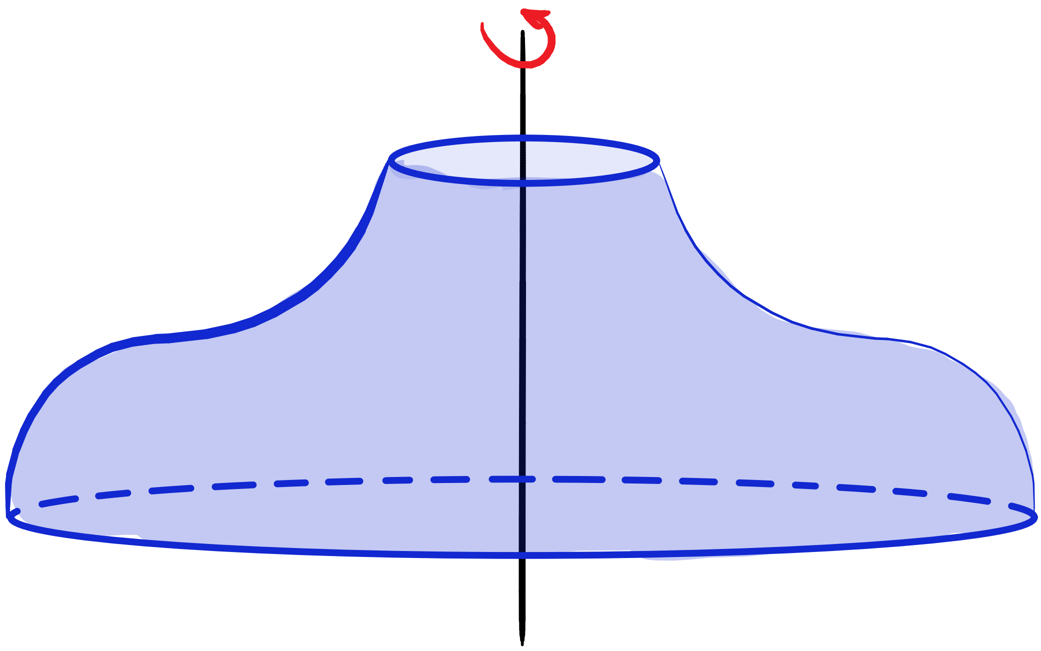
SÓLIDOS E SUPERFÍCIES DE REVOLUÇÃO

SUPERFÍCE DE REVOLUÇÃO

SUPERFÍCIE GERADA PELA ROTAÇÃO DE UMA CURVA EM TORNO DE UM EIXO.



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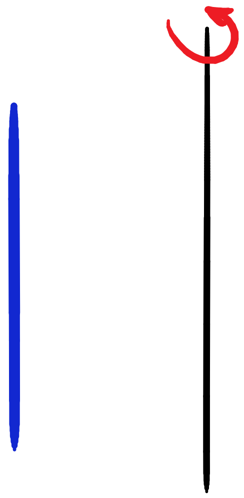

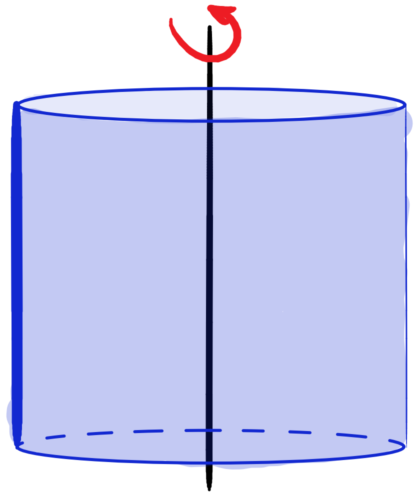

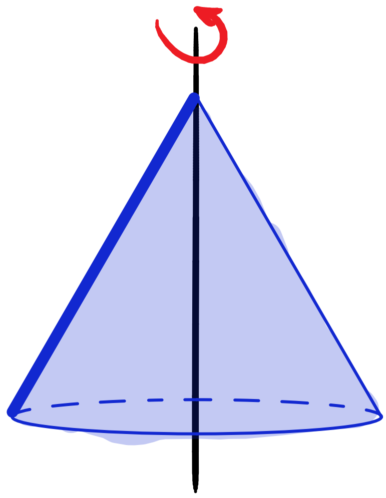

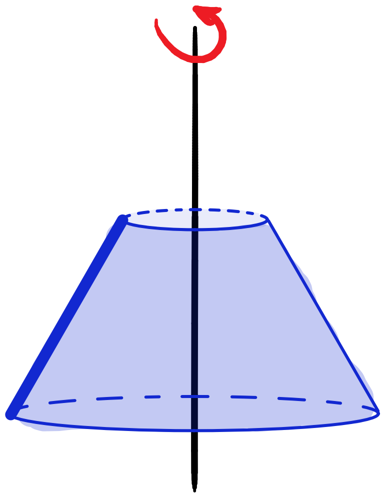

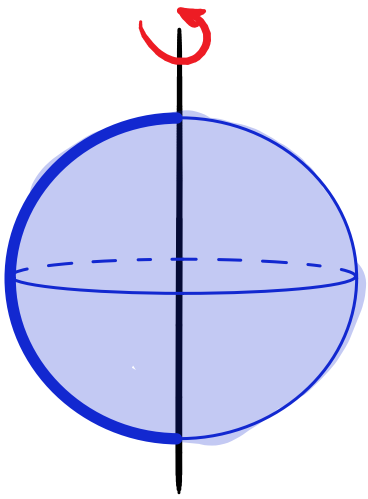

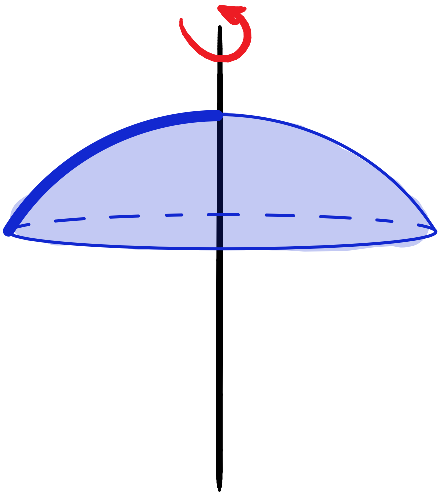

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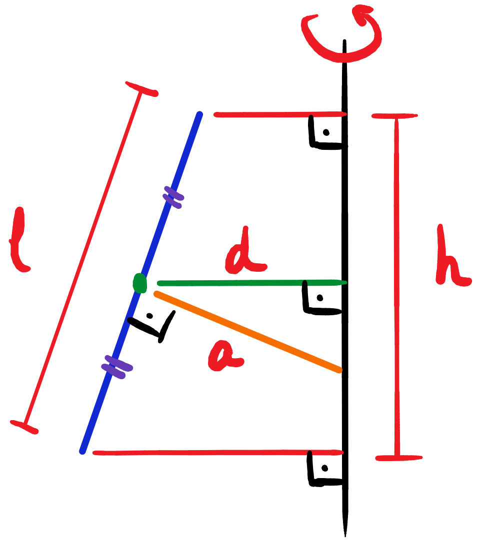
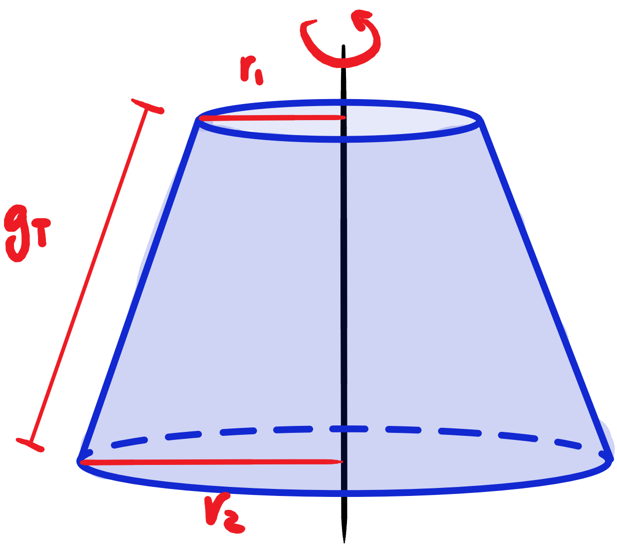
REVOLUÇÃO DE UM SEGMENTO





$$A = \pi . (r_1 + r_2) g_T$$

ÁREA
$$A = 2\pi l d$$
$$A = 2\pi a h$$

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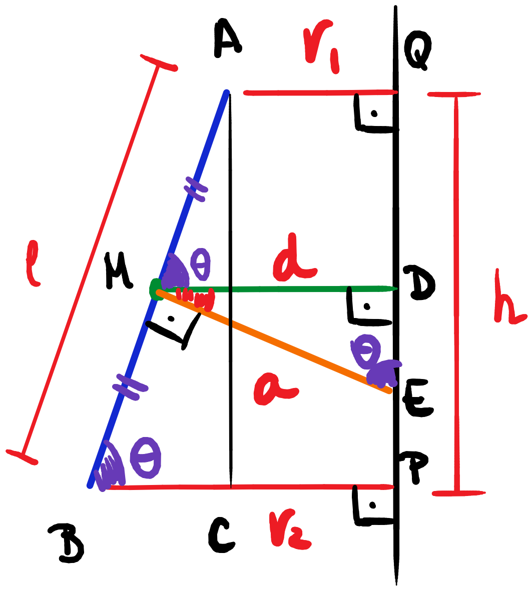

$$\Delta ABC \sim \Delta MED$$

$$\frac{\ell}{a} = \frac{h}{d}$$

$$\ell d = ah$$

* $\overline{MD}$: BASE MÉDIA DE ABPQ.

$$d = \frac{r_1 + r_2}{2} \rightarrow r_1 + r_2 = 2d$$

* ÁREA LATERAL TRONCO DE CONE:

$$A = \pi . (r_1 + r_2) g_T$$

$$A = \pi . 2d . \ell \rightarrow \boxed{A = 2\pi \ell d} \quad \boxed{A = 2\pi ah}$$

