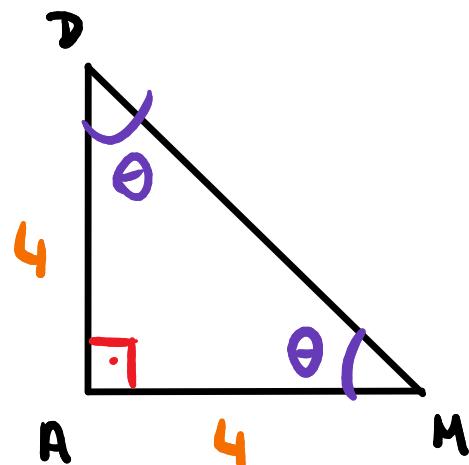
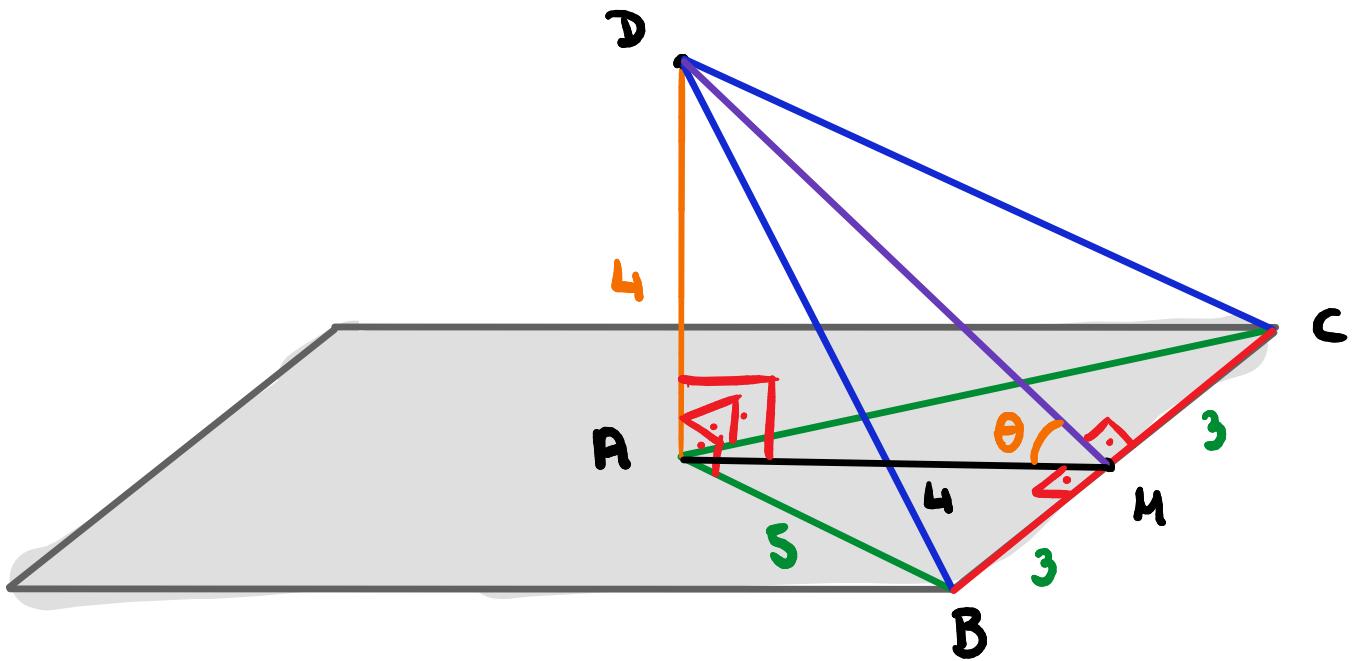
$$\cos \theta = \frac{y}{x} = \sqrt{3}(2 - \sqrt{3}) = \underline{2\sqrt{3} - 3}$$



## EXEMPLO

SEJA O TRIÂNGULO ABC, COM  $AB = AC = 5$  E  $BC = 6$ , LOCALIZADO NUM PLANO  $\alpha$ . A PARTIR DO VÉRTICE A, ERGUE-SE O SEGMENTO AD PERPENDICULARMENTE AO PLANO  $\alpha$ , COM  $AD = 4$ . CALCULE O ÂNGULO FORMADO ENTRE O PLANO  $\alpha$  E O PLANO DO TRIÂNGULO BCD.





$$2\theta = 90^\circ \rightarrow \underline{\theta = 45^\circ}$$

$$\operatorname{tg} \theta = \frac{4}{4} = 1$$

$$\underline{\theta = 45^\circ}$$

