EQUAÇÕES TRIGONOMÉTRICAS

· Tipo 01 : $\sum_{n} \cos(nx) + \sum_{m} \operatorname{sen}(mx) = 0$ ·

$\operatorname{sen}(5x) + \operatorname{sen}(2x) = 0$

$\operatorname{sen}(x) + \cos(7x) = 0$

$\operatorname{sen}(3x) + \cos(2x) = 3\operatorname{sen}(6x)$

EXEMPLO 01

Resolva a equação $\operatorname{sen}(2x) + \cos(4x) = 0$

R :

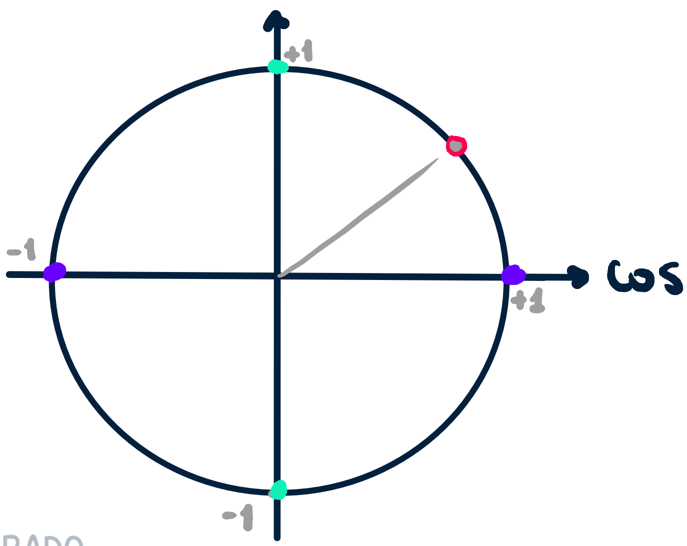
SOLUÇÃO 01

$$\operatorname{sen}(2x) + \cos(4x) = 0 \quad \therefore \quad \operatorname{sen}(2x) + \operatorname{sen}\left(\pi/2 - 4x\right) = 0$$

$$2 \cdot \operatorname{sen}\left(\frac{\pi}{4} - x\right)\cos\left(3x - \pi/4\right) = 0$$

(i) $\operatorname{sen}\left(\frac{\pi}{4} - x\right) = 0 \Rightarrow \frac{\pi}{4} - x = k\pi$

$$\boxed{x = \pi/4 - k\pi}$$



(ii) $\cos\left(3x - \pi/4\right) = 0$

$$3x - \pi/4 = \pi/2 + k\pi$$

$$\boxed{x = \frac{\pi}{4} + \frac{k\pi}{3}}$$

SOLUÇÃO 02

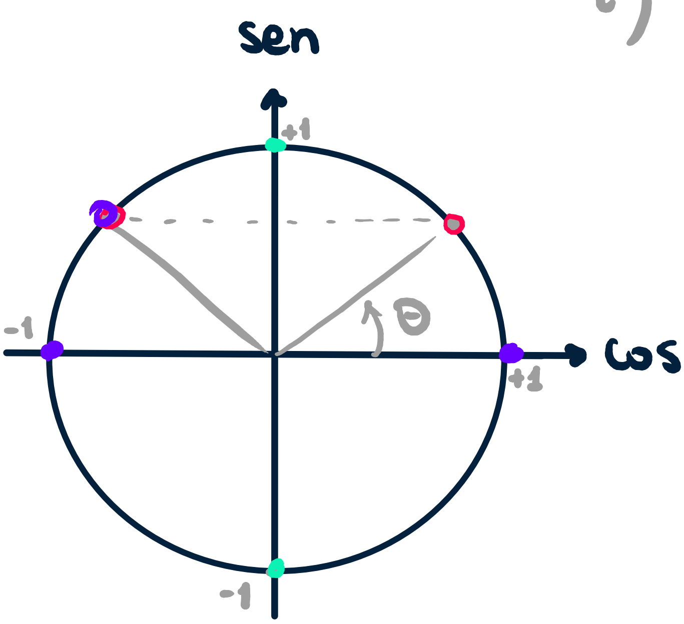
$$\text{sen}(2x) + \cos(4x) = 0 \quad \therefore \quad \text{sen}(2x) = -\cos(4x)$$

$$\text{sen}(2x) = -\,\text{sen}\left(\frac{\pi}{2} - 4x\right) \quad \therefore \text{sen}(2x) = \text{sen}\left(4x - \frac{\pi}{2}\right)$$

i) $2x = \left(4x - \frac{\pi}{2}\right) + 2K\pi$

$$2x = \frac{\pi}{2} - 2K\pi$$

$$\boxed{x = \frac{\pi}{4} - K\pi}$$



ii) $2x = \pi - \left(4x - \frac{\pi}{2}\right) + 2K\pi$

$$2x + 4x = \frac{3\pi}{2} + 2K\pi$$

$$6x = \frac{3\pi}{2} + 2K\pi$$

$$\boxed{x = \frac{\pi}{4} + \frac{K\pi}{3}}$$

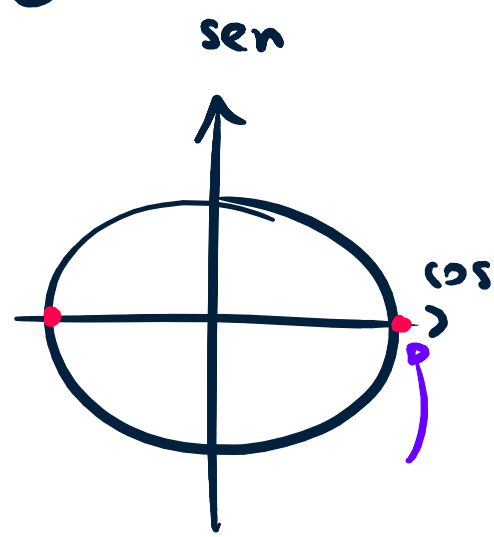
EXEMPLO 02

Resolva a equação $\text{sen}(2x) + \text{sen}(6x) = 2 \cdot \text{sen}(4x)$

R: $$\underbrace{\text{sen}(2x) + \text{sen}(6x)}_{2 \cdot \text{sen}(4x) \cdot \cos(2x)} = 2 \cdot \text{sen}(4x)$$