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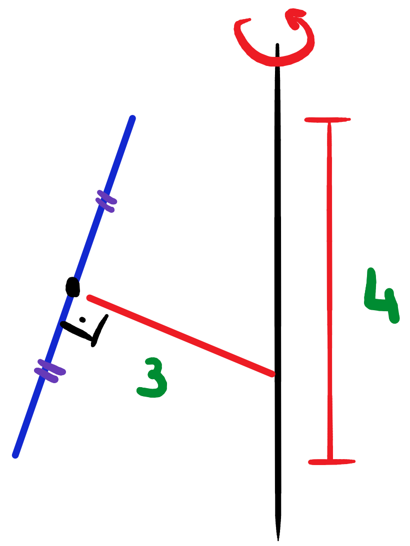
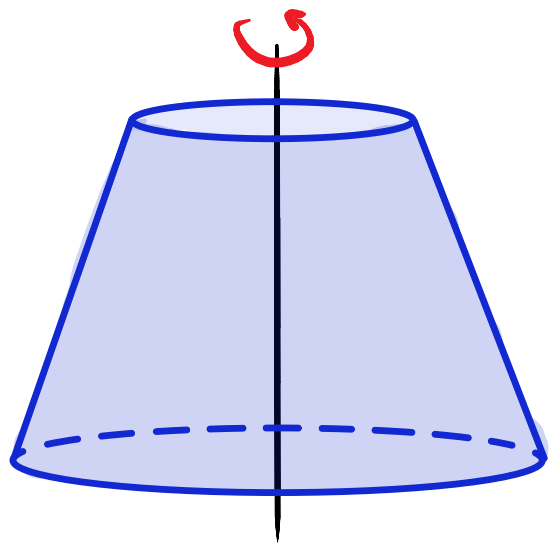
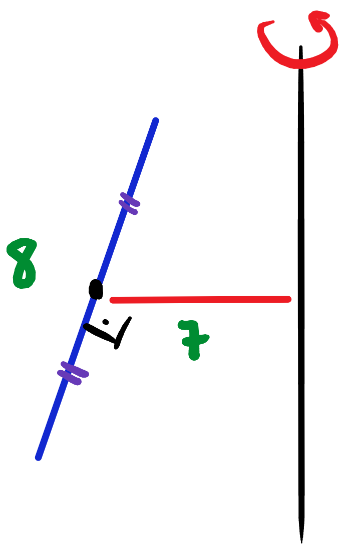
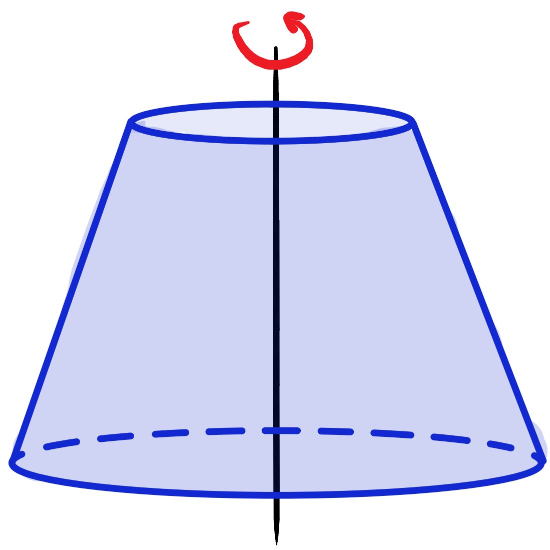




$$A = 2\pi a h = 2\pi . 3 . 4 \rightarrow \underline{A = 24\pi}$$





$$A = 2\pi \ell d = 2\pi . 8 . 7 \rightarrow \underline{A = 112\pi}$$

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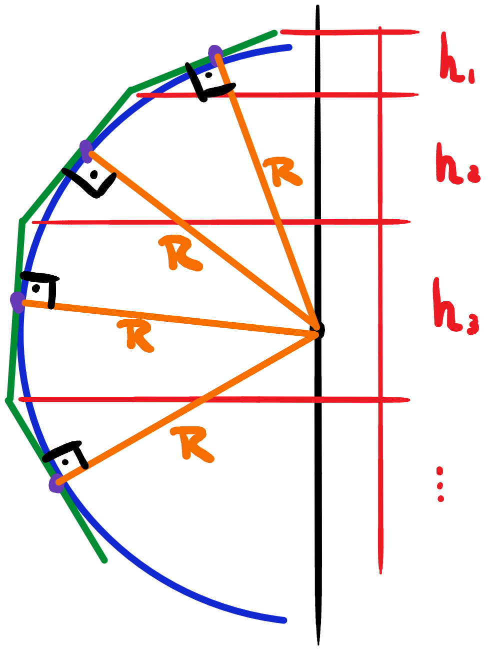
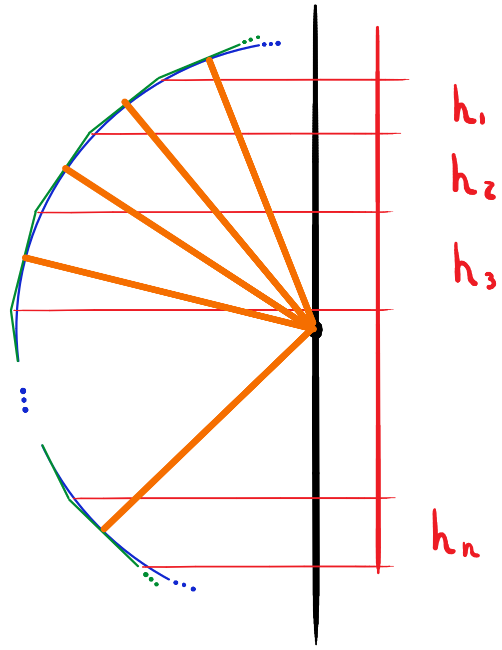
CALOTA ESFÉRICA





$$A_c = A_1 + A_2 + A_3 + \dots + A_n$$

$$= 2\pi R h_1 + 2\pi R h_2 + 2\pi R h_3 + \dots + 2\pi R h_n$$

$$= 2\pi R \underbrace{(h_1 + h_2 + h_3 + \dots + h_n)}_{H}$$

$$\boxed{A_c = 2\pi R H}$$

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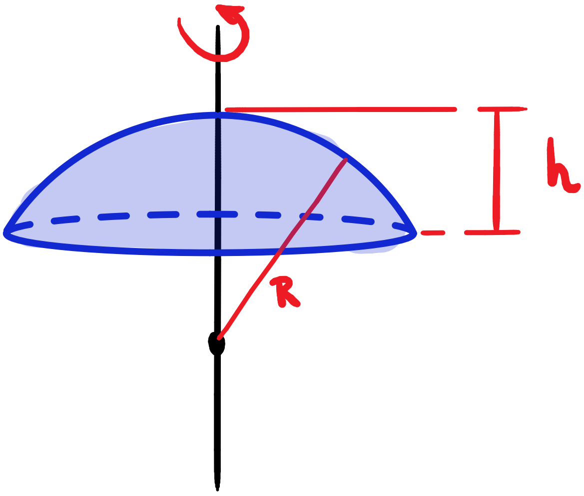
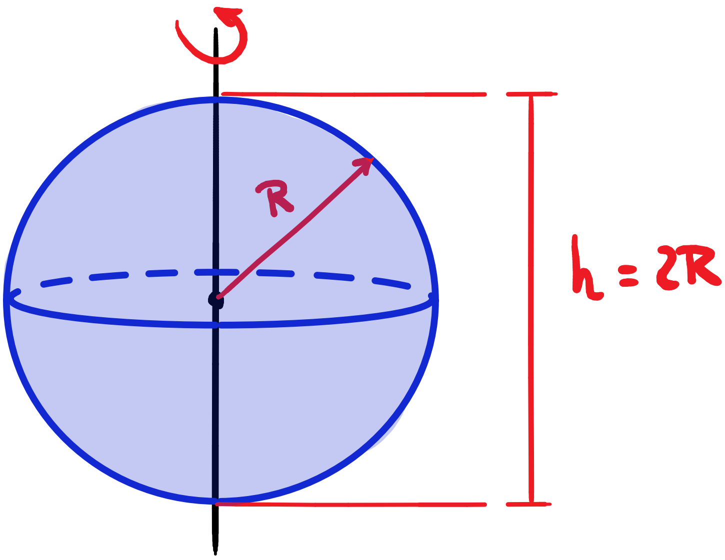


$$\boxed{A = 2\pi R h}$$



$$A = 2\pi R h$$

$$A = 2\pi R \cdot 2R$$

$$\boxed{A = 4\pi R^2}$$

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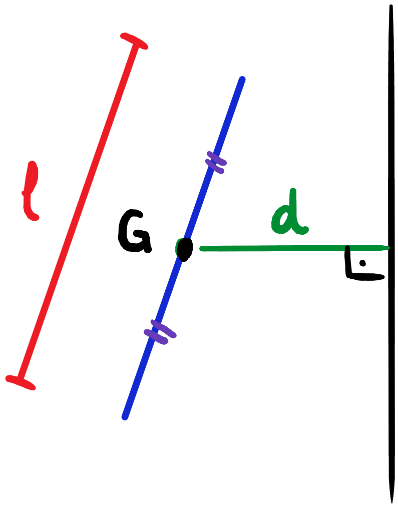
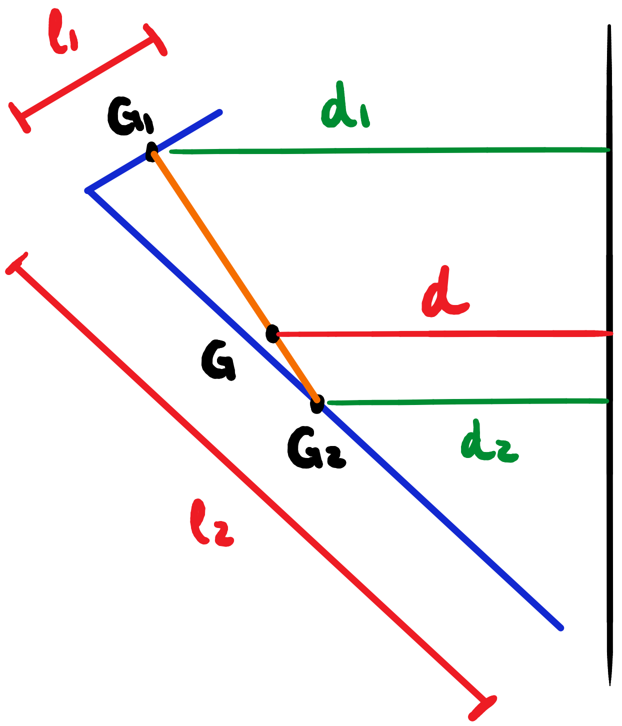
CENTRO DE GRAVIDADE (G)

