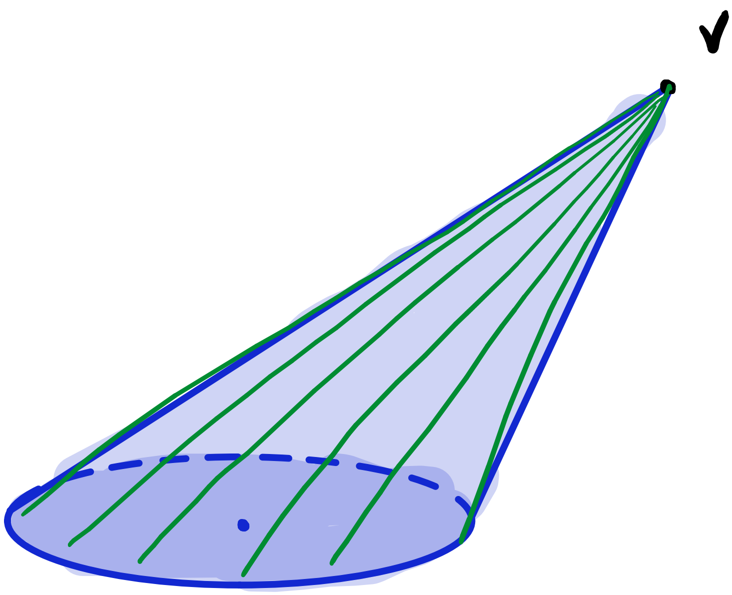
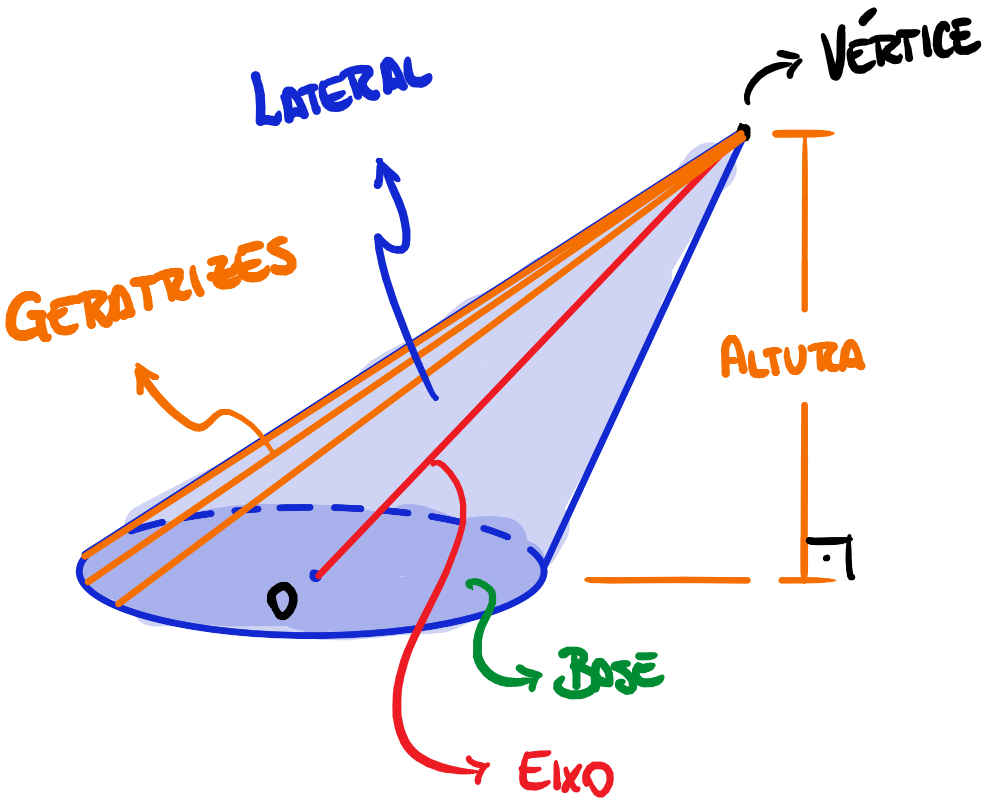
CONES

DEFINIÇÃO

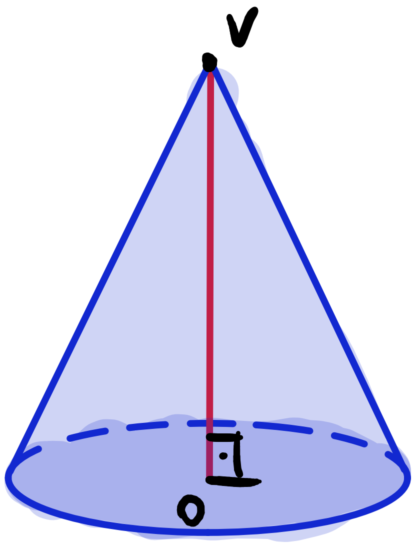
SÓLIDO QUE POSSUI UM CÍRCULO COMO BASE E UM VÉRTICE FORA DO PLANO DESSA BASE.



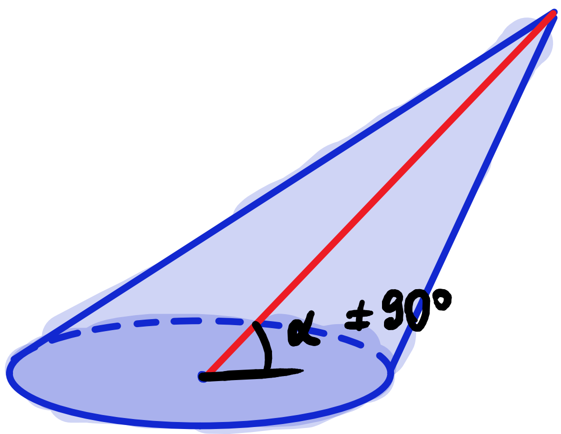
ELEMENTOS DO CONE



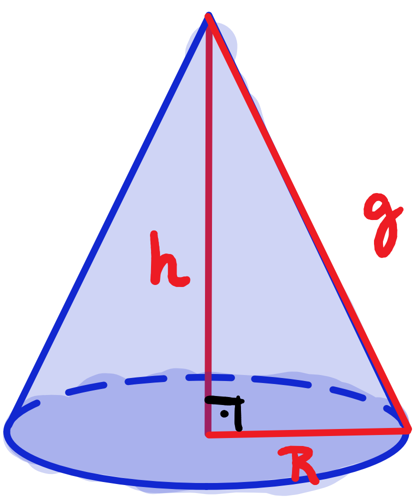
Classificação



Cone Reto

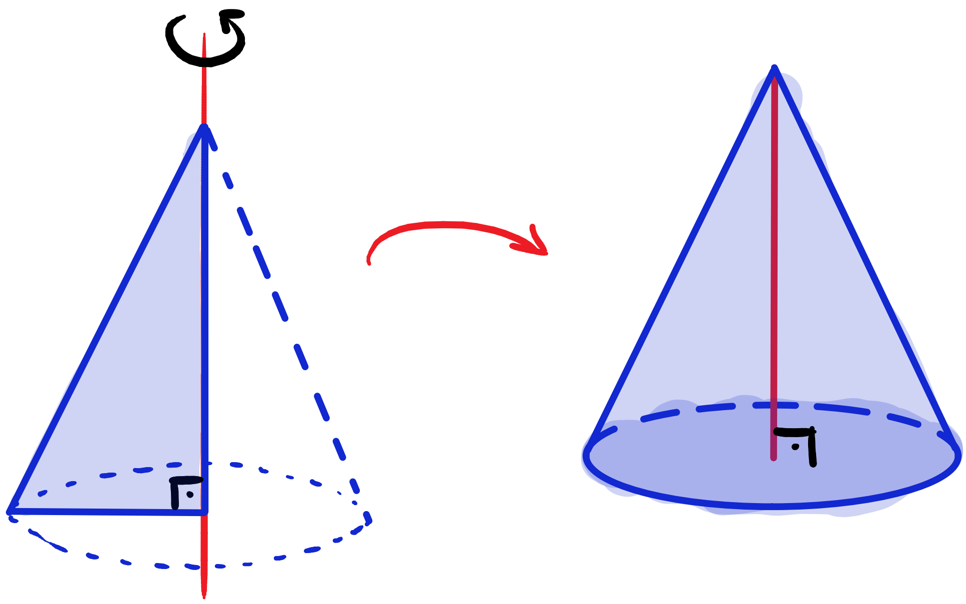


Cone Oblíquo



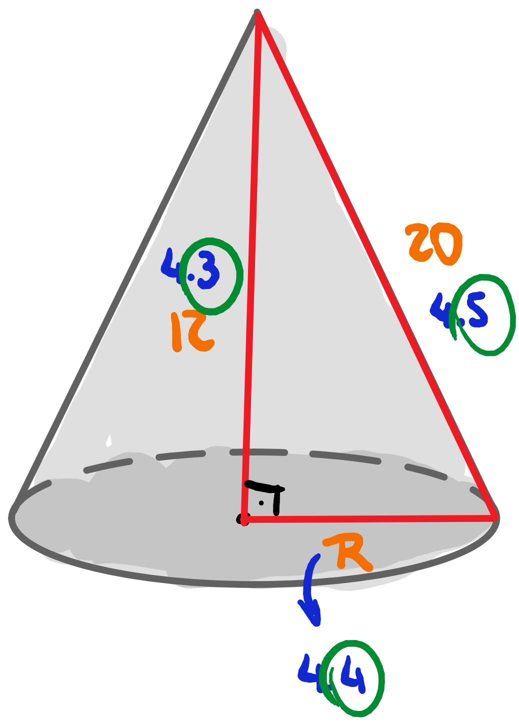
$$\boxed{g^2 = R^2 + h^2}$$

CONE CIRCULAR RETO = **CONE DE REVOLUÇÃO**



EXEMPLO

UM CONE RETO POSSUI ALTURA 12 E GERATRIZ 20. CALCULE O RAIO DA BASE DESSE CONE.



$$R^2 = 20^2 - 12^2$$

$$R^2 = 400 - 144$$

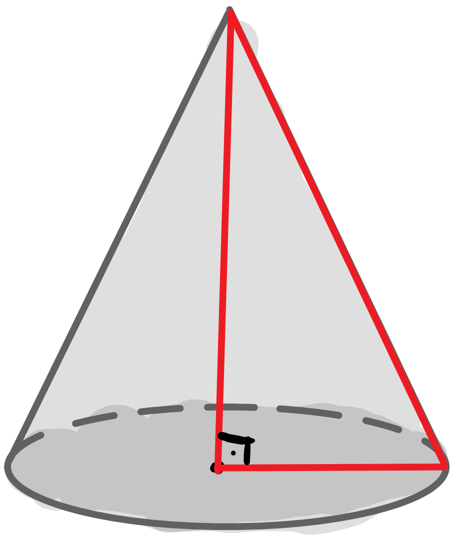
$$R^2 = 256$$

$$R = \sqrt{256}$$

$$R = 16$$

EXEMPLO

O RAIO, A ALTURA E A GERATRIZ DE UM CONE RETO ESTÃO EM PROGRESSAO ARITMÉTICA DE RAZÃO 2. CALCULE A ALTURA DESSE CONE.