$$2\,\text{sen}(4x) \cdot \cos(2x) - 2\,\text{sen}(4x) = 0$$

$$2\,\text{sen}(4x)\left[\cos(2x) - 1\right] = 0$$



(i) $\text{sen}(4x) = 0 \;\therefore\; 4x = k\pi$

$$\boxed{x = k \cdot \pi/4} \quad k \in \mathbb{Z}$$

(ii) $\cos(2x) - 1 = 0 \;\therefore\; \cos(2x) = 1 \;\therefore\; 2x = 2k\pi$

$$\boxed{x = k \cdot \pi \;\; (k \in \mathbb{Z})}$$

$$S = \{x \in \mathbb{R} \mid x = k\pi \text{ ou } x = k \cdot \pi/4,\; k \in \mathbb{Z}\}$$

• TIPO 02 : $a\,\mathrm{sen}(x) + b\cos(x) = c$ •

$\mathrm{sen}\,x + \sqrt{2}\cos x = 1$

$2\,\mathrm{sen}\,x + 3\cos x = 4$

$\sqrt{3}\,\mathrm{sen}\,x + \cos x = 1/3$

EXEMPLO 01

Resolva a equação $\operatorname{sen} x + \sqrt{3}\cos x = 1$

R :

SOLUÇÃO 01

$$\operatorname{sen}(x) = \frac{2tg(x/2)}{1 - tg^2(x/2)} \quad ; \quad \cos(x) = \frac{1 - tg^2(x/2)}{1 + tg^2(x/2)}$$

$$\operatorname{sen} x + \sqrt{3}\cos x = 1$$

$$\frac{2tg(x/2)}{1 - tg^2(x/2)} + \sqrt{3}\cdot\left[\frac{1 - tg^2(x/2)}{1 + tg^2(x/2)}\right] = 1$$

$$\times\,[1 - tg^2(x/2)][1 + tg^2(x/2)]$$

$$2tg(x/2)[1 + tg^2(x/2)] + \sqrt{3}\,[1 - tg^2(x/2)]^2 = [1 + tg^2(x/2)][1 - tg^2(x/2)]$$

$$y = tg(x/2) \rightarrow \quad 2y(1+y^2) + \sqrt{3}(1-y^2)^2 = (1+y^2)(1-y^2)$$

$$2y + 2y^3 + \sqrt{3}(1 - 2y + y^2) = 1 - y^4 \qquad (\ldots)$$

SOLUÇÃO 02

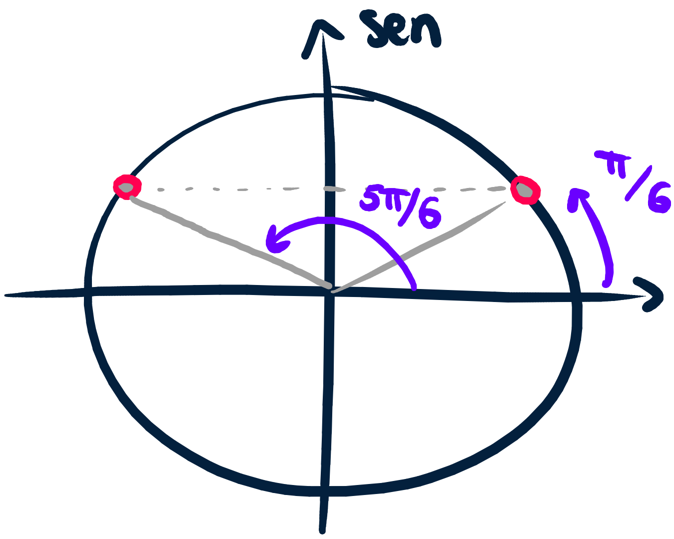
$$\operatorname{sen} x + \sqrt{3}\cos x = 1$$

$\times \frac{1}{2}$

$$\frac{1}{2}\operatorname{sen} x + \frac{1}{2}\sqrt{3}\cos x = 1 \cdot \frac{1}{2}$$

$$\operatorname{sen} x \cdot \cos(\pi/3) + \operatorname{sen}(\pi/3)\cos x = \frac{1}{2}$$

$$\operatorname{sen}(x + \pi/3) = 1/2$$



(i) $x + \pi/3 = \pi/6 + 2K\pi$

$$\boxed{x = -\pi/6 + 2K\pi}$$

(ii) $x + \pi/3 = 5\pi/6 + 2K\pi \therefore \boxed{x = \frac{\pi}{2} + 2K\pi}$

$K \in \mathbb{Z}$

CASO GERAL : $\underbrace{a\,\mathrm{sen}\,x + b\cos x}_{d\,.\,\mathrm{sen}(x+\varphi)} = c$

$$a\,\mathrm{sen}\,x + b\cos x =$$

$$= \sqrt{a^2+b^2}\left[\overbrace{\frac{a}{\sqrt{a^2+b^2}}}^{\cos\varphi}.\,\mathrm{sen}\,x + \overbrace{\frac{b}{\sqrt{a^2+b^2}}}^{\mathrm{sen}\,\varphi}.\,\cos x\right]$$

