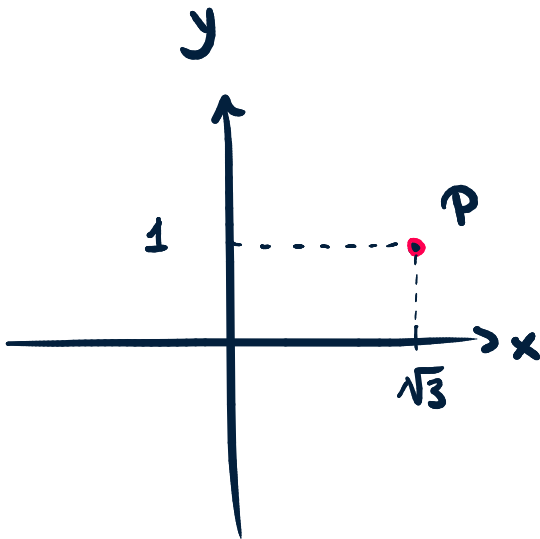
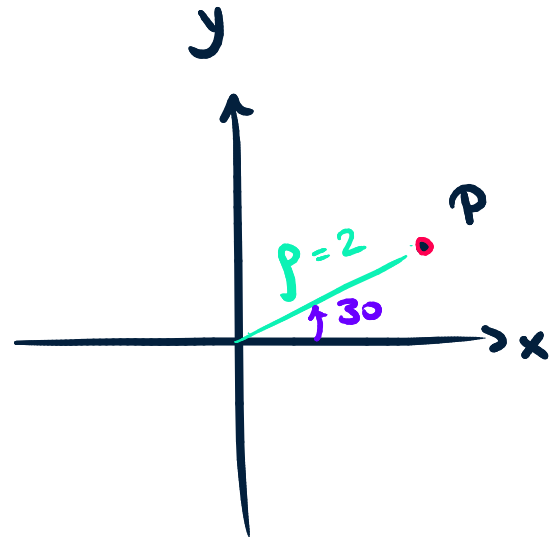