$PA(R, h, g)$

$PA(h-2\ ,\ h\ ,\ h+2)$

$\downarrow \quad \downarrow \quad \downarrow$

$6 \quad 8 \quad 10$

$$g^2 = R^2 + h^2$$

$$(h+2)^2 = (h-2)^2 + h^2$$

$$\cancel{h^2} + 4h + \cancel{4} = \cancel{h^2} - 4h + \cancel{4} + h^2$$

$$0 = h^2 - 8h$$

$$h(h-8) = 0$$

$$\cancel{\boxed{h = 0}}$$

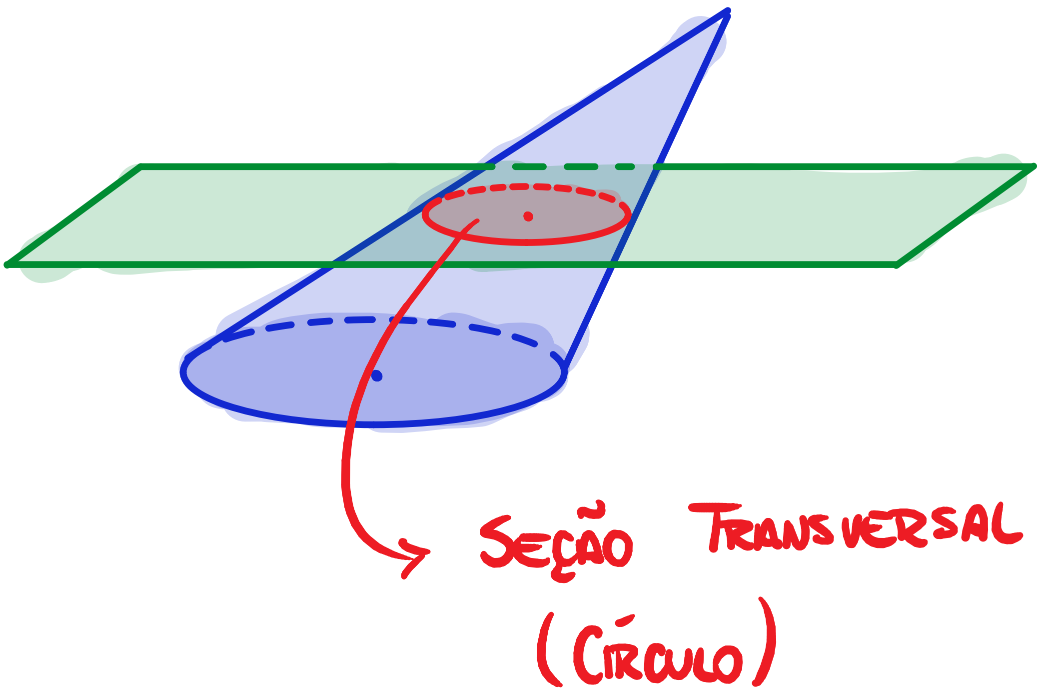
ou

$$\boxed{h = 8}$$

Seções de um Cone

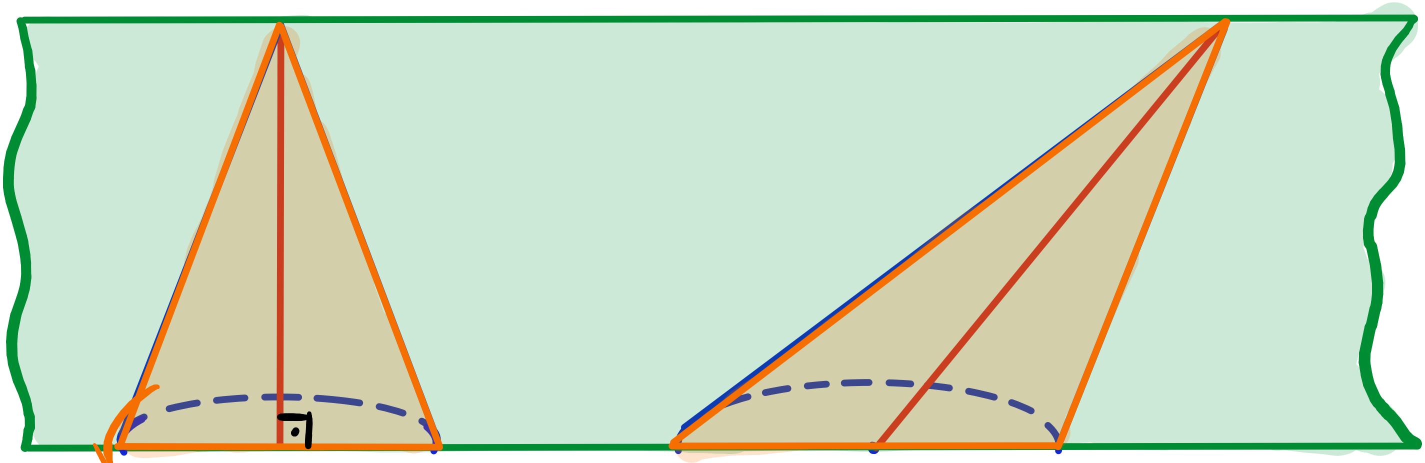
Seção Transversal

Seção paralela à base.



SEÇÃO MERIDIANA

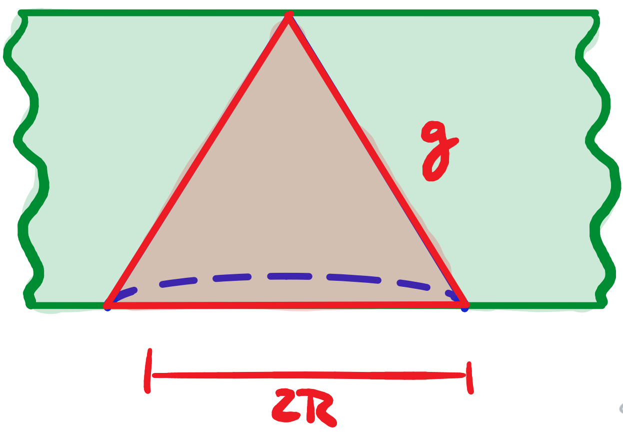
SEÇÃO QUE CONTÉM O EIXO DO CONE.



TRIÂNGULO ISÓSCELES

TRIÂNGULO

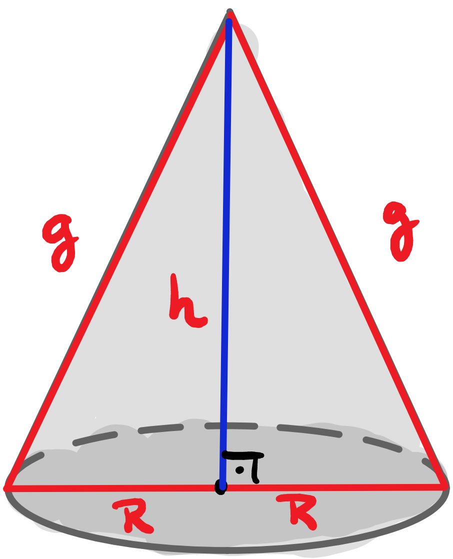
CONE EQUILÁTERO $\Rightarrow$ CONE CUJA SEÇÃO MERIDIANA É UM $\triangle$ EQUILÁTERO



$$\boxed{g = 2R}$$

EXEMPLO

A SEÇÃO MERIDIANA DE UM CONE RETO É UM TRIÂNGULO ISÓCELES CUJO PERÍMETRO É 36. SE A GERATRIZ É 8 UNIDADES MAIOR QUE O RAIO DA BASE, CALCULE A ALTURA DESSE CONE.



$$2g + 2R = 36$$

$$\begin{cases} g + R = 18 \\ g - R = 8 \end{cases}$$

$$2g = 26$$

$$g = 13$$

$$13 + R = 18$$

$$R = 5$$

$$h^2 = g^2 - R^2$$

$$h^2 = 13^2 - 5^2$$

$$h^2 = 18 \cdot 8 = 144$$

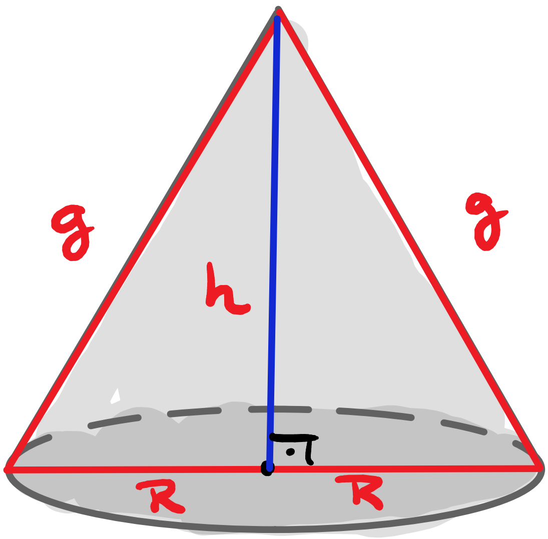
$$h = \sqrt{144}$$

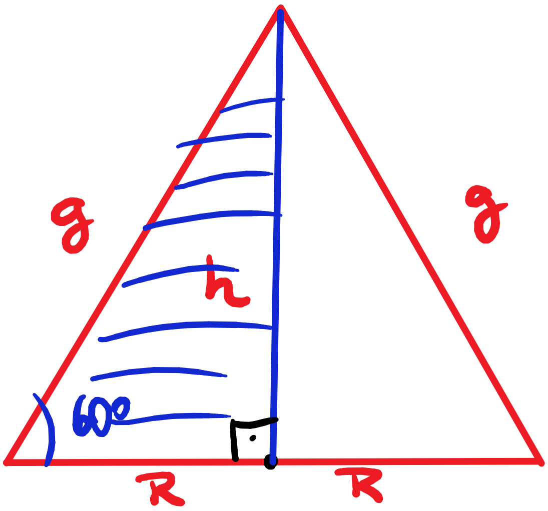
$$\boxed{h = 12}$$

UNIVERSO NARRADO

EXEMPLO

CALCULE A ALTURA DE UM CONE EQUILÁTERO DE GERATRIZ 10.