1 SEGMENTO



POLIGONAL - 2 SEGMENTOS

DENSIDADE LINEAR: $\rho \left({}^{um}/_{uc} \right)$



MOMENTOS: $M = M_1 + M_2$

$$\cancel{\rho}(l_1 + l_2) \cdot d = \cancel{\rho} \cdot l_1 \cdot d_1 + \cancel{\rho} \cdot l_1 \cdot d_2$$

$$\boxed{d = \frac{l_1 d_1 + l_2 d_2}{l_1 + l_2}}$$

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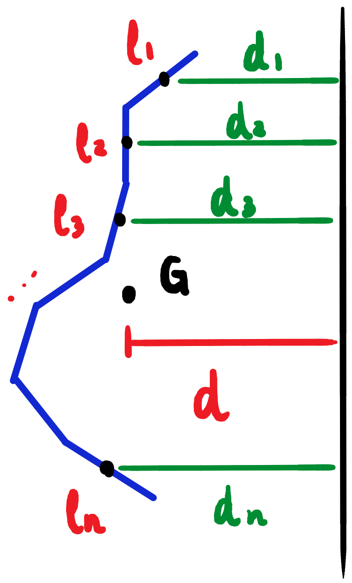
POLIGONAL - n SEGMENTOS



$$M = M_1 + M_2 + \dots + M_n$$

$$\cancel{\rho}(l_1 + l_2 + \dots + l_n) \cdot d = \cancel{\rho} \cdot l_1 \cdot d_1 + \cancel{\rho} \cdot l_2 \cdot d_2 + \dots + \cancel{\rho} \cdot l_n \cdot d_n$$

$$d = \frac{l_1 d_1 + l_2 d_2 + \dots + l_n d_n}{l_1 + l_2 + \dots + l_n}$$

$$\boxed{d = \frac{l_1 d_1 + l_2 d_2 + \dots + l_n d_n}{l}}$$

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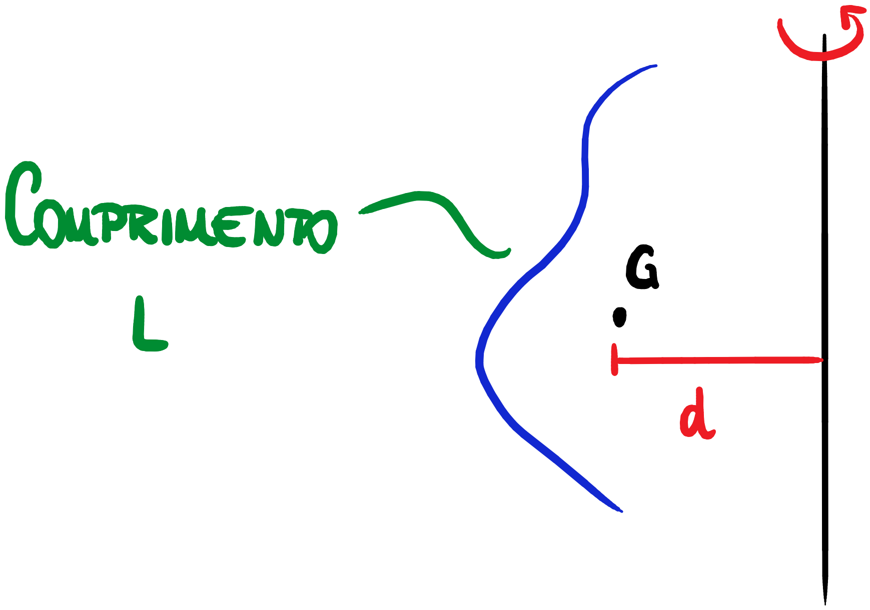
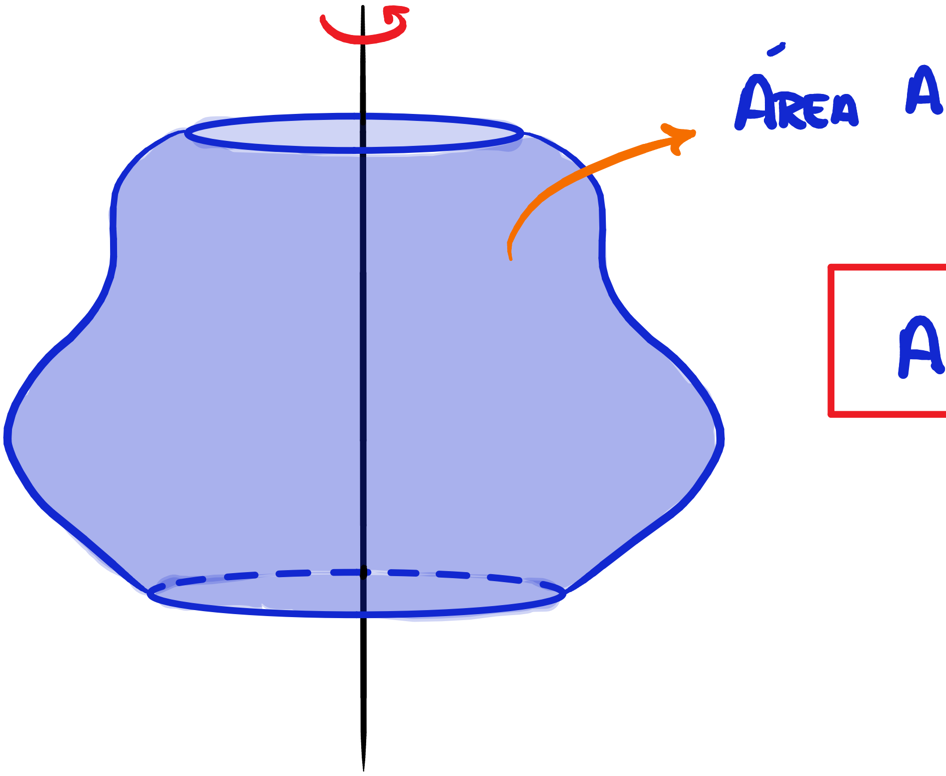
TEOREMA - PAPPUS GULDIN #1





$$\boxed{A = 2\pi \, l \, d}$$

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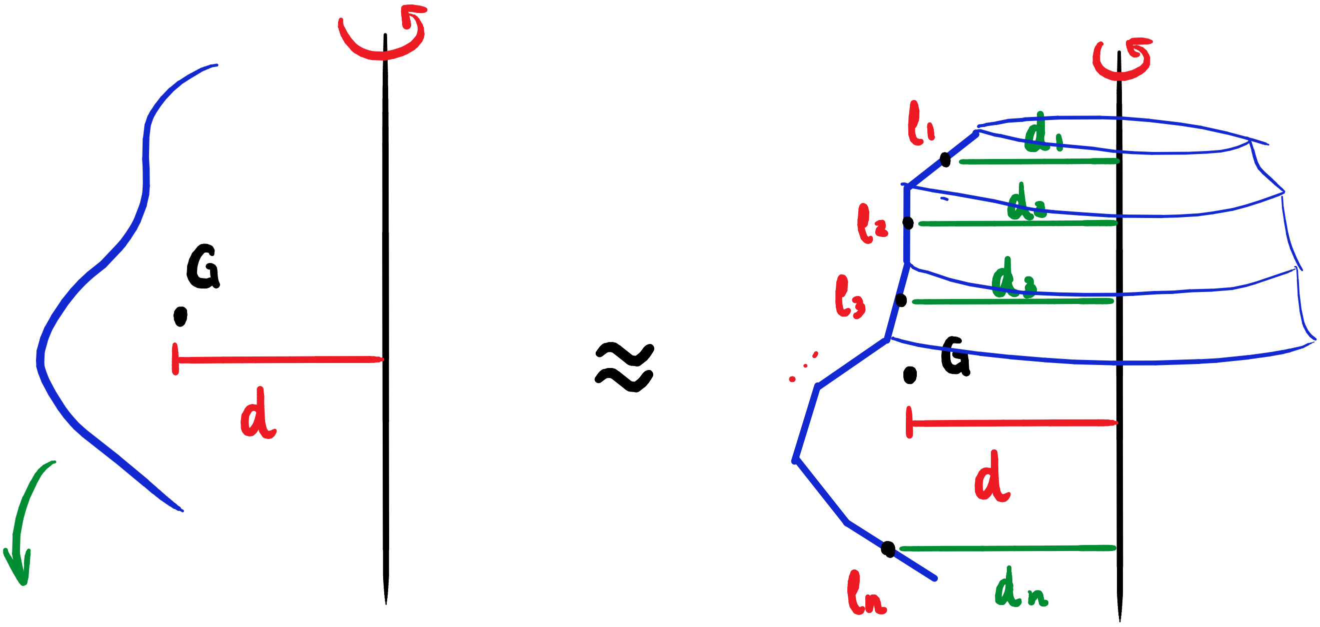