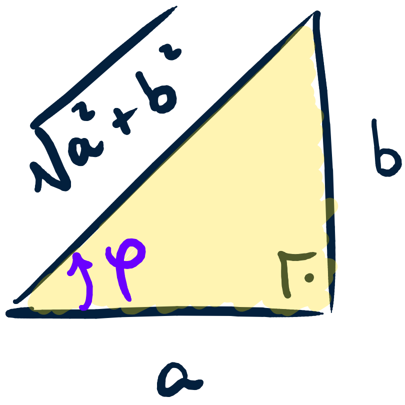
Note que: $\left(\frac{a}{\sqrt{a^2+b^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2+b^2}}\right)^2 = \frac{a^2+b^2}{a^2+b^2} = 1$

$$\cos^2\varphi + \mathrm{sen}^2\varphi = 1$$

$$a\,\mathrm{sen}\,x + b\cos x = \sqrt{a^2+b^2}\left[\mathrm{sen}\,x\,.\cos\varphi + \mathrm{sen}\,\varphi\cos x\right]$$

$$= \sqrt{a^2+b^2}\;\mathrm{sen}(x+\varphi)\text{ , onde}$$

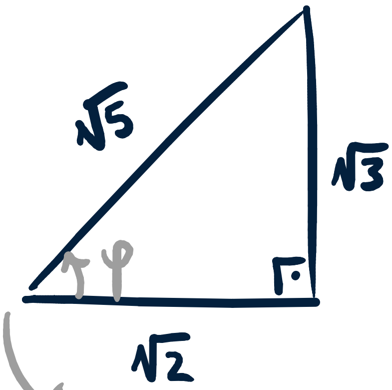
$$\tan\varphi = \frac{b}{\cancel{\sqrt{a^2+b^2}}}\cdot\frac{\cancel{\sqrt{a^2+b^2}}}{a} \;\therefore\; \boxed{\varphi = \arctan(b/a)}$$



EXEMPLO 02

Resolva a equação $\sqrt{2}\,\text{sen}\,x + \sqrt{3}\cos x = 2$

R: $\sqrt{2}\,\text{sen}\,x + \sqrt{3}\cos x = 2$



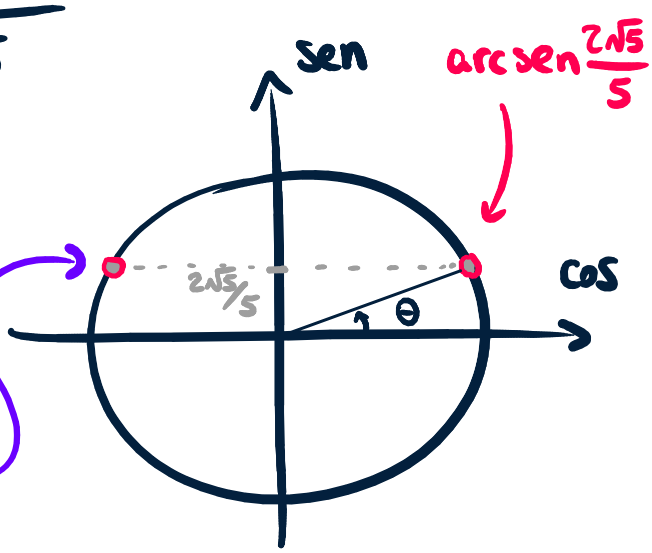
$$\sqrt{5}\left[\frac{\sqrt{2}}{\sqrt{5}}\text{sen}\,x + \frac{\sqrt{3}}{\sqrt{5}}\cos x\right] = 2$$

$\cdot \cos\varphi = \dfrac{\sqrt{2}}{\sqrt{5}}$

$\cdot \text{sen}\,\varphi = \dfrac{\sqrt{3}}{\sqrt{5}}$

$$\sqrt{5}\left[\text{sen}\,x \cdot \cos\varphi + \text{sen}\,\varphi \cos x\right] = 2$$

$$\text{sen}(x+\varphi) = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$



(i) $x + \varphi = \theta + 2k\pi$

$$\boxed{x = \text{arcsen}\left(2\sqrt{5}/5\right) + 2k\pi - \varphi}$$

(ii) $x + \varphi = (\pi - \theta) + 2k\pi$

$$\boxed{x = \pi - \text{arcsen}\left(2\sqrt{5}/5\right) + 2k\pi - \varphi}, \quad k \in \mathbb{Z}$$

onde $\varphi = \text{arctan}\left(\sqrt{3}/\sqrt{2}\right)$

. TIPO 03 : $a\,sen^2x + b\,senx\cos x + c\cdot\cos^2x = d$.

$$-2\,sen^2x + 5\,senx\cos x + 3\cos^2x = 3$$

$$sen^2x + 2\,senx\cos x - 5\cos^2x = 1$$

$$\sqrt{2}\,sen^2x + 3\,senx\cos x - \cos^2x = 2$$

EXEMPLO 01

Resolva : $\operatorname{sen}^2 x + 4 \operatorname{sen}x \cos x + 2\cos^2 x = 1$

R :

SOLUÇÃO 01 : ARCO - METADE

$$\operatorname{sen}^2 x = \frac{1-\cos(2x)}{2} \;;\; \cos^2 x = \frac{1+\cos(2x)}{2} \;;\; \operatorname{sen}x \cos x = \frac{\operatorname{sen}(2x)}{2}$$

$$\operatorname{sen}^2 x + 4 \operatorname{sen}x \cos x + 2\cos^2 x = 1$$

$$\frac{1-\cos(2x)}{2} + 4.\frac{\operatorname{sen}(2x)}{2} + 2\left[\frac{1+\cos(2x)}{2}\right] = 1$$