$$\operatorname{sen} 60^\circ = \frac{h}{g}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{g}$$

$$h = \frac{g\sqrt{3}}{2}$$

$$g = 2R$$

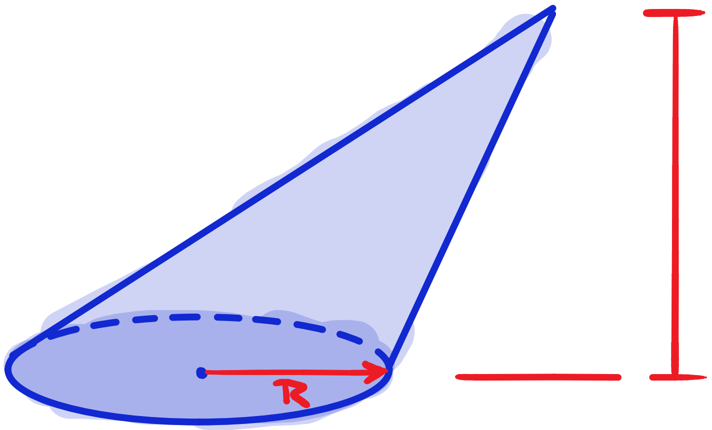
$$h = \frac{g\sqrt{3}}{2}$$

$$h = \frac{10\sqrt{3}}{2}$$

$$h = 10\sqrt{3}$$

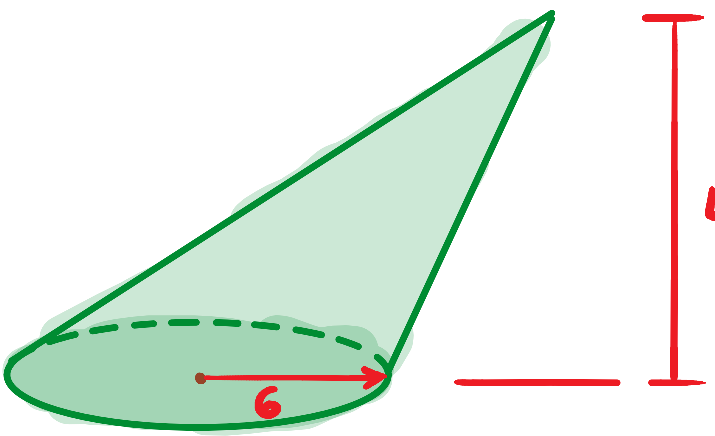
VOLUME

" " " PIRÂMIDE DE BASE CIRCULAR " " "



$$V = \frac{1}{3} \cdot A_b \cdot h$$

$$\boxed{V = \frac{1}{3} \pi R^2 h}$$



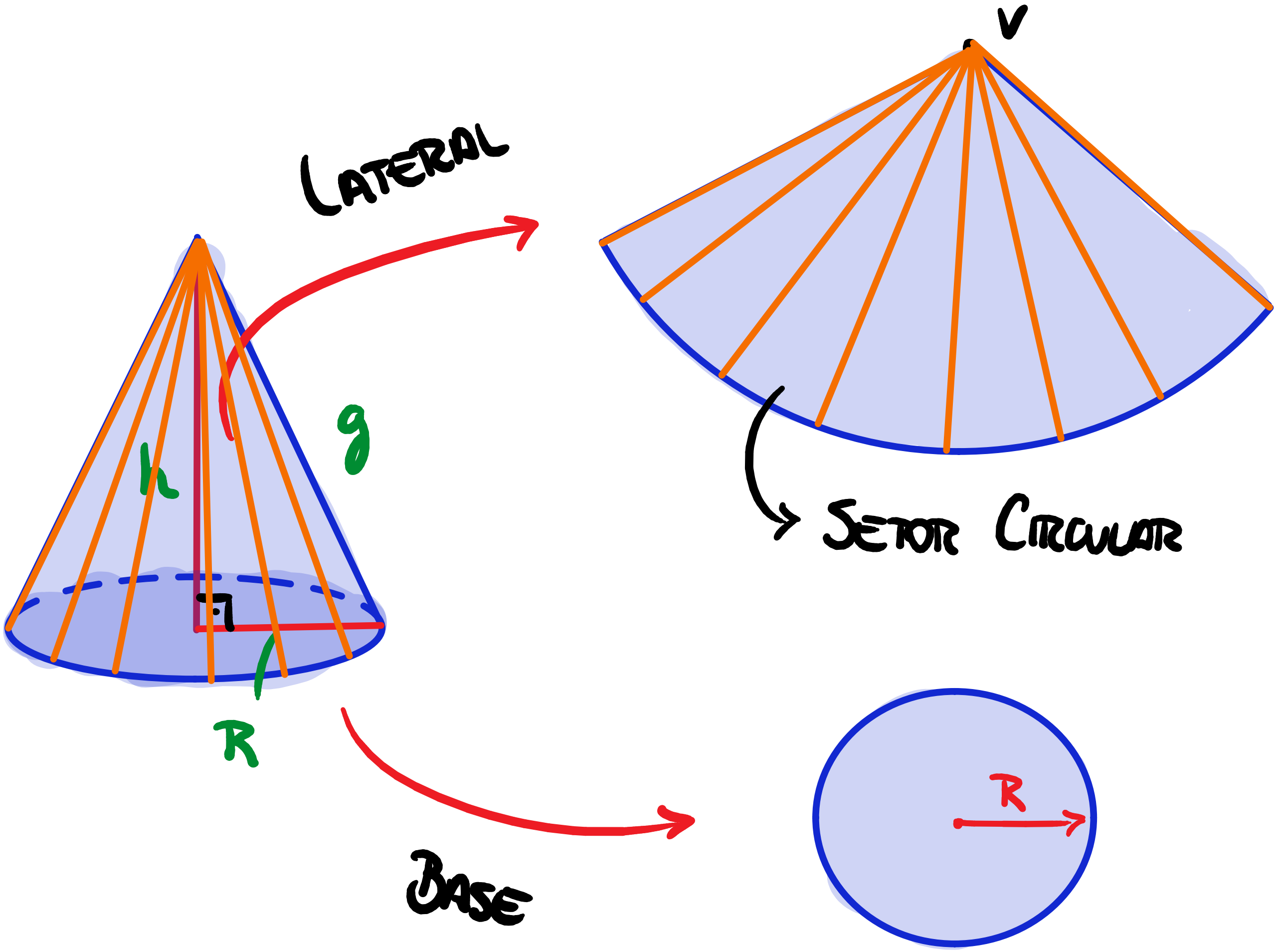
$$V = \frac{1}{3} \pi R^2 \cdot h$$

$$V = \frac{1}{3} \cdot \pi\, 6^2 \cdot 4$$

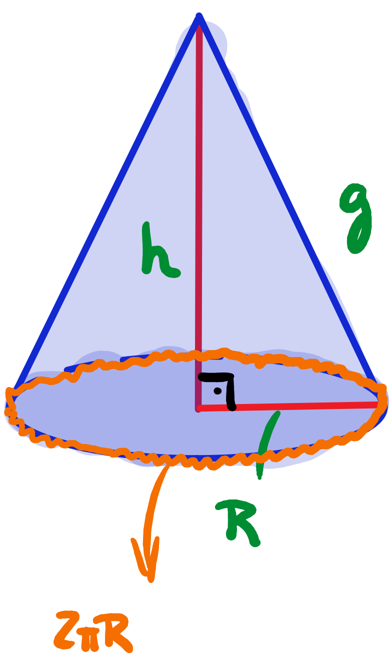
$$V = \frac{1}{\cancel{3}} \cdot \pi \cdot \overset{2}{\cancel{6}} \cdot 6 \cdot 4$$