COMPRIMENTO l

$$A = A_1 + A_2 + A_3 + \dots + A_n$$

$$= 2\pi l_1 d_1 + 2\pi l_2 d_2 + \dots + 2\pi l_n d_n$$

$$= 2\pi (l_1 d_1 + l_2 d_2 + \dots + l_n d_n)$$

$$= 2\pi\, l . d$$

$$\boxed{A = 2\pi\, l\, d}$$

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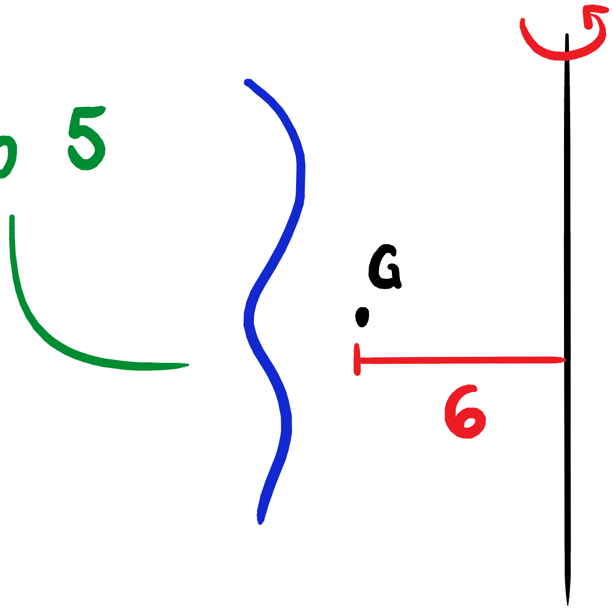
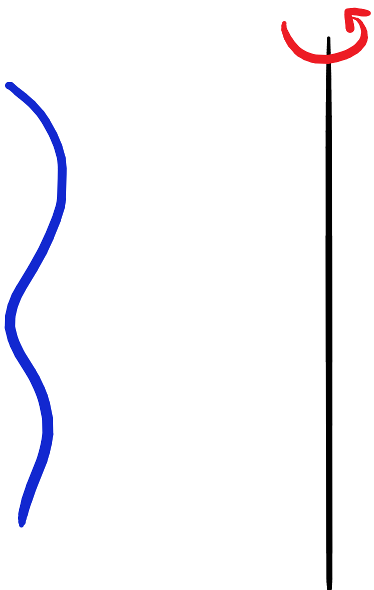
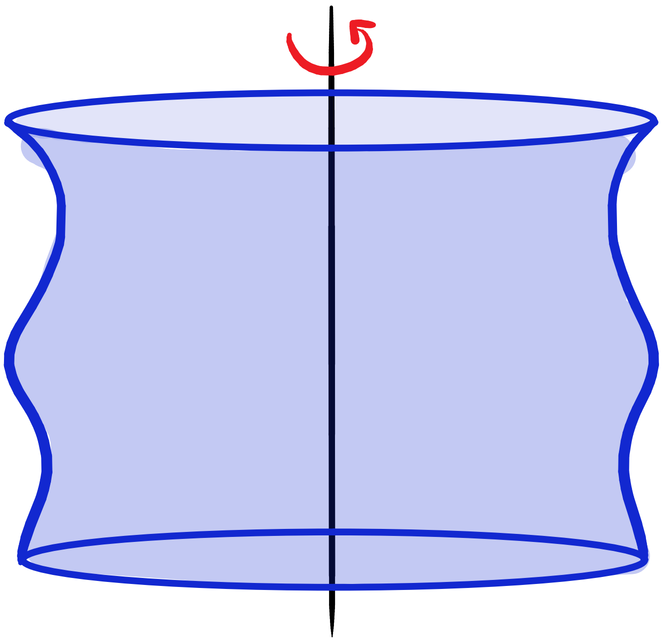
COMPRIMENTO 5







$$A = 2\pi \ell d$$

$$A = 2\pi\, 5 . 6 \rightarrow \boxed{A = 60\pi}$$

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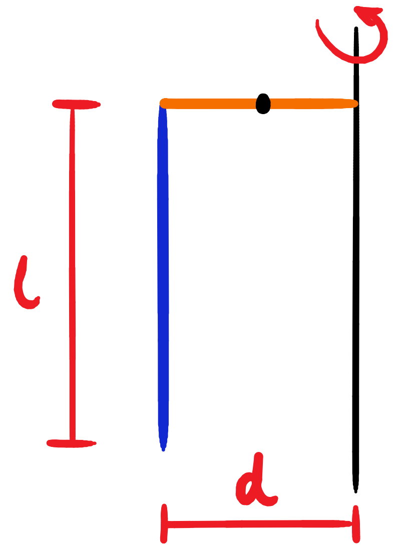
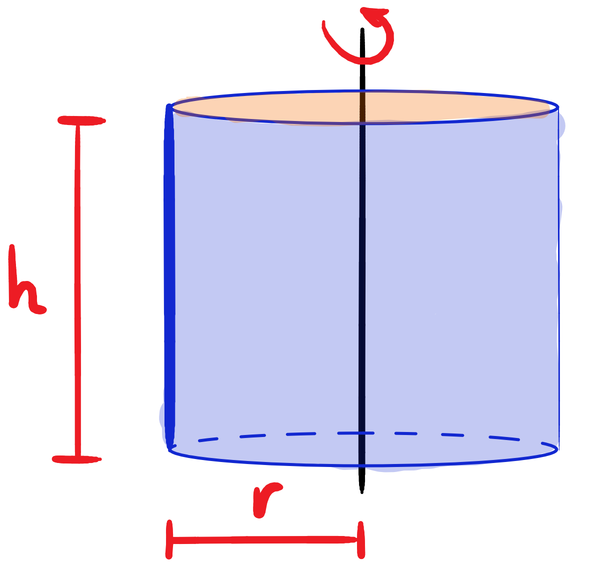
EXEMPLO

DEMONSTRE AS FÓRMULAS DA ÁREA DA BASE E DA ÁREA LATERAL DE UM CILINDRO RETO.





$$A_l = 2\pi\, l\, d$$

$$A_l = 2\pi\, h\, r$$

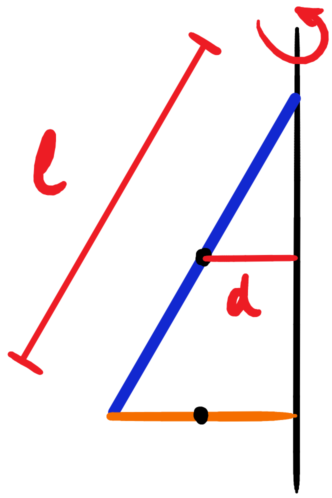
$$\underline{A_l = 2\pi\, r\, h}$$

$$A_b = \cancel{2}\pi \cdot r \cdot \frac{r}{\cancel{2}}$$

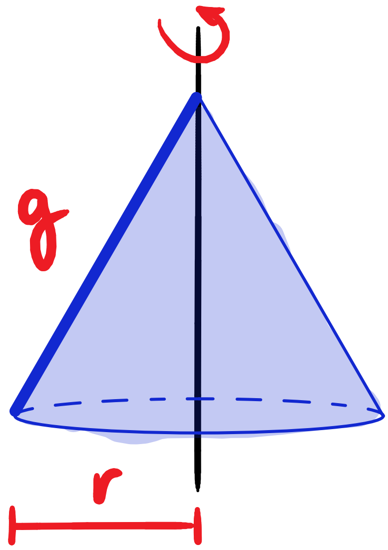
$$\boxed{A_b = \pi r^2}$$

EXEMPLO

DEMONSTRE AS FÓRMULAS DA ÁREA LATERAL E DA ÁREA DA BASE DE UM CONE RETO.



$$d = \frac{r}{2}$$



$$A_b = 2\pi r \cdot \frac{r}{2}$$

$$\underline{A_b = \pi r^2}$$

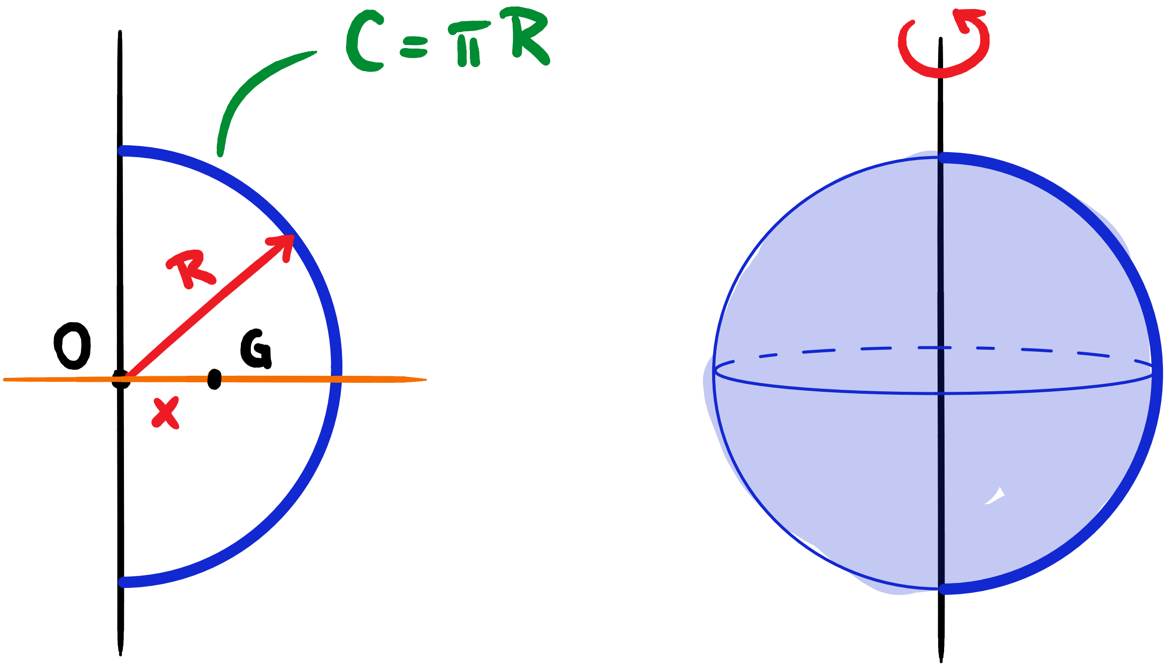
$$A_L = 2\pi g \cdot \frac{r}{2}$$

$$\underline{A_L = \pi r g}$$

EXEMPLO

SEJA UMA SEMICIRCUNFERÊNCIA DE RAIO R.

CALCULE A DISTÂNCIA ENTRE O CENTRÓIDE E O CENTRO DESSA SEMICIRCUNFERÊNCIA.