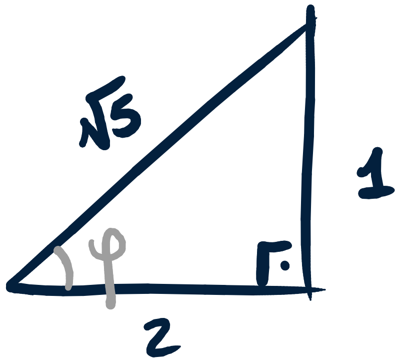
$\times 2$

$$1-\cos(2x) + 2\operatorname{sen}(2x) + 2 + 2\cos(2x) = 2$$

$$2\operatorname{sen}(2x) + \cos(2x) = 1$$

$$a \operatorname{sen}\theta + b\cos\theta = c$$

$$\sqrt{5}\left[\operatorname{sen}(2x)\underbrace{\frac{2}{\sqrt{5}}}_{\cos\varphi} + \underbrace{\frac{1}{\sqrt{5}}}_{\sin\varphi}\cos(2x)\right] = 1$$

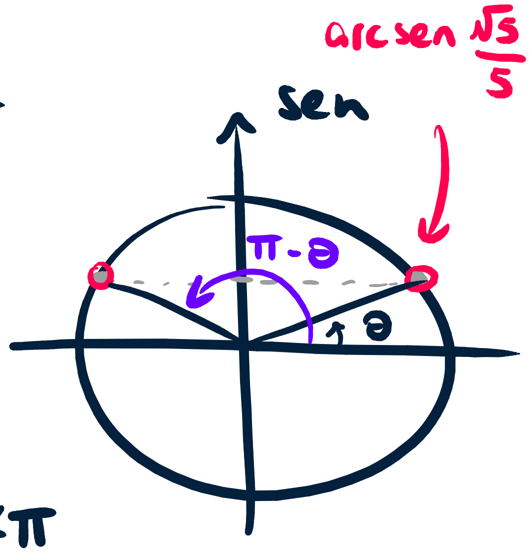


$$\tan\varphi = 1/2$$

$$\varphi = \arctan(1/2)$$

$$\sqrt{5}\left[\operatorname{sen}(2x)\cos\varphi + \operatorname{sen}\varphi\cos(2x)\right] = 1$$

$$\operatorname{sen}(2x+\varphi) = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$



(i) $2x + \varphi = \theta + 2K\pi$

$$2x + \varphi = \operatorname{arc\,sen}\left(\sqrt{5}/5\right) + 2K\pi$$

$$\boxed{x = \frac{1}{2}\left[\operatorname{arc\,sen}\left(\sqrt{5}/5\right) - \operatorname{arctan}\left(1/2\right) + 2K\pi\right]}$$

(ii) $2x+\varphi = (\pi - \theta) + 2K\pi$

$$\boxed{x = \frac{1}{2}\left[\left(\pi - \operatorname{arc\,sen}\left(\sqrt{5}/5\right)\right) - \operatorname{arctan}\left(1/2\right) + 2K\pi\right]}$$

$$K \in \mathbb{Z}$$

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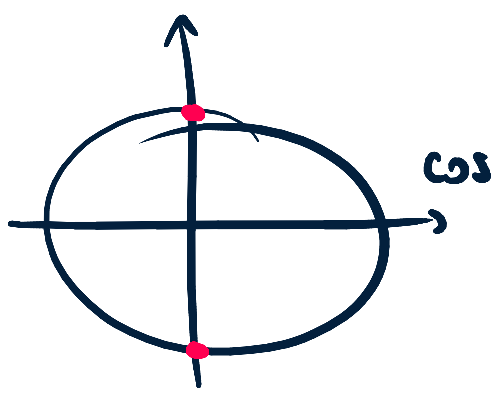
SOLUÇÃO 02 : tg x

$$\sin^2 x + 4\sin x\cos x + 2\cos^2 x = 1$$



(i) $\cos x = 0$ é solução!

$$\boxed{x = \pi/2 + k\pi}$$

(ii) $\sin^2 x + 4\sin x\cos x + 2\cos^2 x = 1$ $\quad \div \cos^2 x$

$$\tan^2 x + \frac{4\sin x\cancel{\cos x}}{\cos x\cdot\cancel{\cos x}} + 2\cdot 1 = \frac{1}{\cos^2 x}$$

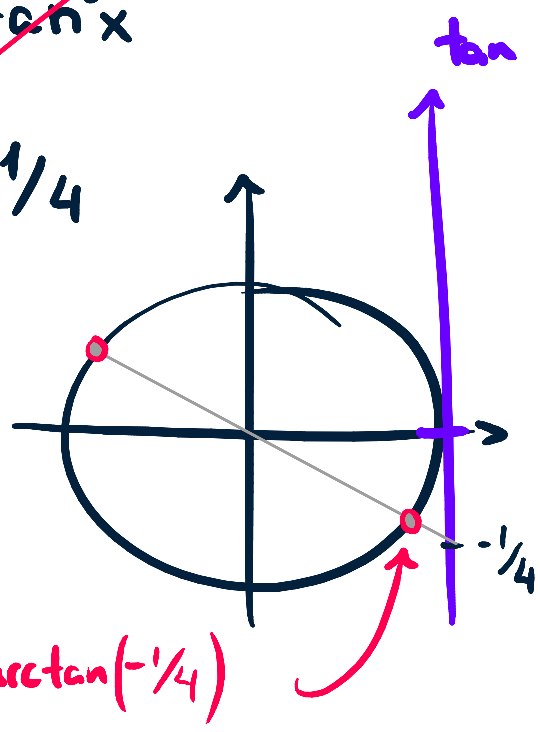
$$\tan^2 x + 4\tan x + 2 = \sec^2 x = 1 + \tan^2 x$$

$$\cancel{\tan^2 x} + 4\tan x + 2 = 1 + \cancel{\tan^2 x}$$

$$4\tan x = -1 \quad \therefore \quad \tan x = -1/4$$

$$\boxed{x = \arctan(-1/4) + k\pi}$$

$k \in \mathbb{Z}$



. TIPO 04 : $a(\text{sen}x + \cos x) + b\,\text{sen}x\cos x = c$.