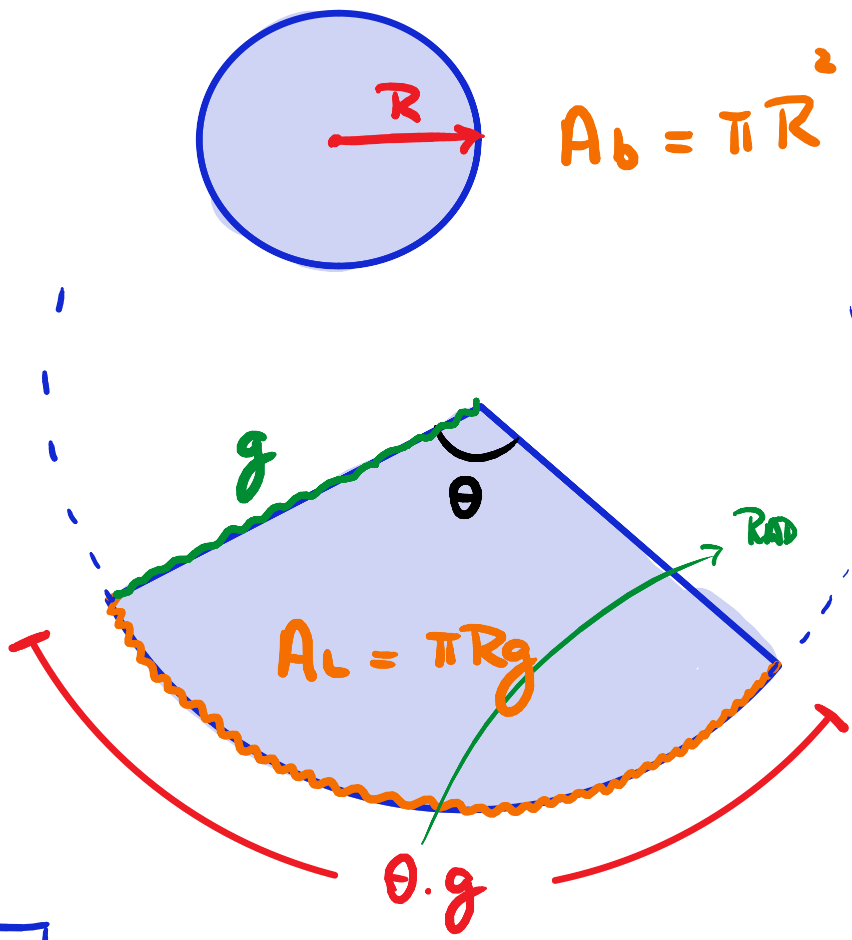
$$\boxed{V = 24\pi}$$

PLANIFICAÇÃO DO CONE RETO



Áreas





$$\theta g = 2\pi R$$

$$\boxed{\theta = \frac{2\pi R}{g}}$$

Ângulo		Área
2π	___	πg^2
θ	___	A_L

$$A_L = \frac{\theta \cdot \cancel{\pi} g^2}{2\cancel{\pi}} = \frac{\theta g^2}{2}$$

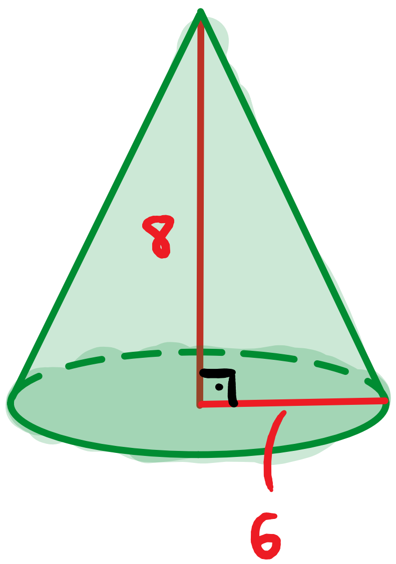
$$A_L = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\pi R}{\cancel{g}} \cdot g^{\cancel{2}}$$

$$\boxed{A_L = \pi R g}$$

Área da Base: $A_b = \pi R^2$

Área Lateral: $A_L = \pi R g$

Área Total: $A_T = A_b + A_L$



$A_b = \pi \cdot 6^2 \rightarrow A_b = 36\pi$

$A_L = \pi R g \rightarrow A_L = \pi \cdot 6 \cdot 10$

$A_L = 60\pi$

$g^2 = R^2 + h^2$

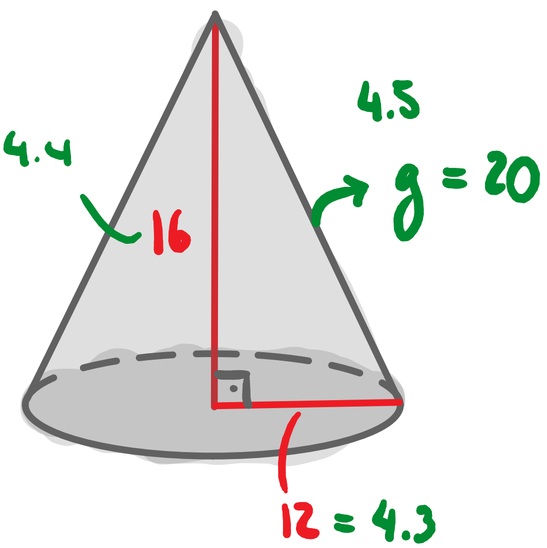
$g^2 = 6^2 + 8^2$

$g^2 = 100$

$g = 10$

EXEMPLO

SEJA UM CONE CUJO RAIO DA BASE É 12 E ALTURA É 16. CALCULE SEU VOLUME E ÁREA TOTAL.



$$V = \frac{1}{3}\pi R^2 . h$$

$$V = \frac{1}{3} . \pi . 12^2 . 16$$

$$V = \frac{1}{\cancel{3}} . \pi . \overset{4}{\cancel{12}} . 12 . 16$$

$$\underline{V = 768\pi}$$

$$A_b = \pi R^2 \rightarrow A_b = \pi . 12^2 \rightarrow \underline{A_b = 144\pi}$$

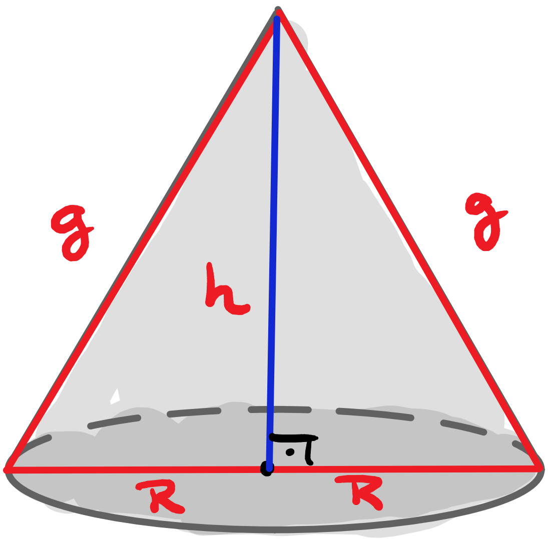
$$A_L = \pi R g \rightarrow A_L = \pi . 12 . 20 \rightarrow \underline{A_L = 240\pi}$$

$$A_T = A_b + A_L$$

$$A_T = 144\pi + 240\pi \rightarrow \underline{A_T = 384\pi}$$

EXEMPLO

CALCULE O VOLUME DE UM CONE EQUILÁTERO DE RAIO DA BASE R.



$$g = 2R$$

$$h = \frac{L\sqrt{3}}{2}$$

$$h = \frac{2R\sqrt{3}}{2}$$

$$h = R\sqrt{3}$$

$$V = \frac{1}{3} \cdot \pi R^2 h$$

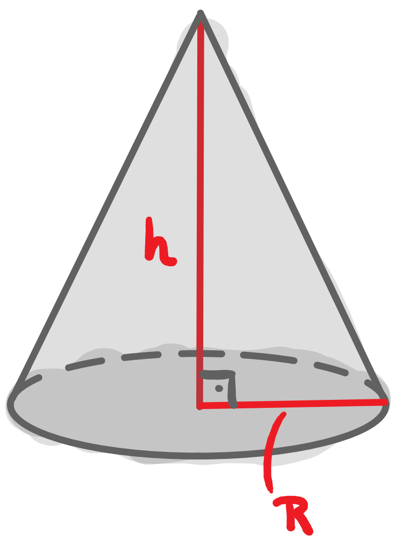
$$V = \frac{1}{3} \cdot \pi \cdot R^2 \cdot R\sqrt{3}$$

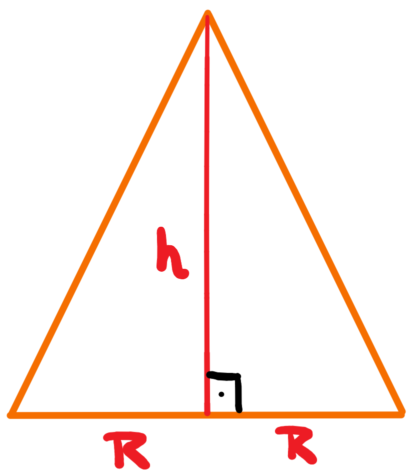
$$\boxed{V = \frac{1}{3} \pi R^3 \sqrt{3}}$$

EXEMPLO

A SEÇÃO MERIDIANA DE UM CONE RETO É UM TRIÂNGULO CUJA ÁREA É IGUAL A ÁREA DA BASE DO CONE. CALCULE:

- VOLUME DO CONE SABENDO QUE O RAIO DA BASE MEDE 3.





$$\pi R^{\cancel{2}} = \frac{1}{\cancel{2}} \cdot \cancel{2}\cancel{R} \cdot h$$

$$\underline{h = \pi R}$$

$$R = 3 \quad ; \quad h = 3\pi$$

$$V = \frac{1}{3} \cdot \pi R^2 \cdot h \rightarrow V = \frac{1}{\cancel{3}} \cdot \pi \cdot 3^2 \cdot \cancel{3}\pi$$

$$\underline{V = 9\pi^2}$$

EXEMPLO

NUM CONE RETO A GERATRIZ É O DOBRO DO RAIO DA BASE. CALCULE O VOLUME DO CONE SABENDO QUE SUA AREA LATERAL É 72π.