· FORMA TRIGONOMÉTRICA ·



$$P = (x, y) = (\sqrt{3}, 1)$$

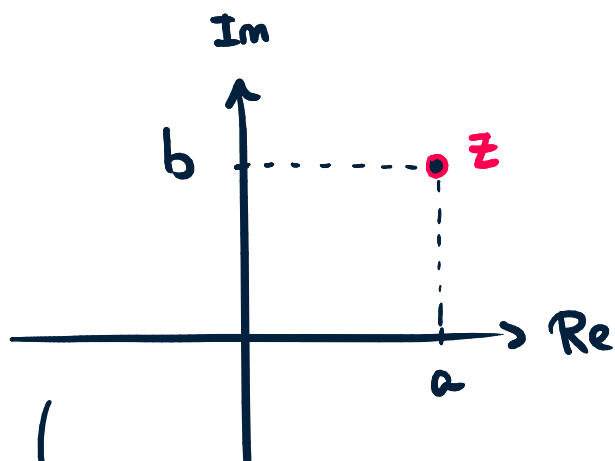
COORDENADAS CARTESIANAS



$$P = (\rho, \theta) = (2, \pi/6)$$

COORDENADAS POLARES





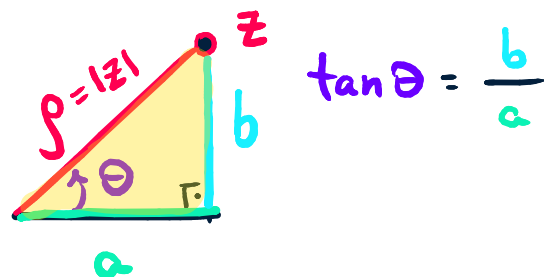
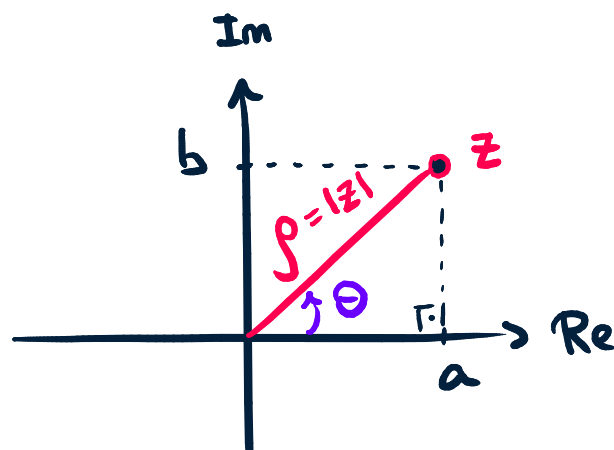
$$z = a + b \cdot i$$

$$z = \rho \cdot \cos \theta + \rho \cdot \sin \theta \cdot i$$

$$z = \rho \cdot (\cos \theta + i \sin \theta)$$

$$\rho = \sqrt{a^2 + b^2}$$

$$\theta = \arctan\left(\frac{\text{Im}(z)}{\text{Re}(z)}\right)$$



$$\cos \theta = \frac{a}{\rho} \therefore a = \rho \cdot \cos \theta$$

$$\sin \theta = \frac{b}{\rho} \therefore b = \rho \cdot \sin \theta$$

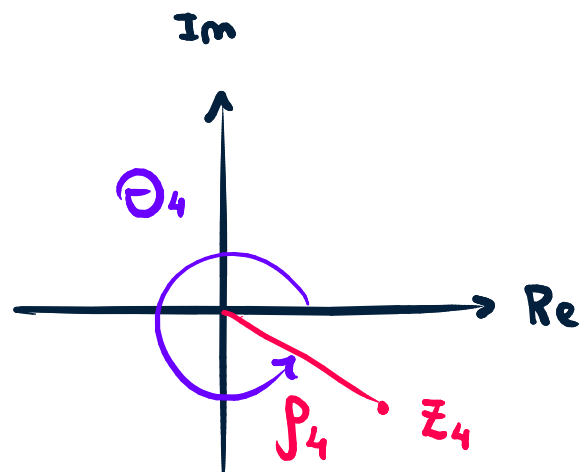
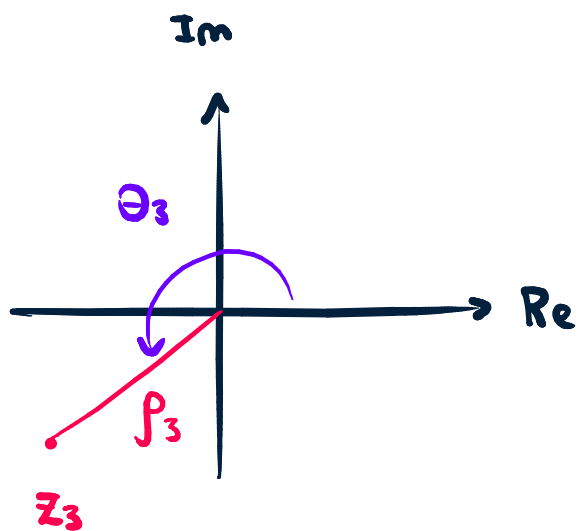
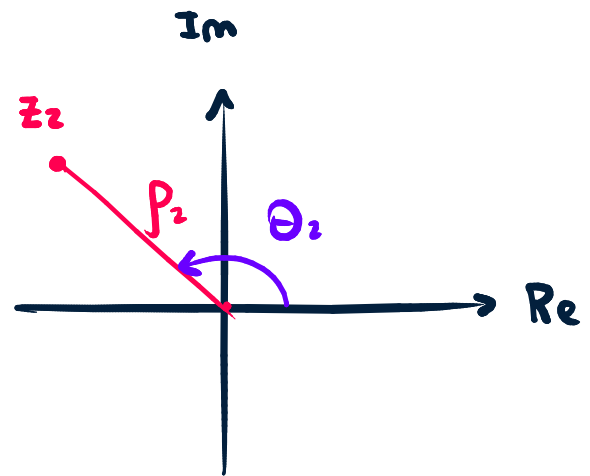
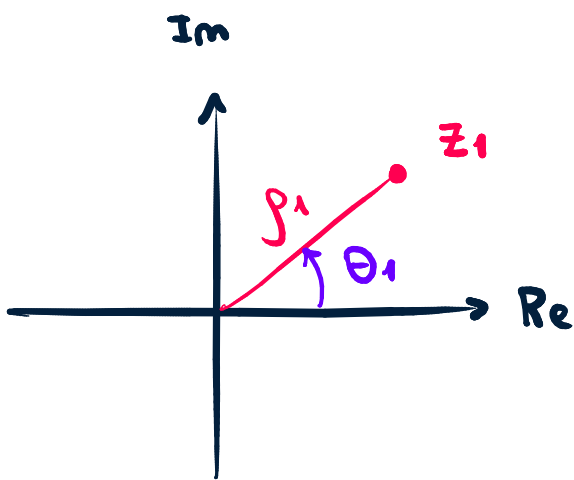
Podemos escrever o n° complexo z na forma