$$\text{sen}x + \sqrt{2}\,\text{sen}x\cos x + \cos x = 1$$

$$3\,\text{sen}x - 2\,\text{sen}x\cos x + 3\cos x = 0$$

$$5(\text{sen}x + \cos x) + 3\,\text{sen}x\cos x = 3$$

TÉCNICA

$$a(\operatorname{sen}x + \cos x) + b\operatorname{sen}x\cos x = c$$

↳ Substituição : $\boxed{y = \operatorname{sen}x + \cos x}$ quadrado

$$y^2 = (\operatorname{sen}x + \cos x)^2$$

$$y^2 = \operatorname{sen}^2 x + 2\operatorname{sen}x\cdot\cos x + \cos^2 x$$

$$y^2 = 1 + 2\operatorname{sen}x\cos x \longrightarrow \operatorname{sen}x\cdot\cos x = \frac{y^2-1}{2}$$

Voltando à equação :

$$a(\operatorname{sen}x + \cos x) + b\operatorname{sen}x\cos x = c$$

$$a\cdot y + b\cdot\frac{(y^2-1)}{2} = c$$

$\times 2$

$$2ay + b\cdot y^2 - b = 2c$$

$\div b$

$$y^2 + \frac{2a}{b}\cdot y - 1 - \frac{2c}{b} = 0$$

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EXEMPLO

Resolva: $\operatorname{sen}x + \cos x + \sqrt{2}\operatorname{sen}x\cos x = 1$

R: $\underbrace{\operatorname{sen}x + \cos x}_{y} + \sqrt{2}\operatorname{sen}x\cos x = 1$

$y = \operatorname{sen}x + \cos x \rightarrow y^2 = 1 + 2\operatorname{sen}x\cos x$

$\operatorname{sen}x \cdot \cos x = \dfrac{y^2-1}{2}$

$\operatorname{sen}x + \cos x + \sqrt{2}\operatorname{sen}x\cos x = 1$

$y + \sqrt{2}\cdot\dfrac{(y^2-1)}{2} = 1$

$2y + \sqrt{2}y^2 - \sqrt{2} = 2 \quad \therefore \quad \sqrt{2}y^2 + 2y - (2+\sqrt{2}) = 0$

$\Delta = 4 + 4\cdot\sqrt{2}(2+\sqrt{2}) = 4[1 + 2\sqrt{2} + 2] = 4(3+2\sqrt{2})$

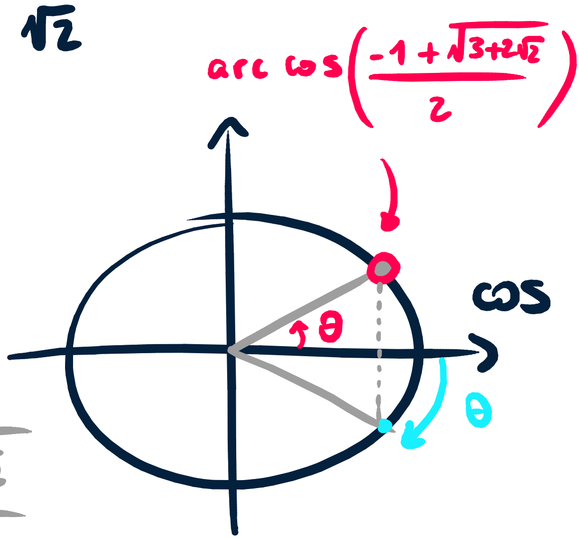
$y = \dfrac{-2 \pm 2\sqrt{3+2\sqrt{2}}}{2\sqrt{2}} = \dfrac{-1 \pm \sqrt{3+2\sqrt{2}}}{\sqrt{2}}$

$\hookrightarrow \operatorname{sen}x + \cos x = y \quad \vdots \quad 2\operatorname{sen}(\pi/4)\cdot\cos(x - \pi/4) = y$

$\operatorname{sen}x + \operatorname{sen}(\pi/2 - x) = y \quad \vdots \quad \cancel{2}\cdot\dfrac{\sqrt{2}}{\cancel{2}}\cos(x - \pi/4) = y$

$$\sqrt{2}\cos\left(x-\pi/4\right) = y = \frac{-1\pm\sqrt{3+2\sqrt{2}}}{\sqrt{2}}$$

$$\cos\left(x-\pi/4\right) = \frac{-1\pm\sqrt{3+2\sqrt{2}}}{2}$$



CASO 01 : $\cos\left(x-\pi/4\right) = \frac{-1+\sqrt{3+2\sqrt{2}}}{2}$

(i) $x-\pi/4 = \theta + 2k\pi \therefore \boxed{x = \frac{\pi}{4} + \arccos\left(\frac{-1+\sqrt{3+2\sqrt{2}}}{2}\right) + 2k\pi}$

(ii) $x-\pi/4 = (2\pi - \theta) + 2k\pi \therefore \boxed{x = \frac{9\pi}{4} - \arccos\left(\frac{- +\sqrt{3+2\sqrt{2}}}{2}\right) + 2k\pi}$