$$g^2 = R^2 + h^2$$

$$g = 2.R$$

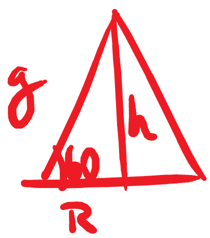
$$A_L = \pi R g \rightarrow \cancel{\pi} R g = 72\cancel{\pi}$$

$$R.2R = 72$$

$$R^2 = 36$$

$$R = 6$$

$$g = 12$$



$$12^2 = 6^2 + h^2$$

$$h^2 = 144 - 36$$

$$h^2 = 108$$

$$\boxed{h = 6\sqrt{3}}$$

108	2
54	2
27	3
9	3
3	3
1	

$$V = \frac{1}{3} \cdot \pi R^2 \cdot h$$

$$V = \frac{1}{\cancel{3}} \pi \cdot 6^2 \cdot \cancel{6}^{2}\sqrt{3}$$

$$V = 72\pi$$

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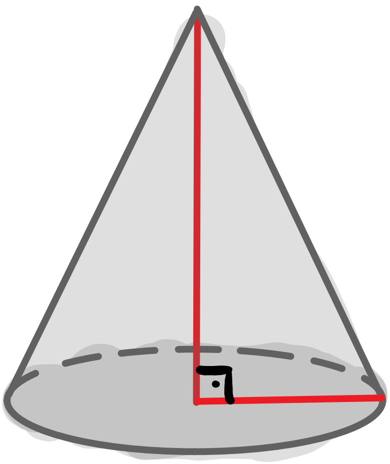
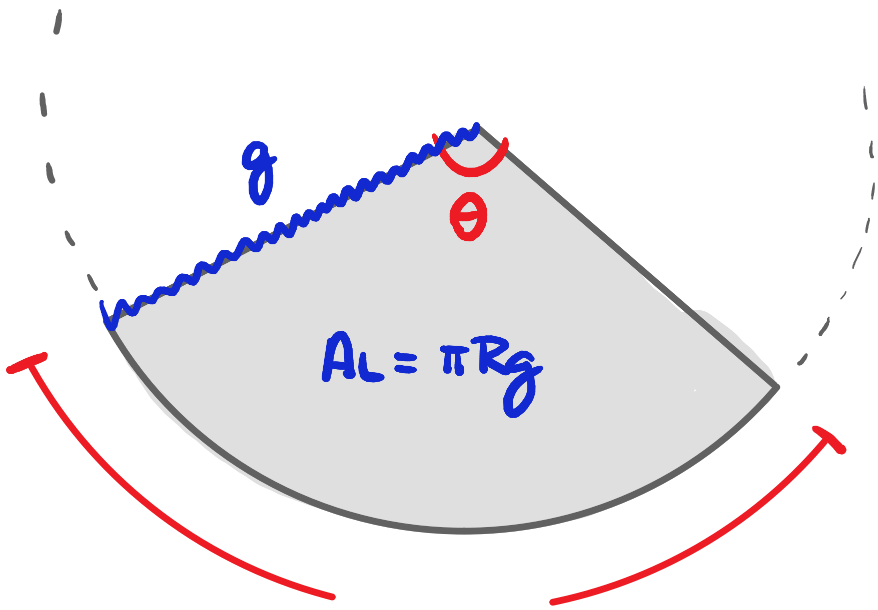
EXEMPLO

DETERMINE O ÂNGULO DO SETOR CIRCULAR RESULTANTE DA PLANIFICAÇÃO DE UM CONE EQUILÁTERO.

$g = 2R$





ÂNGULO		ÁREA
2π	—	πg^2
θ	—	$\pi R g$

$$\theta \cdot \cancel{\pi} g^{\cancel{2}} = 2\cancel{\pi} \cdot \pi R \cancel{g}$$

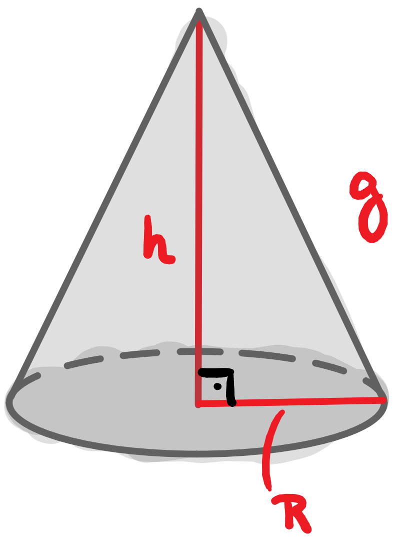
$$\boxed{\theta = \frac{2\pi R}{g}}$$

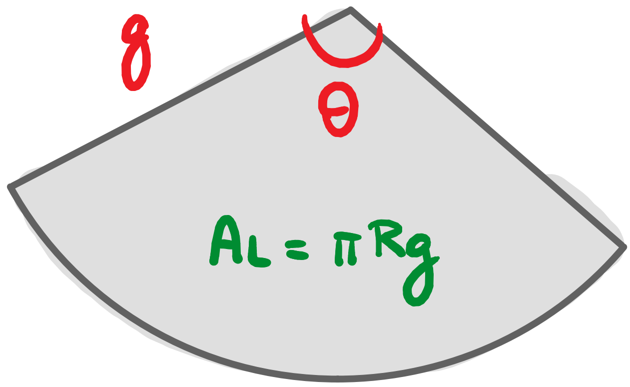
$$\theta = \frac{\cancel{2}\pi \cancel{R}}{\cancel{2}\cancel{R}} = \pi \text{ RAD}$$

$$\boxed{\theta = 180^\circ}$$

EXEMPLO

A PLANIFICAÇÃO DA LATERAL DE UM CONE GERA UM SETOR CIRCULAR DE RAIO 3 E ÂNGULO 120°. CALCULE O VOLUME DESSE CONE.





ÂNGULO		ÁREA
$360°$	—	$\cancel{\pi} g^{\cancel{2}}$
θ	—	$\cancel{\pi} R \cancel{g}$

$\rightarrow$

$$\theta g = 360 R$$

$$\theta = \frac{360 \cdot R}{g}$$

$g = 3 \qquad \theta = 120^\circ$

$$\theta g = 360R \rightarrow R = \frac{\theta}{360} \cdot g$$

$$R = \frac{\cancel{120}}{\cancel{360}_{3}} \cdot 3 \rightarrow R = 1$$

$$g^2 = R^2 + h^2 \rightarrow h^2 = 3^2 - 1^2$$

$$h^2 = 8$$

$$\underline{h = 2\sqrt{2}}$$

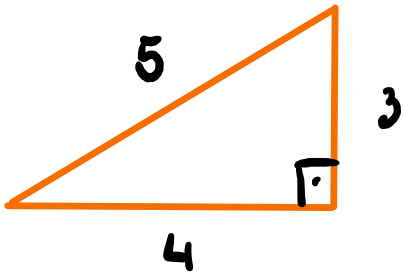
$$V = \frac{1}{3} \cdot \pi R^2 \cdot h$$

$$V = \frac{1}{3} \cdot \pi \cdot 1^2 \cdot 2\sqrt{2} \rightarrow \boxed{V = \frac{2\pi\sqrt{2}}{3}}$$

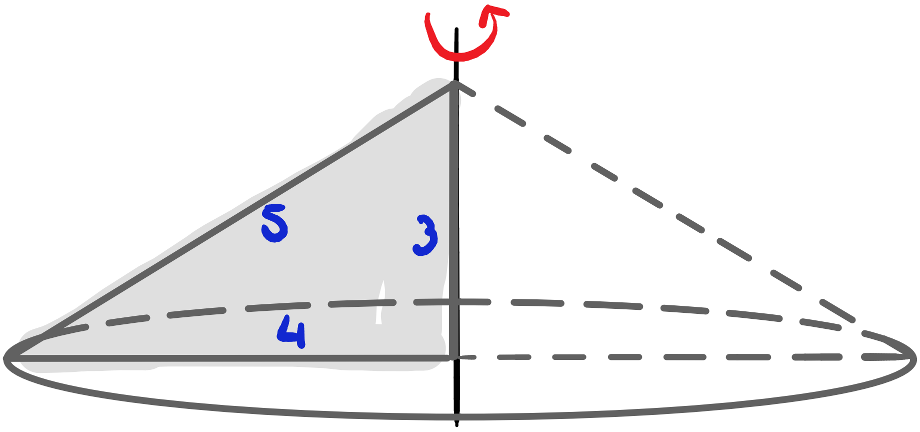
EXEMPLO

CALCULE O VOLUME DO SÓLIDO GERADO PELA ROTAÇÃO DE UM TRIÂNGULO DE LADOS 3, 4 E 5 EM TORNO:

a. DO SEU MENOR LADO.
b. DO SEU MAIOR LADO.



a



$R = 4$

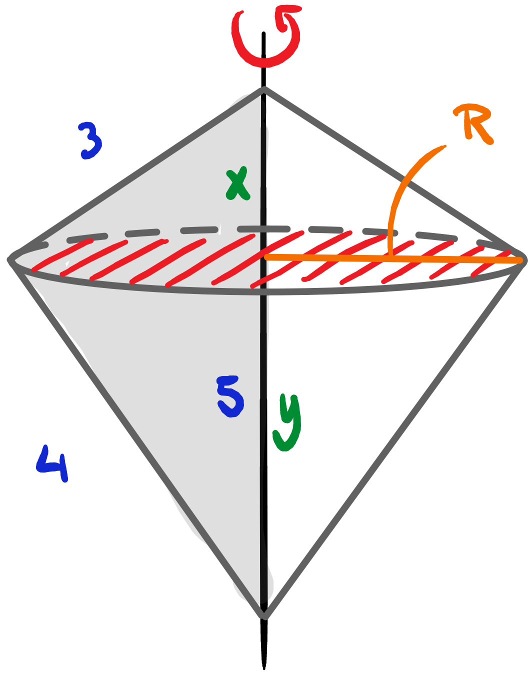
$h = 3$

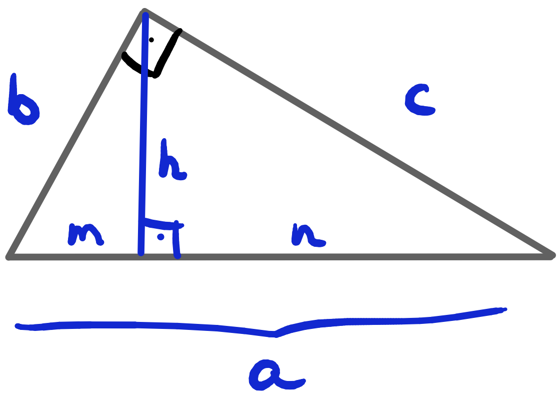
$$V = \frac{1}{3} \pi R^2 . h$$

$$V = \frac{1}{\cancel{3}} . \pi . 4^2 . \cancel{3}$$

$$V = 16\pi$$

b





$$ah = bc$$

$$5 \cdot R = 4 \cdot 3$$

$$R = \frac{12}{5}$$

$$V_T = V_1 + V_2$$

$$= \frac{1}{3}\pi R^2 x + \frac{1}{3}\pi R^2 y$$

$$= \frac{1}{3}\pi R^2 \underbrace{(x+y)}_{5}$$

$$= \frac{5\pi R^2}{3}$$

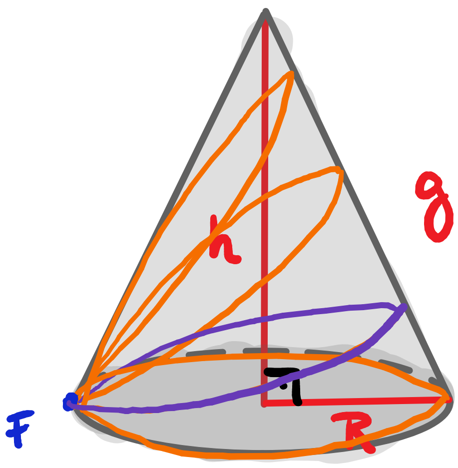
$$V_T = \frac{5\pi}{3} \cdot \left(\frac{12}{5}\right)^2$$