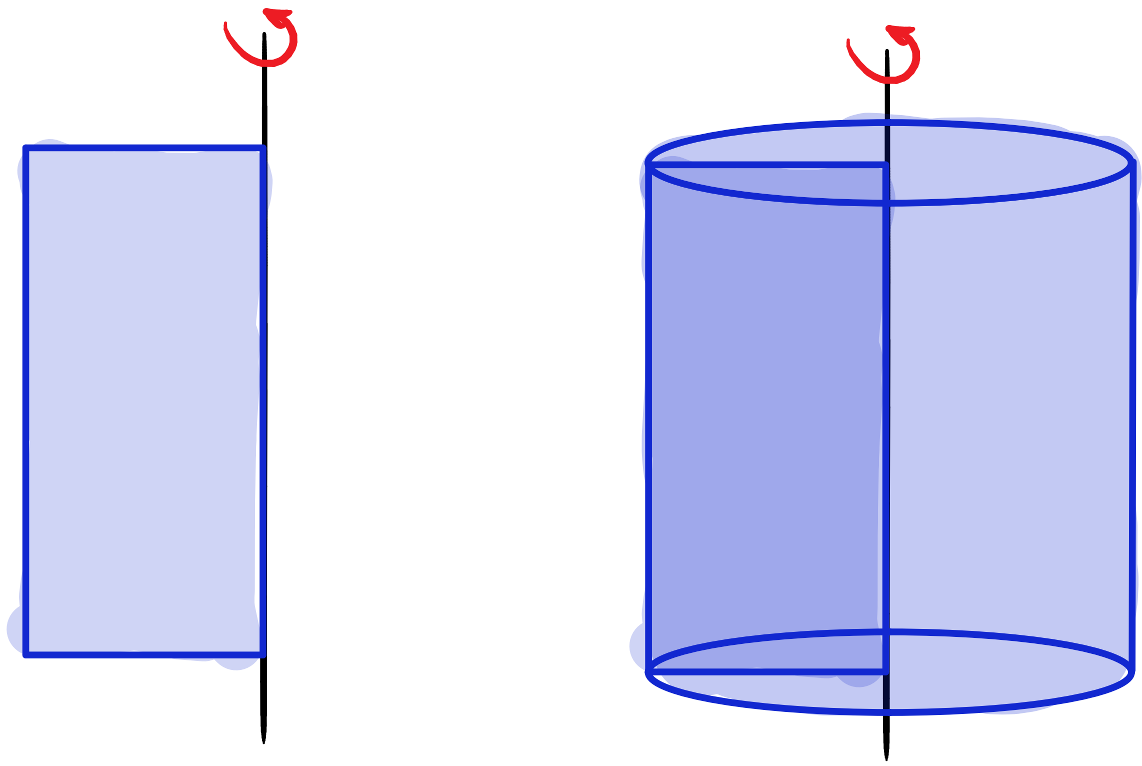
$$A = 2\pi \cdot \ell \cdot d$$

$$^{2}\cancel{4}\cancel{\pi}R^{\cancel{2}\,1} = \cancel{2}\cancel{\pi} \cdot \pi \cancel{R} \cdot x$$

$$x = \frac{2R}{\pi}$$

SÓLIDO DE REVOLUÇÃO

SÓLIDO GERADO PELA ROTAÇÃO DE UMA FIGURA PLANA EM TORNO DE UM EIXO.



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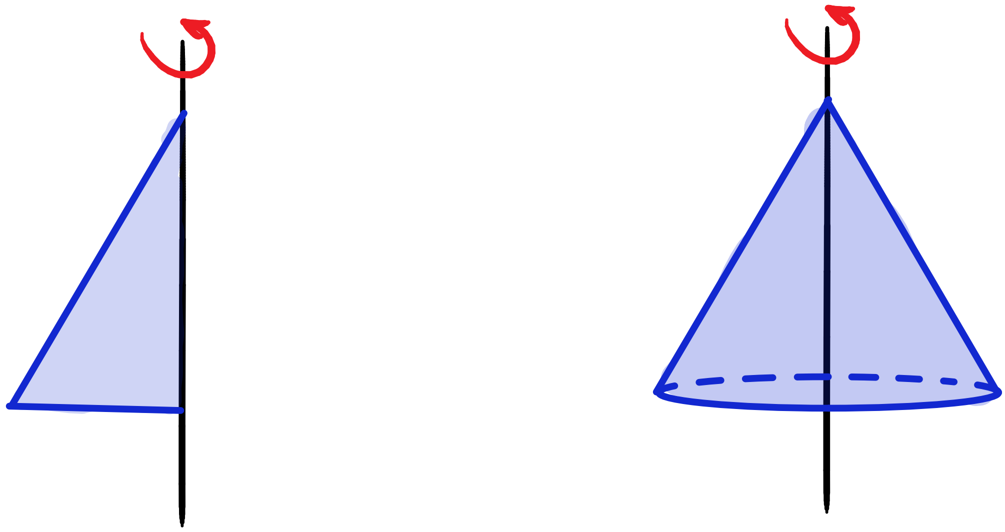

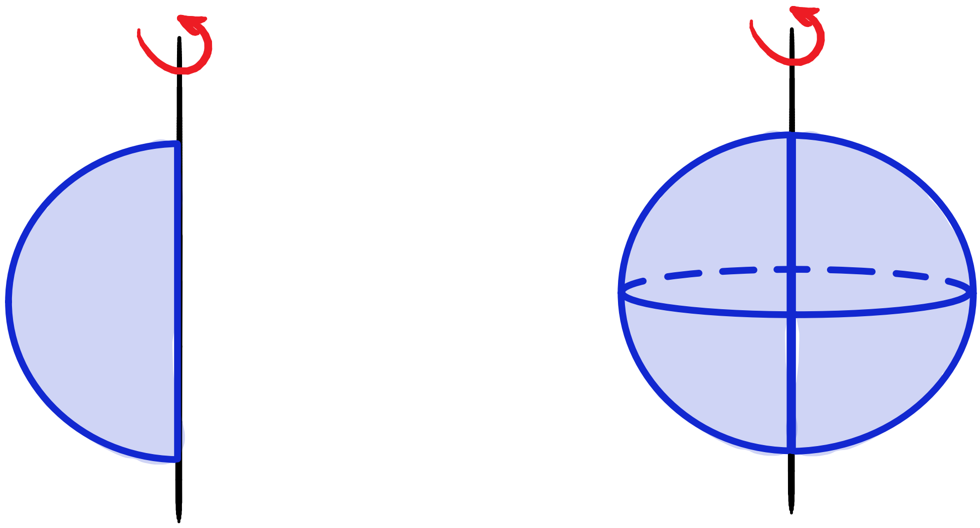

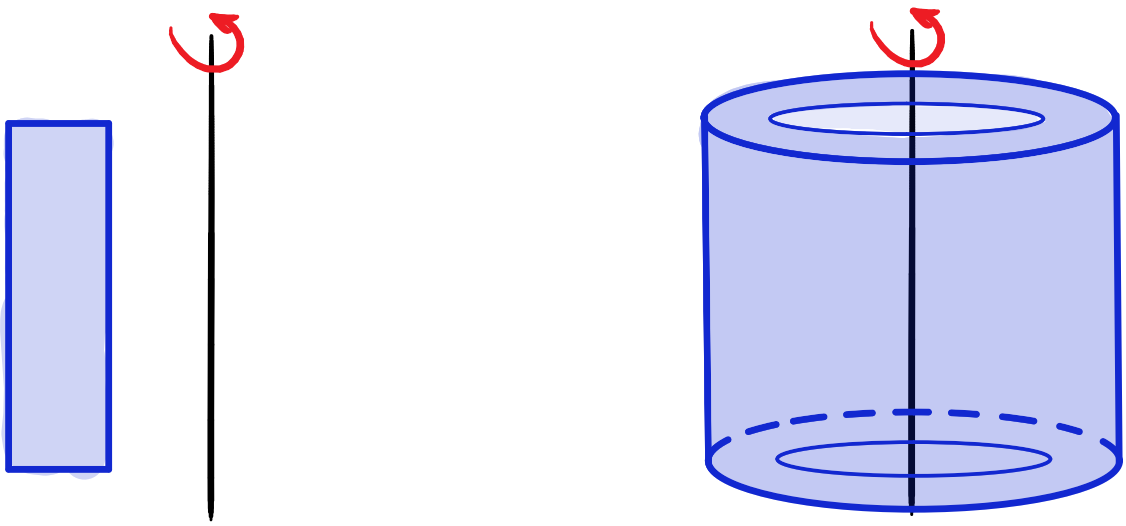

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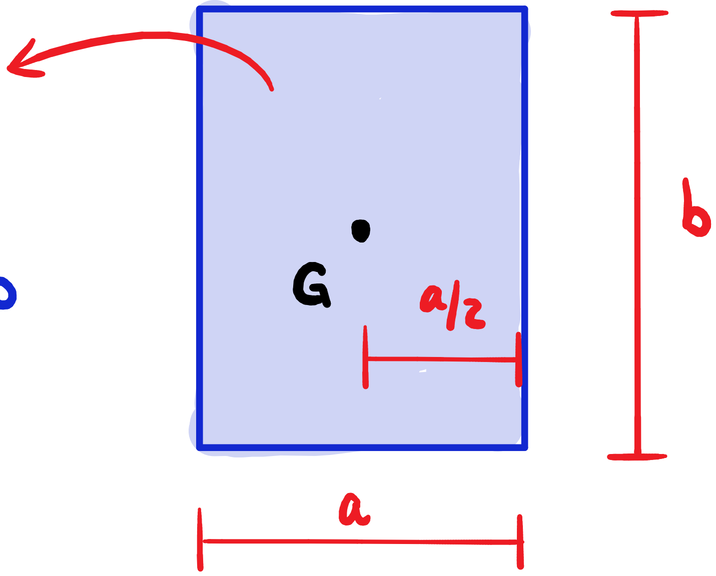
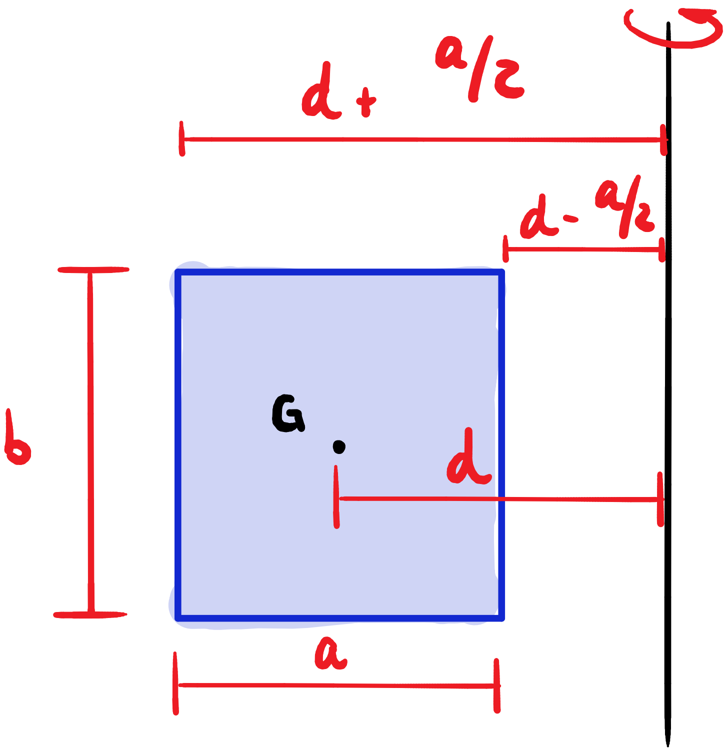
ROTAÇÃO DO RETÂNGULO