CASO 02 : $\cos\left(x-\pi/4\right) = \frac{-1-\sqrt{3+2\sqrt{2}}}{2}$

$$\boxed{x = \frac{\pi}{4} + \arccos\left(\frac{-1-\sqrt{3+2\sqrt{2}}}{2}\right) + 2k\pi}$$

$$\boxed{x = \frac{9\pi}{4} - \arccos\left(\frac{-1-\sqrt{3+2\sqrt{2}}}{2}\right) + 2k\pi}$$

OUTROS CASOS

CASO 01: $\text{sen}^4 x + \cos^4 x = a$

R: $\text{sen}^4 x + \cos^4 x = 1 - \dfrac{\text{sen}^2(2x)}{2}$

PROVA: $\text{sen}^4 + \cos^4 x = (\text{sen}^2 + \cos^2 x)^2 - 2\,\text{sen}^2 x \cos^2 x$

$$= 1^2 - \frac{4 \cdot \text{sen}^2 x \cos^2 x}{2}$$

$$= 1 - \frac{(\overbrace{2\,\text{sen}\,x \cos x}^{\text{sen}(2x)})^2}{2}$$

$$= 1 - \frac{\text{sen}^2(2x)}{2}$$

CASO 02: $\operatorname{sen}^6 x + \cos^6 x = a$

R: $\operatorname{sen}^6 x + \cos^6 x = 1 - \dfrac{3\operatorname{sen}^2(2x)}{4}$

PROVA:

$$\operatorname{sen}^6 x + \cos^6 x = \overbrace{(\operatorname{sen}^2 x + \cos^2 x)}^{1}(\operatorname{sen}^4 x - \operatorname{sen}^2 x \cos^2 x + \cos^4 x)$$

$$(a^3 + b^3) = (a+b)(a^2 - a \cdot b + b^2)$$

$$\operatorname{sen}^6 x + \cos^6 x = 1 \cdot \left[\underbrace{\operatorname{sen}^4 x + \cos^4 x}_{1 - \frac{\operatorname{sen}^2(2x)}{2}} - \frac{\overbrace{4\operatorname{sen}^2 x \cos^2 x}^{(2\operatorname{sen} x \cos x)^2}}{4} \right]$$

$$= 1 - \frac{\operatorname{sen}^2(2x)}{2} - \frac{\operatorname{sen}^2(2x)}{4}$$

$$= 1 - \frac{3}{4}\operatorname{sen}^2(2x)$$

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CASO 03 : $\arcsin(2x) = \arcsin(\sqrt{2}\,x) + \arcsin(x)$

$(x \geq 0)$

R :

$\alpha = \arcsin(2x) \Longrightarrow \operatorname{sen}\alpha = 2x$

$\beta = \arcsin(\sqrt{2}x) \Longrightarrow \operatorname{sen}\beta = x\sqrt{2}$

$\gamma = \arcsin(x) \Longrightarrow \operatorname{sen}\gamma = x$

$\arcsin(2x) = \arcsin(\sqrt{2}\,x) + \arcsin(x)$

$\alpha = \beta + \gamma$

$\operatorname{sen}(\alpha) = \operatorname{sen}(\beta+\gamma)$

$\operatorname{sen}\alpha = \operatorname{sen}\beta\cos\gamma + \operatorname{sen}\gamma\cos\beta$

(i) $\operatorname{sen}\gamma = x \quad \therefore \quad \cos^2\gamma + \operatorname{sen}^2\gamma = 1 \therefore \cos^2\gamma = 1 - x^2$

$$\boxed{\cos\gamma = +\sqrt{1-x^2}}$$

(ii) $\operatorname{sen}\beta = x\sqrt{2} \quad \therefore \quad \boxed{\cos\beta = +\sqrt{1-2x^2}}$

$$\arcsin(2x) = \arcsin(\sqrt{2}\,x) + \arcsin(x)$$

$$\operatorname{sen}\alpha = \operatorname{sen}\beta\cos\gamma + \operatorname{sen}\gamma\cos\beta$$

$$2x = x\sqrt{2}\cdot\sqrt{1-x^2} + x\cdot\sqrt{1-2x^2}$$

(i) $\boxed{x=0}$

(ii) $2 = \sqrt{2}\sqrt{1-x^2} + \sqrt{1-2x^2}$

$$2^2 = 2(1-x^2) + 2\sqrt{2}\sqrt{1-x^2}\sqrt{1-2x^2} + (1-2x^2)$$

$$4 = 2 - 2x^2 + 2\sqrt{2}\sqrt{(1-x^2)(1-2x^2)} + 1 - 2x^2$$

$$1 + 4x^2 = 2\sqrt{2}\sqrt{(1-x^2)(1-2x^2)}$$

$$1^2 + 8x^2 + 16x^4 = 4\cdot 2\,(1-x^2)(1-2x^2)$$

$$1^2 + 8x^2 + 16x^4 = 8(1-2x^2-x^2+2x^4)$$

$$1 + 8x^2 + \cancel{16x^4} = \cancel{16x^4} - 24x^2 + 8$$

$$32x^2 = 7 \;\therefore\; x^2 = 7/32 \qquad \boxed{x = +\sqrt{7/32}}$$

APROFUNDAMENTO

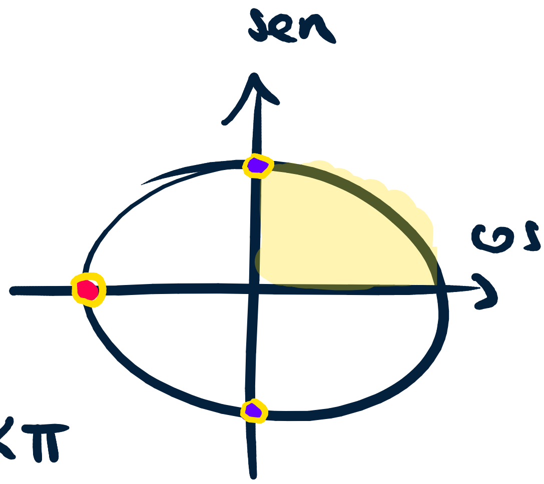
EXEMPLO 01 (ITA 2008)