• algebraica : $z = a + b i = (a, b)$

• trigonometrica : $z = |z| \cdot (\cos \theta + i \sin \theta)$
 $= \rho \cdot (\cos \theta + i \sin \theta)$
 $= (\rho, \theta)$



OBS.: ARGUMENTO DE UM N.º COMPLEXO (θ)



$\theta \rightarrow$ ângulo entre 0 e 2π medido no sentido anti-horário.

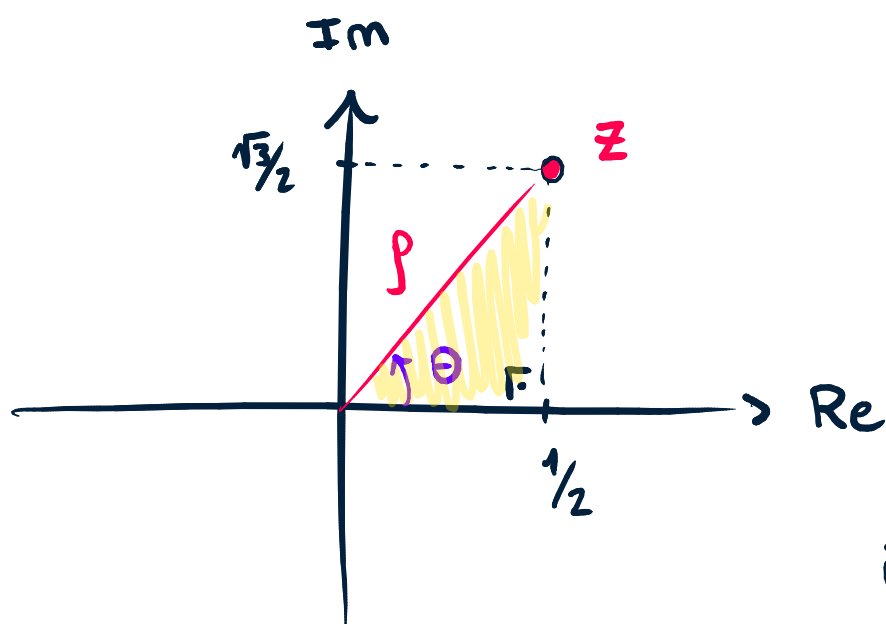


EXEMPLO 01

Escreva na forma trigonométrica

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$$

R:



$$\begin{aligned} \text{i) } \rho^2 &= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{1}{4} + \frac{3}{4} = 1 \end{aligned}$$

$$\rho = 1$$

$$\text{ii) } \cos \theta = \frac{1/2}{1} = \frac{1}{2}$$

$$\theta = 60^\circ = \pi/3$$

$$z = \rho \cdot (\cos \theta + i \cdot \sin \theta) = 1 \cdot (\cos \pi/3 + i \sin \pi/3)$$