ÁREA

$A = ab$





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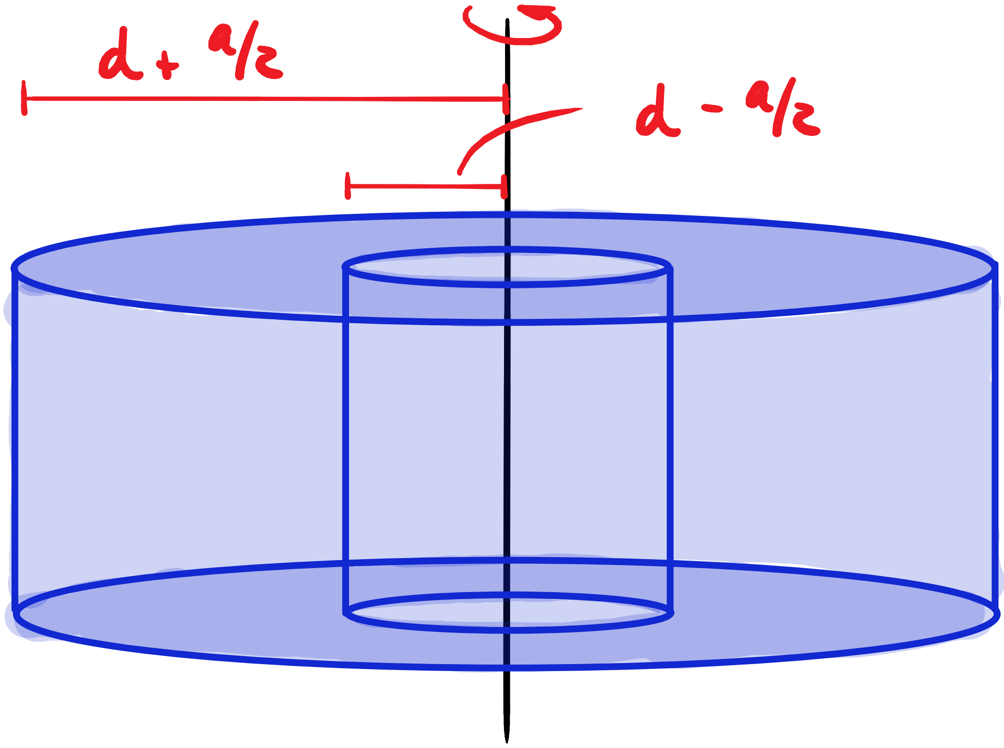

$$V_{SOL} = V_{CIL.G} - V_{CIL.P}$$

$$= \pi\left(d + \frac{a}{2}\right)^2 \cdot b - \pi\left(d - \frac{a}{2}\right)^2 \cdot b$$

$$= \pi b\left[\left(d + \frac{a}{2}\right)^2 - \left(d - \frac{a}{2}\right)^2\right]$$

$$= \pi b \cdot (2d \cdot a) = 2\pi \underbrace{ab}_{A} d$$

$$\boxed{V_s = 2\pi A d}$$

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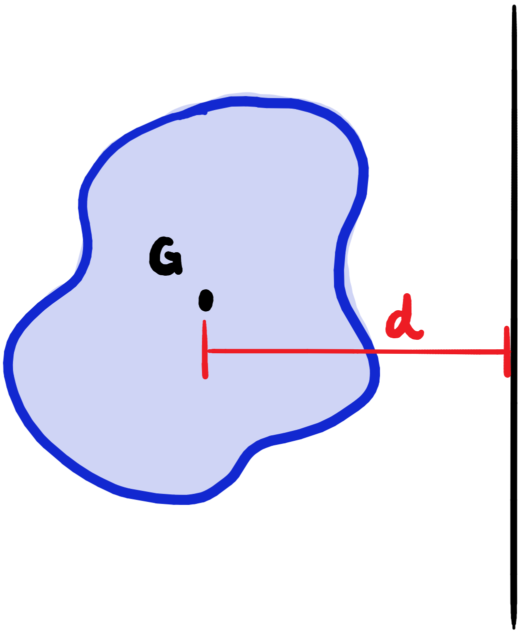
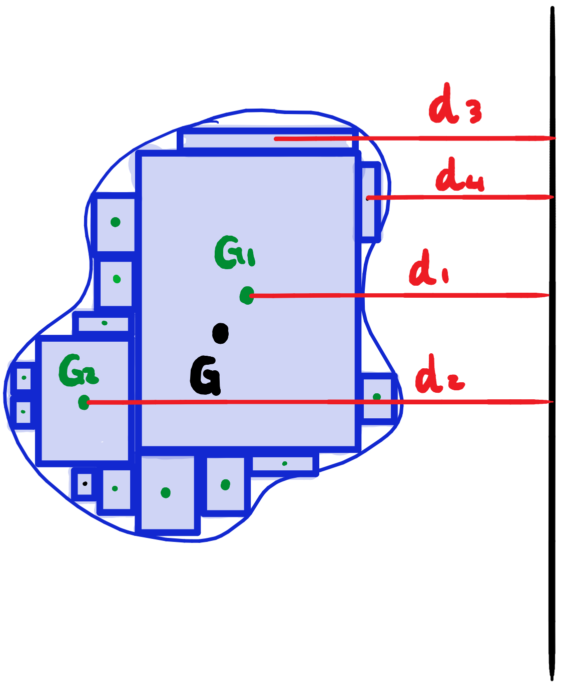
CENTRO DE GRAVIDADE



$$M = A.d$$



$$M = A_1 d_1 + A_2 d_2 + A_3 . d_3 + \dots$$

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$$Ad = A_1 d_1 + A_2 d_2 + A_3 d_3 + \dots + A_n d_n$$

$$\boxed{d = \frac{A_1 d_1 + A_2 d_2 + A_3 d_3 + \dots + A_n d_n}{A}}$$

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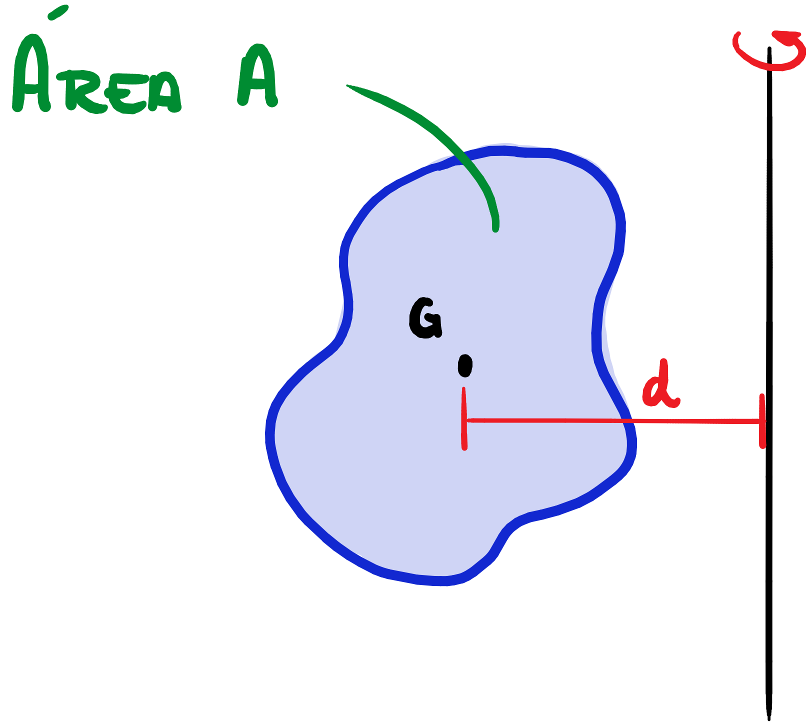
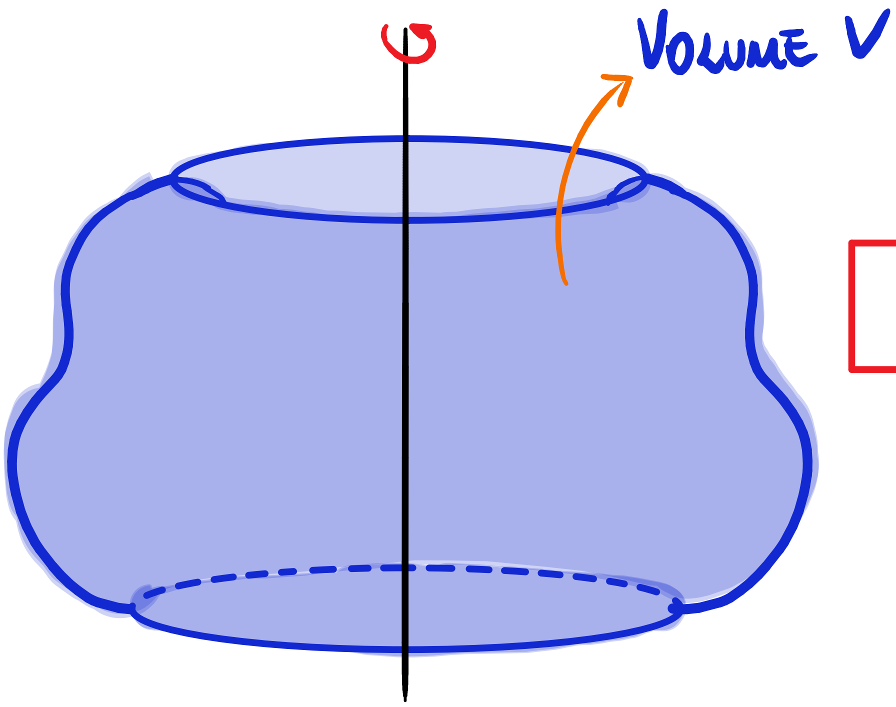
TEOREMA - PAPPUS GULDIN #2