A soma das soluções da equação

$\cos(3x) + 2\cos(6x) + \cos(9x) = 0$, $x \in [0, \pi/2]$ é:

R:

$$\underbrace{\cos(3x) + \cos(9x)}_{2\cos(6x)\cdot\cos(3x)} + 2\cos(6x) = 0$$



$$2\cos(6x)\left[\cos(3x) + 1\right] = 0$$

(i) $\cos(6x) = 0 \therefore 6x = \frac{\pi}{2} + k\pi$

$$\boxed{x = \frac{\pi}{12} + \frac{k\pi}{6}}$$

$k = 0$: $\boxed{x_1 = \pi/12}$

$k = 1$: $\boxed{x_2 = 3\pi/12}$

$k = 2$: $\boxed{x_3 = 5\pi/12}$

(ii) $\cos(3x) = -1 \therefore 3x = \pi + 2k\pi$

$$\boxed{x = \frac{\pi}{3} + \frac{2k\pi}{3}}$$

$k = 0$: $\boxed{x_4 = \pi/3 = 4\pi/12}$

R: $\sum_{i=1}^{4} x_i = 13\pi/12$

EXEMPLO 02 (ITA 2012)

Encontre x que maximize a expressão

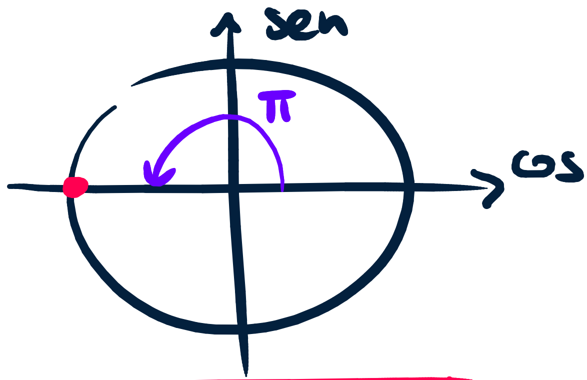
$\operatorname{sen}(2x) - \sqrt{3}\cos(2x)$.

R: $Y = 2\left[\frac{1}{2}\cdot\operatorname{sen}(2x) - \frac{\sqrt{3}}{2}\cos(2x)\right]$

$$Y = 2\left[\operatorname{sen}\pi/6 \cdot \operatorname{sen}(2x) - \cos\pi/6 \cdot \cos(2x)\right]$$

$$Y = -2\left[\cos\pi/6 \cdot \cos(2x) - \operatorname{sen}\pi/6 \cdot \operatorname{sen}(2x)\right]$$

$$Y = -2\cos(2x + \pi/6) \Rightarrow Y_{máx} = +2$$



$$\cos(2x + \pi/6) = -1$$

$$2x + \pi/6 = \pi + 2K\pi$$

$$2x = 5\pi/6 + 2K\pi \;\therefore\; \boxed{x = \frac{5\pi}{12} + K\pi}$$

$K \in \mathbb{Z}$

EXEMPLO 03 (IME 2005)

Resolva a equação $2\,\text{sen}(11x) + \cos(3x) + \sqrt{3}\,\text{sen}(3x) = 0$

R:

$$2\,\text{sen}(11x) + \underbrace{\cos(3x) + \sqrt{3}\,\text{sen}(3x)} = 0$$

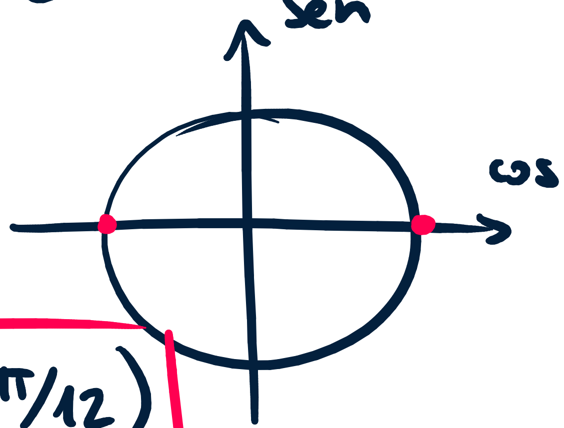
$$2\cdot\left[\frac{1}{2}\cdot\cos(3x) + \frac{\sqrt{3}}{2}\,\text{sen}(3x)\right]$$

$$2\cdot\left[\text{sen}\,30^\circ\cos(3x) + \cos 30^\circ\cdot\text{sen}(3x)\right] = 2\cdot\text{sen}(3x + \pi/6)$$

$$2\,\text{sen}(11x) + 2\,\text{sen}\left(3x + \pi/6\right) = 0$$

$$\text{sen}(11x) + \text{sen}\left(3x + \pi/6\right) = 0$$

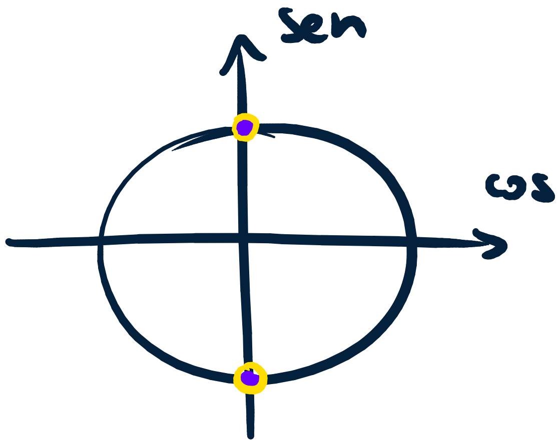
$$2\cdot\overbrace{\text{sen}\left(7x + \pi/12\right)}\underbrace{\cos\left(4x - \pi/12\right)} = 0$$