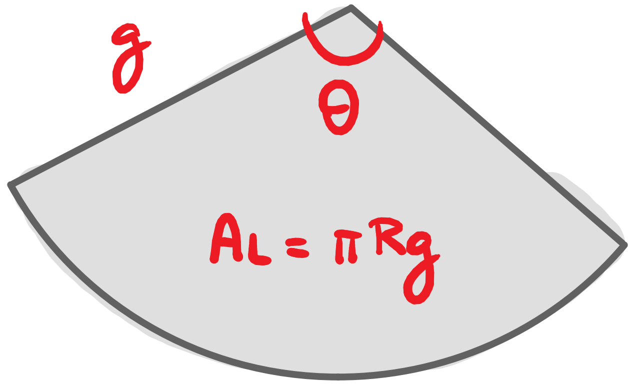
$$V_T = \frac{\cancel{5}\pi}{\cancel{3}} \cdot \frac{\overset{4}{\cancel{12}}}{\cancel{5}} \cdot \frac{12}{5}$$

$$\boxed{V_T = \frac{48\pi}{5}}$$

EXEMPLO

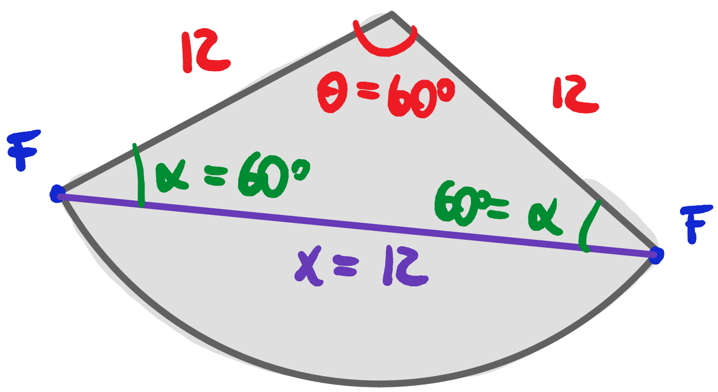
UMA FORMIGA SE ENCONTRA NA BASE DE UM CONE RETO DE RAIO DA BASE 2 E GERATRIZ 12 E DESEJA DAR UMA VOLTA NO CONE, ANDANDO PELA SUPERFICE LATERAL DO MESMO. QUAL A MENOR DISTÂNCIA QUE ELA DEVERÁ PERCORRER?





$2\alpha + 60 = 180$

$\alpha = 60°$



$$2\pi \text{ —— } \cancel{\pi} g^{\cancel{1}}$$

$$\theta \text{ —— } \cancel{\pi} R \cancel{g}$$

$$\rightarrow \theta g = 2\pi R$$

$$\theta = \frac{2\pi R}{g}$$

$$\theta = \frac{\cancel{2}\pi \cdot \cancel{2}}{\cancel{12}\ 3}$$

$$\theta = \frac{\pi}{3} = 60^\circ$$

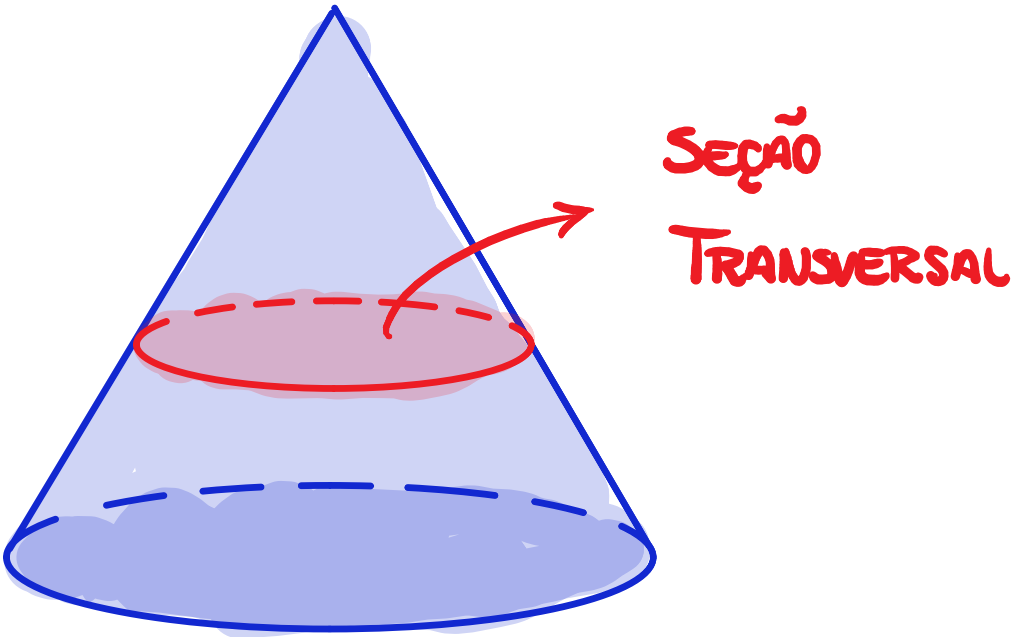
$$x^2 = 12^2 + 12^2 - 2 \cdot 12 \cdot 12 \cdot \cos 60^\circ$$

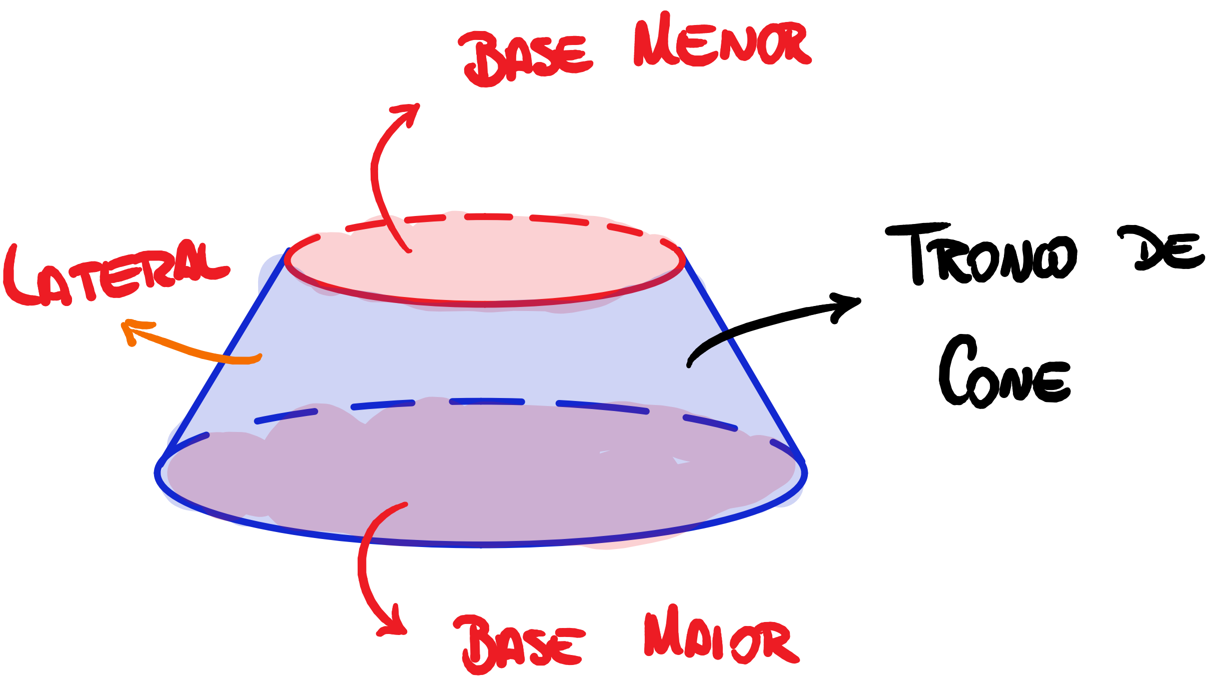
$$x^2 = 2 \cdot 12^2 - \cancel{2} \cdot 12^2 \cdot \frac{1}{\cancel{2}}$$