$$\boxed{V = 2\pi A d}$$

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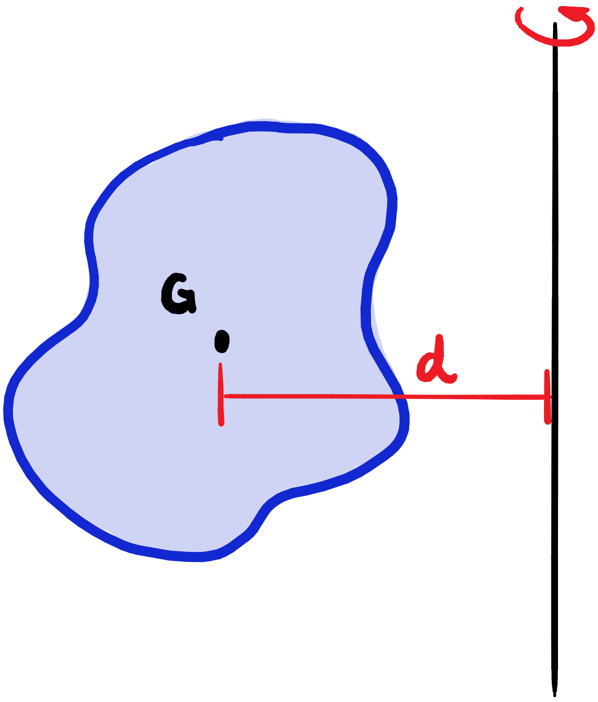

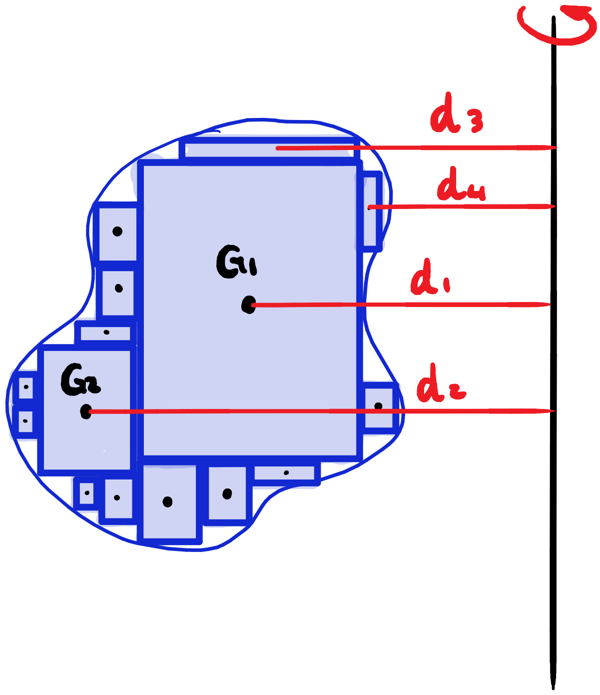

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$= 2\pi A_1 d_1 + 2\pi A_2 d_2 + \dots + 2\pi A_n d_n$$

$$= 2\pi \underbrace{(A_1 d_1 + A_2 d_2 + \dots + A_n d_n)}_{Ad}$$

$$\boxed{V = 2\pi A d}$$

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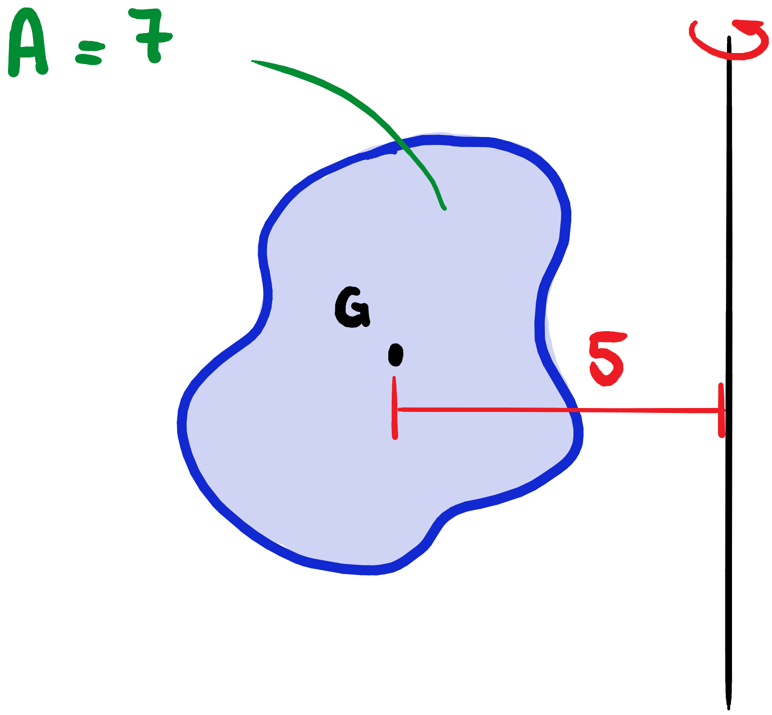

$$V = 2\pi \cdot 7 \cdot 5$$

$$\boxed{V = 70\pi}$$

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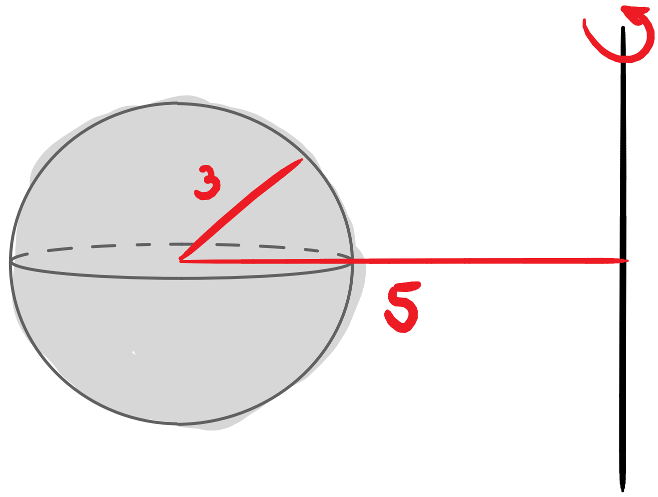
EXEMPLO

CALCULE O VOLUME DO SÓLIDO GERADO PELA ROTAÇÃO DE UM CÍRCULO DE RAIO 3 EM TORNO DE UM EIXO QUE ESTÁ A 5 DE DISTÂNCIA DO CENTRO DESSE CÍRCULO.



$$A = \pi R^2$$

$$A = \pi . 3^2$$

$$A = 9\pi$$

$$V = 2\pi A d$$

$$V = 2\pi . 9\pi . 5$$

$$\boxed{V = 90\pi^2}$$

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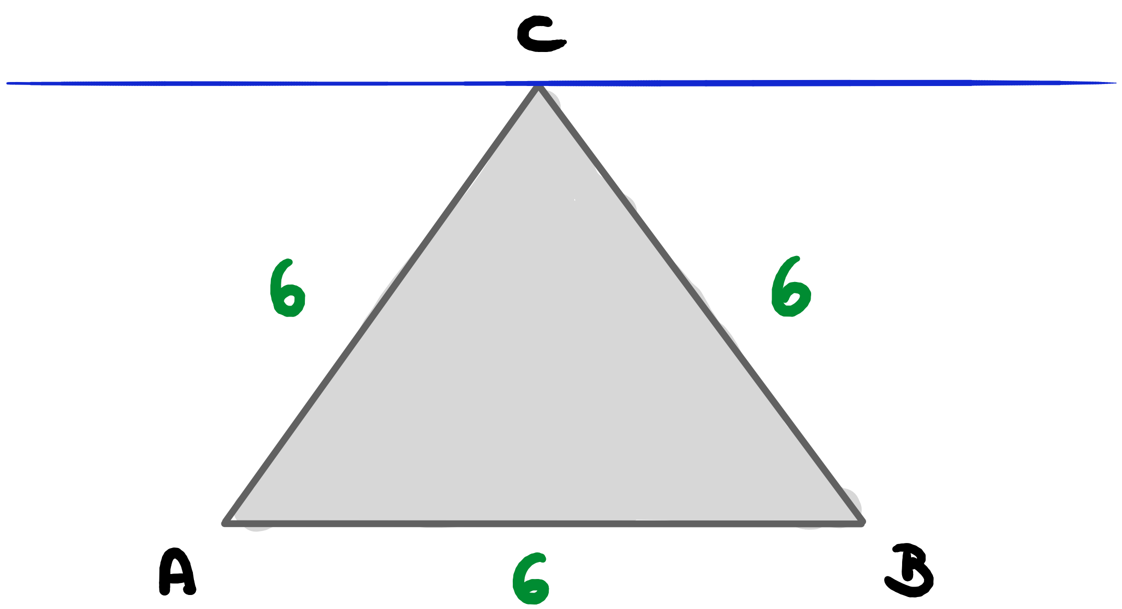
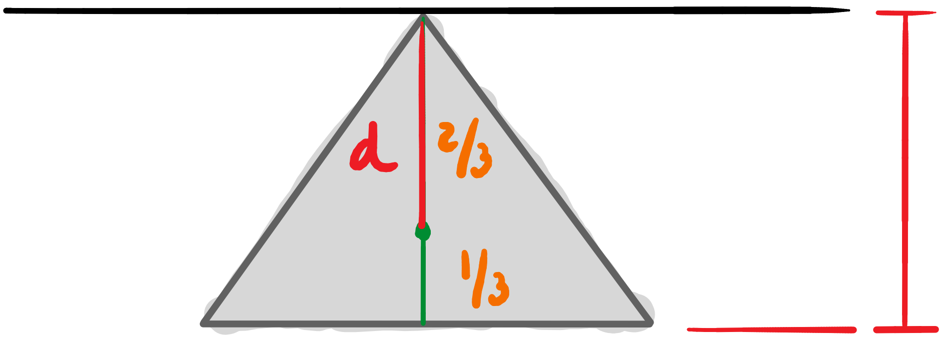
EXEMPLO

SEJA O TRIÂNGULO EQUILÁTERO ABC DE LADO 6. CALCULE O VOLUME DO SÓLIDO GERADO PELA ROTAÇÃO DESSE TRIÂNGULO EM TORNO DA RETA r, QUE PASSA POR C E É PARALELA A AB.