(i) $\text{sen}\left(7x + \pi/12\right) = 0$

$$7x + \pi/12 = k\pi \;\therefore\; \boxed{x = \frac{1}{7}\left(k\pi - \pi/12\right)}$$

(ii) $\cos\left(4x - \frac{\pi}{12}\right) = 0$



$$4x - \frac{\pi}{12} = \frac{\pi}{2} + k\pi$$

$$4x = \frac{7\pi}{12} + k\pi$$

$$\boxed{x = \frac{1}{4}\left(\frac{7\pi}{12} + k\pi\right)}$$

$$S = \left\{ x \in \mathbb{R} \;\middle|\; x = \frac{1}{7}\left(k\pi - \frac{\pi}{12}\right) \text{ ou } x = \frac{1}{4}\left(\frac{7\pi}{12} + k\pi\right),\ k \in \mathbb{Z} \right\}$$

EXEMPLO 04 (ITA 2004)

Mostre que, se $\text{sen}(3\alpha)+\text{sen}(3\beta)+\text{sen}(3\gamma)=0$, sendo α, β, γ ângulos de um Δ, então pelo menos um deles é igual a $60°$.

R:

$$\text{sen}\left[3\pi-(3\alpha+3\beta)\right]$$

$$\alpha+\beta+\gamma=\pi$$

$$3\alpha+3\beta+3\gamma=3\pi$$

$$3\gamma=3\pi-(3\alpha+3\beta)$$

$$\underbrace{\text{sen}(3\alpha)+\text{sen}(3\beta)}+\text{sen}(3\gamma)=0$$

$$2\cdot\text{sen}\left(\frac{3\alpha+3\beta}{2}\right)\cdot\cos\left(\frac{3\alpha-3\beta}{2}\right)$$

$$2\cdot\text{sen}\left(\frac{3\alpha+3\beta}{2}\right)\cos\left(\frac{3\alpha-3\beta}{2}\right)+\underbrace{\text{sen}\left[3\pi-(3\alpha+3\beta)\right]}_{+\text{sen}(3\alpha+3\beta)}=0$$

$$+\text{sen}\left[2\cdot\frac{(3\alpha+3\beta)}{2}\right]$$

$$2\cdot\text{sen}\left(\frac{3\alpha+3\beta}{2}\right)\cos\left(\frac{3\alpha-3\beta}{2}\right)+\underbrace{\text{sen}\left[2\frac{(3\alpha+3\beta)}{2}\right]}_{\text{sen}(2x)=2\,\text{sen}\,x\cos x}=0$$

$$2\cdot sen\left(\frac{3\alpha+3\beta}{2}\right)\cos\left(\frac{3\alpha-3\beta}{2}\right) + sen\left[\frac{2(3\alpha+3\beta)}{2}\right] = 0$$

$$\overbrace{2\cdot sen\left(\frac{3\alpha+3\beta}{2}\right)}\cos\left(\frac{3\alpha-3\beta}{2}\right) + \overbrace{2\, sen\left(\frac{3\alpha+3\beta}{2}\right)}\cos\left(\frac{3\alpha+3\beta}{2}\right) = 0$$

$$\underline{2\cdot sen\left(\frac{3\alpha+3\beta}{2}\right)}\left[\cos\left(\frac{3\alpha-3\beta}{2}\right) + \cos\left(\frac{3\alpha+3\beta}{2}\right)\right] = 0$$

$$2\cdot sen\left(\frac{3\alpha+3\beta}{2}\right)\left[2\cdot\cos\left(\frac{3\alpha}{2}\right)\cos\left(-\frac{3\beta}{2}\right)\right] = 0$$

$$\underbrace{4\, sen\left(\frac{3\pi-3\gamma}{2}\right)}\cos\left(\frac{3\alpha}{2}\right)\cos\left(\frac{3\beta}{2}\right) = 0$$

$$sen\left(\frac{3\pi}{2}-\frac{3\gamma}{2}\right) = \overbrace{sen\frac{3\pi}{2}}^{-1}\cos(3\gamma/2) - sen\frac{3\gamma}{2}\,\cancel{\cos\frac{3\pi}{2}}^{\,0}$$

$$\to -4\cos\left(\frac{3\alpha}{2}\right)\cos\left(\frac{3\beta}{2}\right)\cdot\cos\left(\frac{3\gamma}{2}\right) = 0$$

$$\cos(3\theta/2) = 0 \Rightarrow 3\theta/2 = \pi/2 \therefore \boxed{\theta = \pi/3 = 60^\circ}$$

RESUMO

Tipo 01 : $\sum_n \cos(nx) + \sum_m \operatorname{sen}(mx) = 0$

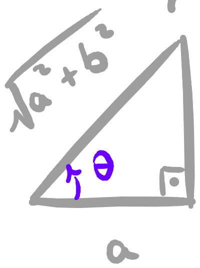
TRANSFORMAR SOMA EM PRODUTO

Tipo 02 : $a\operatorname{sen}(x) + b\cos(x) = c$

$$\times \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

$$\operatorname{sen}\theta = \frac{b}{\sqrt{a^2+b^2}}$$

$$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$$



$$tg\theta = \frac{b}{a}$$

TIPO 03 : $a\operatorname{sen}^2 x + b \operatorname{sen}x\cos x + c\cdot\cos^2 x = d$

ARCO - DOBRO

TIPO 04 : $a(\operatorname{sen}x + \cos x) + b\operatorname{sen}x\cos x = c$

$(\operatorname{sen}x + \cos x) = z$