$$x^2 = 12^2$$

$$x = 12$$

TRONCO DE CONE





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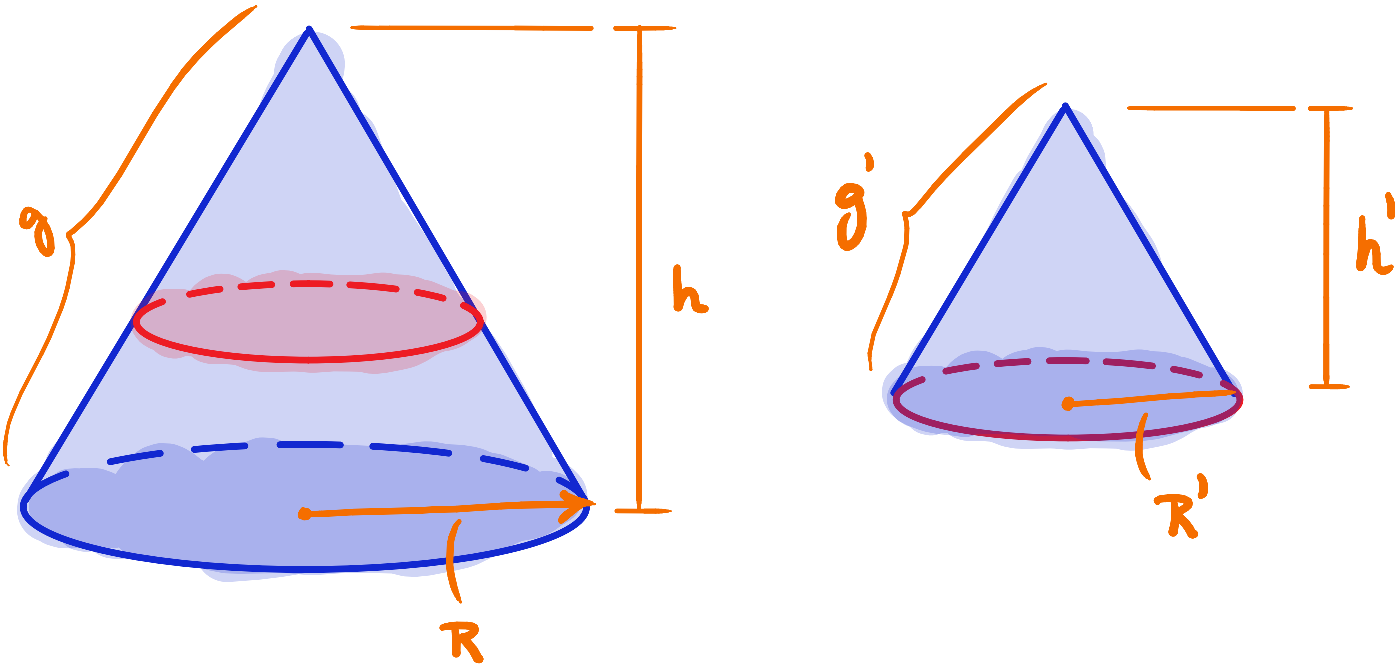
CONES SEMELHANTES

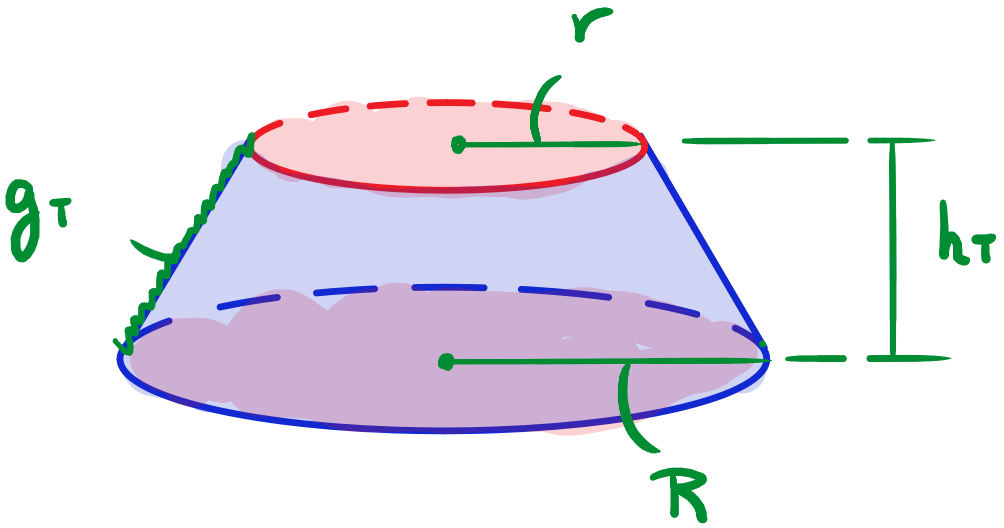


COMPRIMENTO: $$\boxed{\frac{g'}{g} = \frac{h'}{h} = \frac{R'}{R} = K}$$

ÁREA: $$\boxed{\frac{A_b'}{A_b} = \frac{A_L'}{A_L} = \frac{A_T'}{A_T} = K^2}$$

VOLUME: $$\boxed{\frac{V'}{V} = K^3}$$

OBS: $$\boxed{V_{TRONCO} = V - V'}$$



$$V_{TRONCO} = \frac{h_T}{3}\left(A_1 + \sqrt{A_1 A_2} + A_2\right)$$

$$= \frac{h_T}{3}\left(\pi R^2 + \sqrt{\pi R^2 . \pi r^2} + \pi r^2\right)$$

$$= \frac{h_T}{3}\left(\pi R^2 + \pi . R . r + \pi r^2\right)$$

$$\boxed{V_{TRONCO} = \frac{\pi\, h_T}{3}\left(R^2 + R.r + r^2\right)}$$

$$\boxed{A_L = \pi . (R + r) . g_T}$$

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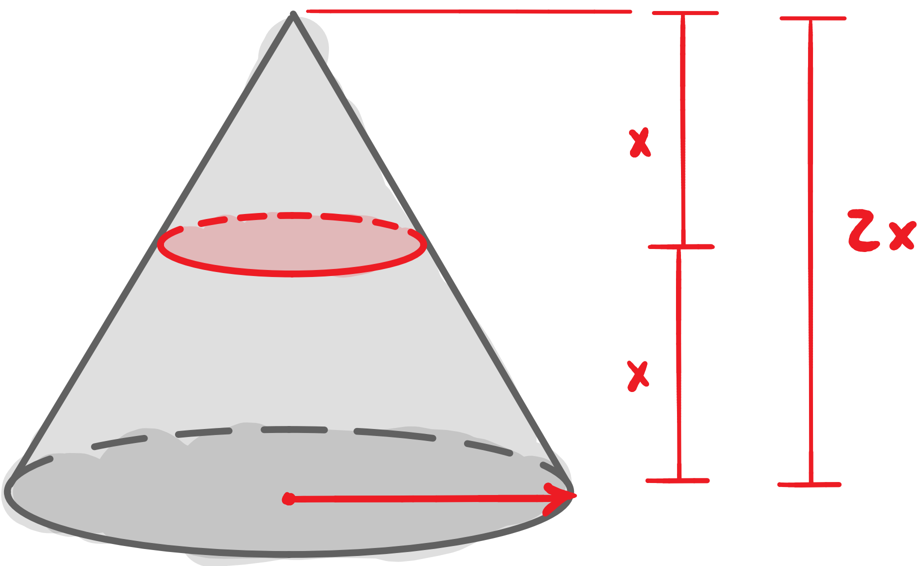
EXEMPLO

SEJA UM CONE DE VOLUME 72 E ÁREA LATERAL 28. PASSA-SE UMA SEÇÃO EQUIDISTANTE DA BASE E DO VÉRTICE DESSE CONE. DETERMINE:

a. O VOLUME DO CONE MENOR.
b. O VOLUME DO TRONCO.
c. A ÁREA LATERAL DO CONE MENOR.
d. A ÁREA LATERAL DO TRONCO.



$$K = \frac{x}{2x}$$

$$K = \frac{1}{2}$$

a) $\frac{V'}{V} = k^3 \rightarrow \frac{V'}{72} = \left(\frac{1}{2}\right)^3 \rightarrow \underline{V' = 8}$

b) $V_T = V - V' \rightarrow V_T = 72 - 9 \rightarrow \underline{V_T = 63}$

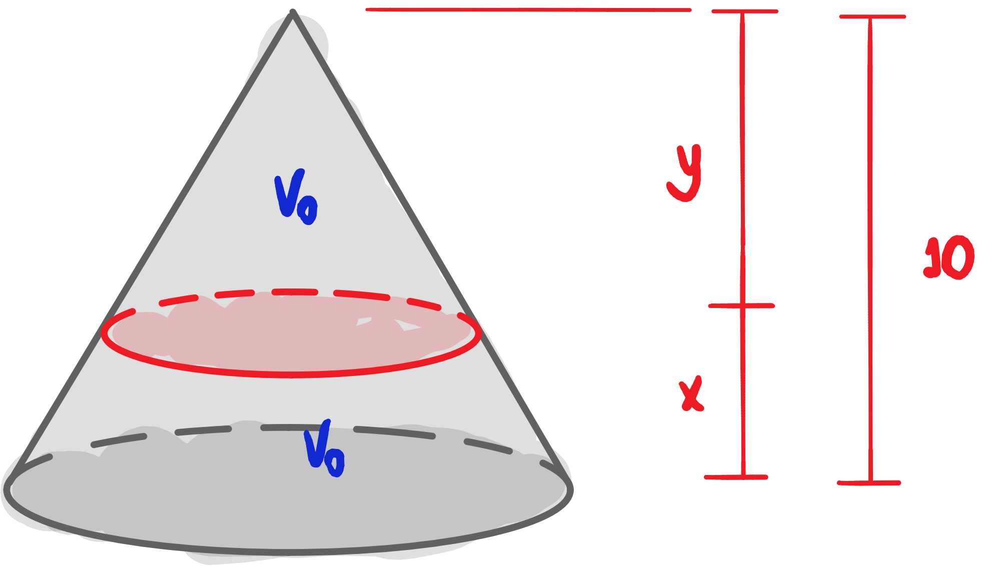
c) $\frac{A_L'}{A_L} = k^2 \rightarrow \frac{A_L'}{28} = \left(\frac{1}{2}\right)^2 \rightarrow \underline{A_L' = 7}$

d) $A_{LT} = A_L - A_L' \rightarrow A_{LT} = 28 - 7$

$\underline{A_{LT} = 21}$

EXEMPLO

SEJA UM CONE DE ALTURA 10. A DISTÂNCIA DA BASE DEVE SE SECCIONAR ESSE CONE DE FORMA A GERAR DOIS SÓLIDOS DE MESMO VOLUME?