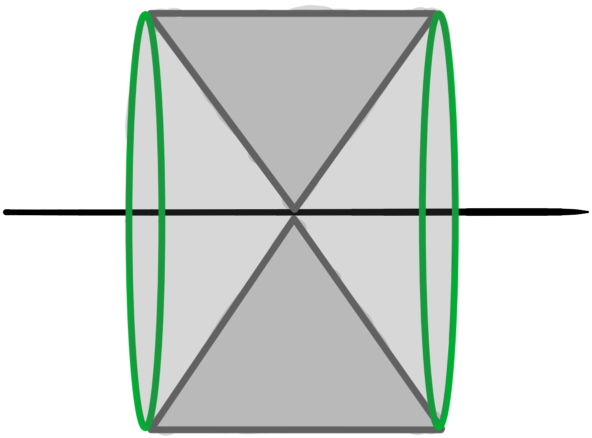
$$h = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$$

$$A = \frac{6^2\sqrt{3}}{4} = \underline{9\sqrt{3}} \; ; \quad d = \frac{2}{3}\,3\sqrt{3}$$

$$\underline{d = 2\sqrt{3}}$$

$$V = 2\pi A d \rightarrow V = 2\pi\, 9\sqrt{3} \cdot 2\sqrt{3}$$

$$\boxed{V = 108\pi}$$



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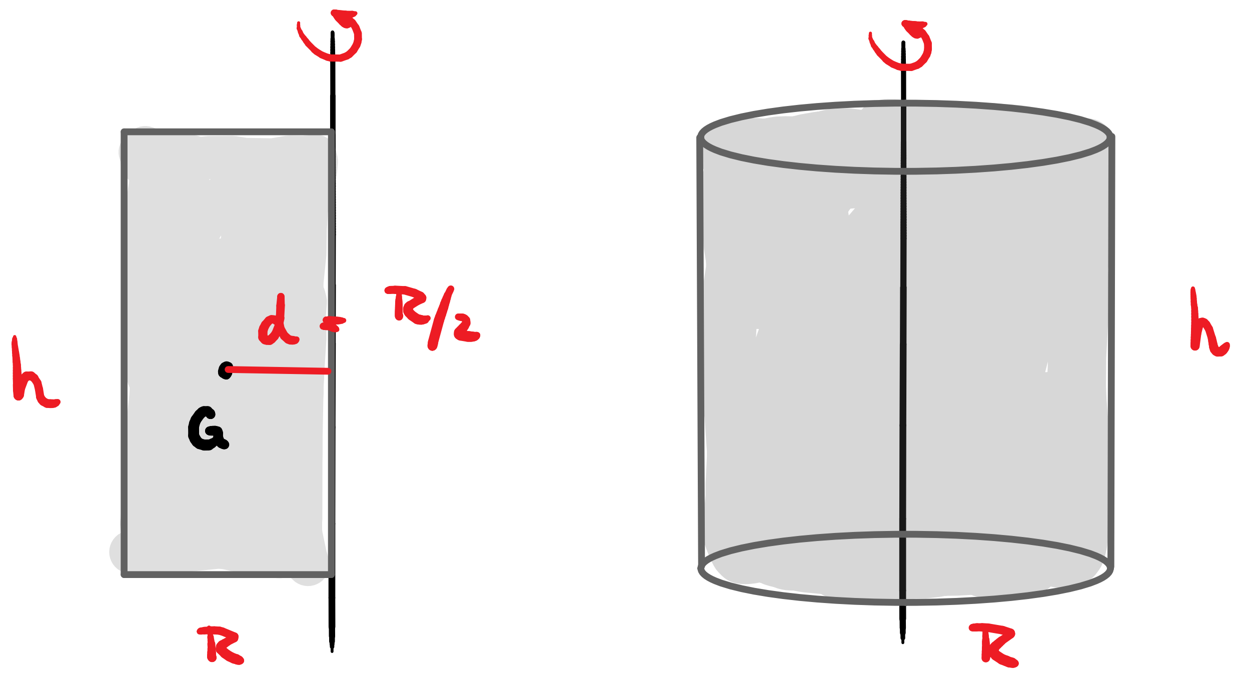
EXEMPLO

DEMONSTRE A FÓRMULA DO VOLUME DE UM CILINDRO CIRCULAR RETO.



$A = Rh \quad ; \quad d = R/2$

$V = \cancel{2}\pi Rh \cdot \frac{R}{\cancel{2}} \rightarrow \boxed{V = \pi R^2 h}$

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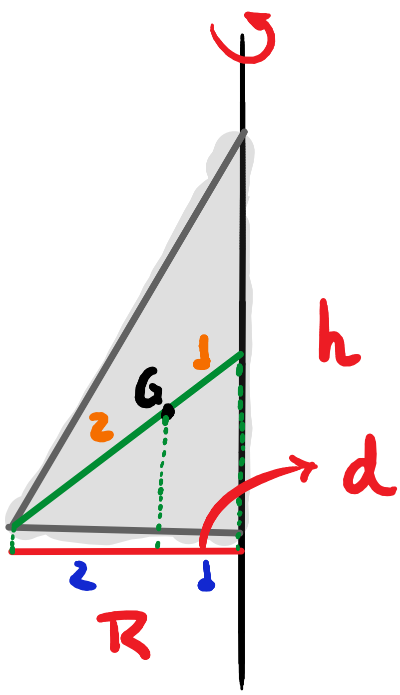
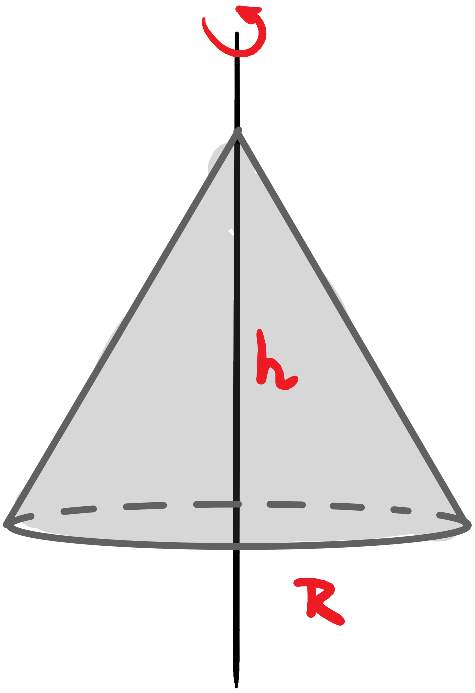
EXEMPLO

DEMONSTRE A FÓRMULA DO VOLUME DE UM CONE CIRCULAR RETO.





$$A = \frac{1}{2} \cdot R \cdot h \qquad d = \frac{1}{3} R$$

$$A = \frac{Rh}{2}$$

$$V = \cancel{2}\pi \cdot \frac{Rh}{\cancel{2}} \cdot \frac{1}{3} R \rightarrow \boxed{V = \frac{1}{3} \pi R^2 h}$$

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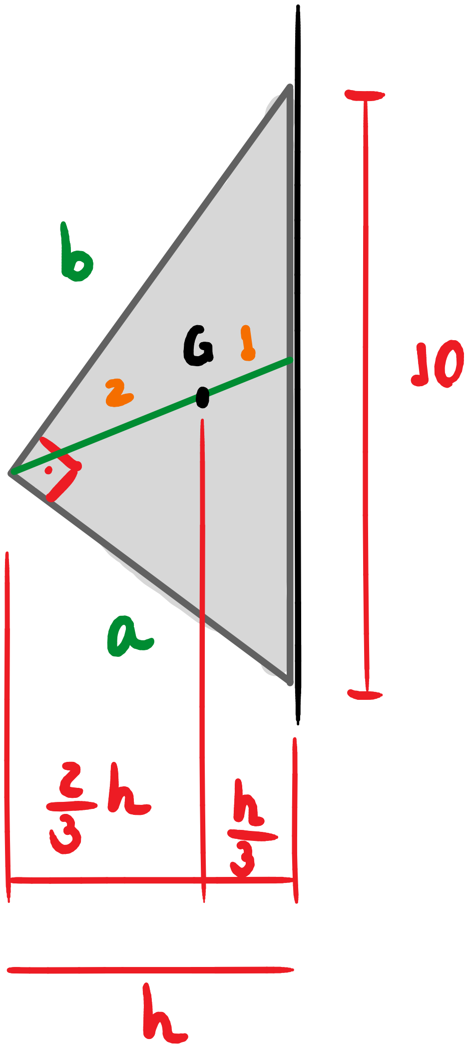
EXEMPLO

UM TRIÂNGULO RETÂNGULO TEM HIPOTENUSA 10. O VOLUME DO SÓLIDO GERADO PELA ROTAÇÃO DESSE TRIÂNGULO EM TORNO DA HIPOTENUSA É 30π. CALCULE O PERÍMETRO DESSE TRIÂNGULO.



$$a + b + 10 = ?$$

$$A = \frac{1}{2}ab$$

$$A = \frac{1}{2} \cdot 10 \cdot h$$

$$V = 2\pi A d$$

$$3\cancel{0}\cancel{\pi} = \cancel{2}\cancel{\pi} \cdot \cancel{5}h \cdot \frac{h}{3}$$

$$h^2 = 9$$

$$\underline{h = 3}$$

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$$ab = 10h$$

$$ab = 30$$

$$a^2 + b^2 = 10^2$$

$$a^2 + 2ab + b^2 = 100 + 2 . 30$$

$$(a+b)^2 = 160$$

$$a + b = \sqrt{160}$$

$$a + b = 4\sqrt{10}$$

PERIMÉTRO : $4\sqrt{10} + 10$

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