$$\frac{V}{V'} = K^3 \rightarrow \frac{2\cancel{V_0}}{\cancel{V_0}} = K^3$$

$$K^3 = 2 \rightarrow \underline{K = \sqrt[3]{2}}$$

$$\frac{10}{y} = \sqrt[3]{2} \rightarrow y = \frac{10}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}}$$

$$y = \frac{10\sqrt[3]{4}}{\sqrt[3]{8} = 2}$$

$$y = 5\sqrt[3]{4}$$

$$x = 10 - y$$

$$\boxed{x = 10 - 5\sqrt[3]{4}}$$

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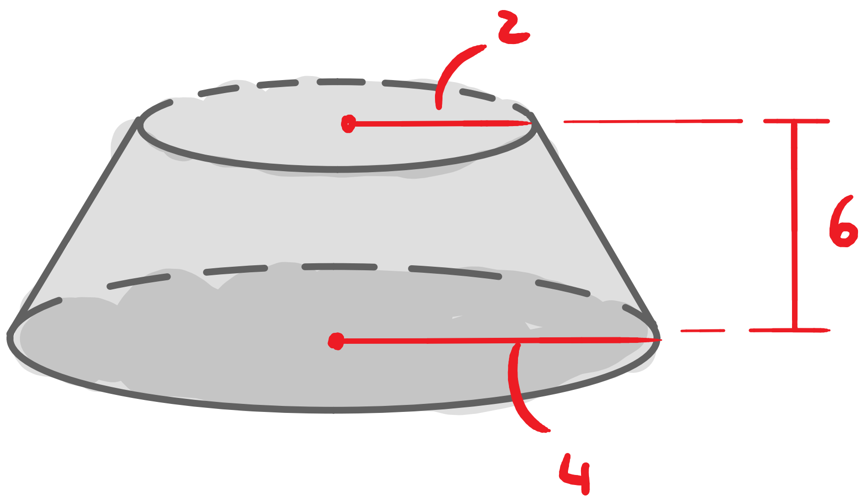
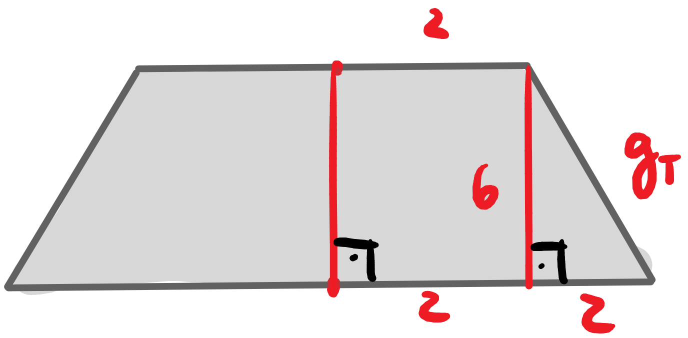
EXEMPLO

CALCULE O VOLUME E A ÁREA LATERAL DO TRONCO DE CONE ABAIXO.



Solução 1



$$g_T^2 = 2^2 + 6^2$$

$$g_T = \sqrt{4 + 36}$$

$$g_T = \sqrt{40}$$

$$g_T = 2\sqrt{10}$$

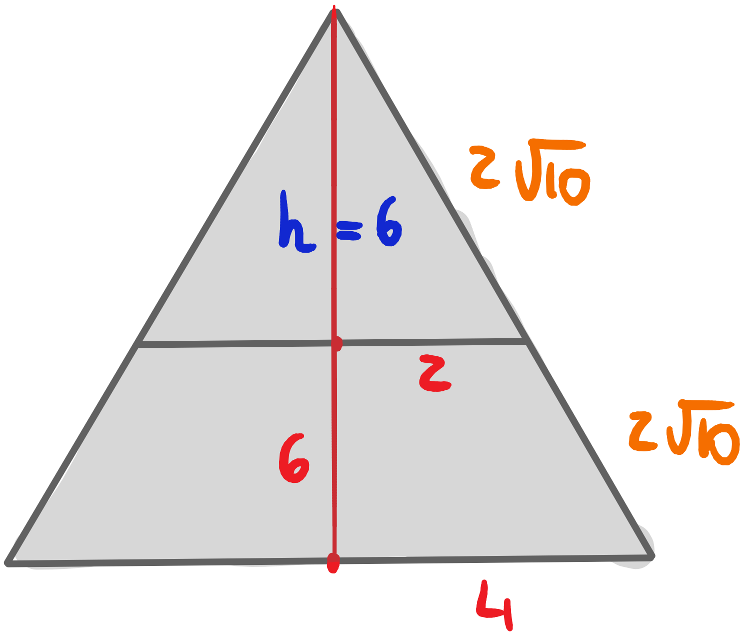
$$V = \frac{\pi h}{3}\left(R^2 + Rr + r^2\right)$$

$$V = \frac{\pi \cancel{6}^{\,2}}{\cancel{3}}\left(4^2 + 4.2 + 2.2\right) \rightarrow V = 56\pi$$

$$A_L = \pi(R+r)g_T \rightarrow A_L = \pi(4+2).2\sqrt{10}$$

$$A_L = 12\pi\sqrt{10}$$

Solução 2



$K = \frac{R}{r} = \frac{4}{2} = 2$

$K^2 = 4$

$K^3 = 8$

$\frac{h+6}{h} = 2 \rightarrow h + 6 = 2h \rightarrow \underline{h = 6}$

Cone Peq: $V' = \frac{1}{\cancel{3}} \pi \cdot 2^2 \cdot \cancel{6}^{2} \rightarrow \underline{V' = 8\pi}$

$\xrightarrow{K^3}$ $\underline{V = 64\pi}$

$V_T = 56\pi$

$A_L' = \pi \cdot 2 \cdot 2\sqrt{10} \rightarrow A_L' = 4\pi\sqrt{10}$

$\xrightarrow{K^2}$ $A_L = 16\pi\sqrt{10}$

$A_T = 12\pi\sqrt{10}$

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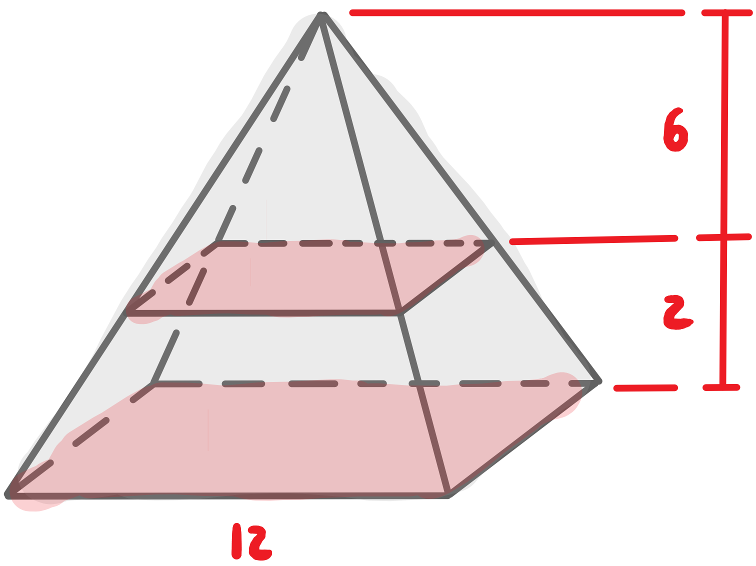
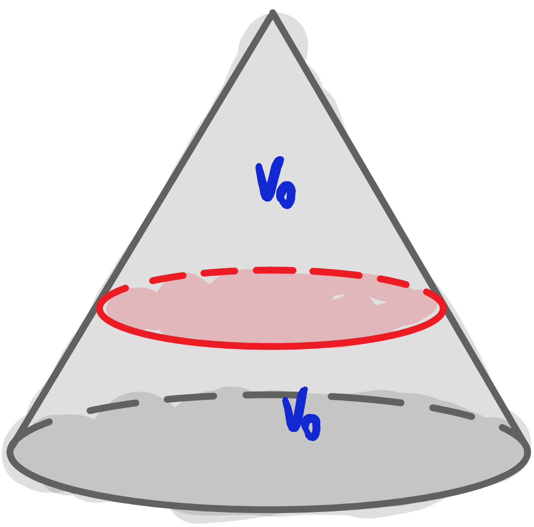
EXEMPLO

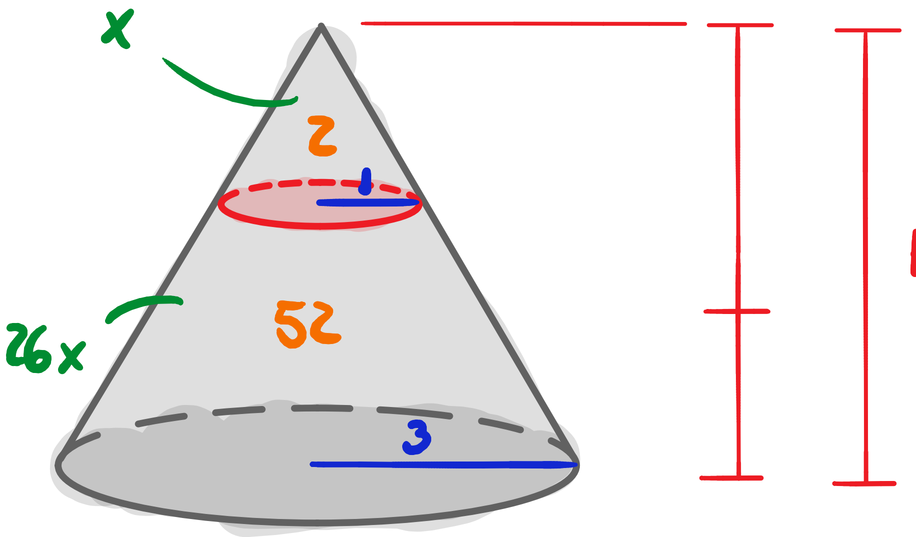
CALCULE O VOLUME E AREA LATERAL DO TRONCO GERADO A PARTIR DO CONE RETO ABAIXO. ..igual pg 36 piramides..





EXEMPLO

UM TRONCO DE CONE TEM BASES COM RAIOS 1 E 3. CALCULE A ALTURA DO CONE QUE DEU ORIGEM AO TRONCO SABENDO QUE O VOLUME DESSE TRONCO É 52.



$V_{TOTAL} = 54$

$$K = \frac{3}{1} \rightarrow \underline{K = 3} \rightarrow \underline{K^3 = 3^3 = 27}$$

$$V = \frac{1}{3}\pi R^2 H \rightarrow 54 = \frac{1}{\cancel{3}}\pi \cdot 3^{\cancel{2}} \cdot H$$

$$18 = \pi \cdot H \rightarrow \underline{H = \frac{18}{\pi}}$$