$$z = \cos(\pi/3) + i \sin(\pi/3)$$

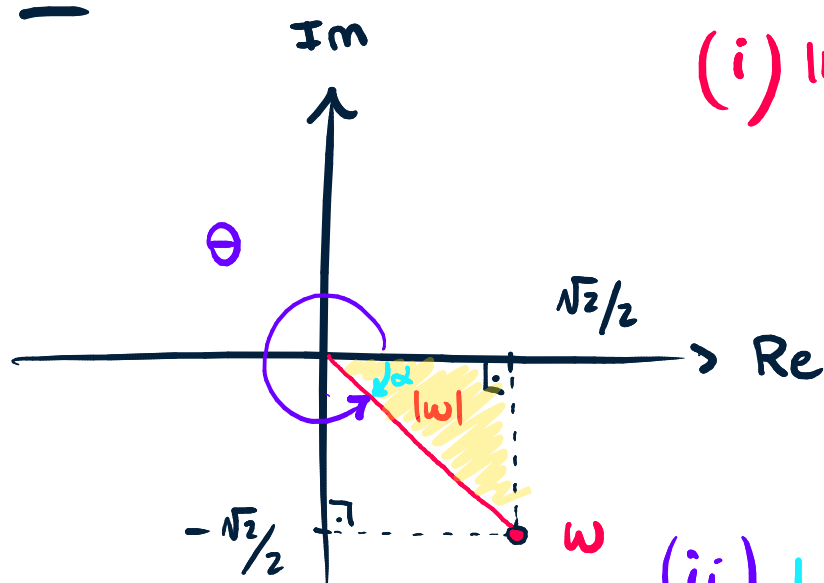


EXEMPLO 02

Escreva na forma trigonométrica

$$w = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$$

R:



$$(i) |w|^2 = \rho^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 \\ = \frac{1}{2} + \frac{1}{2} = 1$$

$$\rho = 1$$

$$(ii) \operatorname{tg} \alpha = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$$

$$\alpha = 45^\circ = \pi/4 \text{ rad}$$

$$\theta + \alpha = 2\pi \therefore$$

$$\theta = 7\pi/4$$

$$Z = \rho \cdot (\cos \theta + i \cdot \operatorname{sen} \theta) = 1 \cdot (\cos 7\pi/4 + i \cdot \operatorname{sen} 7\pi/4)$$

$$Z = \cos 7\pi/4 + i \cdot \operatorname{sen} 7\pi/4$$



EXEMPLO 03

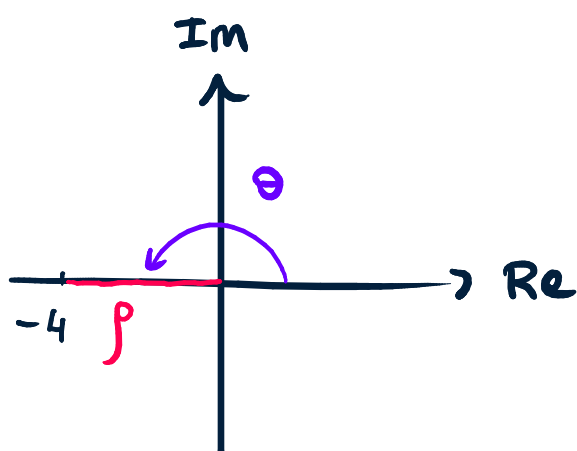
Escreva na forma trigonométrica

$$w = -4$$

R :

$$(i) \rho = 4$$

$$(ii) \theta = \pi \text{ rad}$$



$$w = \rho \cdot (\cos \theta + i \cdot \sin \theta)$$

$$= 4 \cdot (\cos \pi + i \cdot \underbrace{\sin \pi}_{\text{zero}}) = 4 \cdot \cos \pi \quad \Bigg|$$



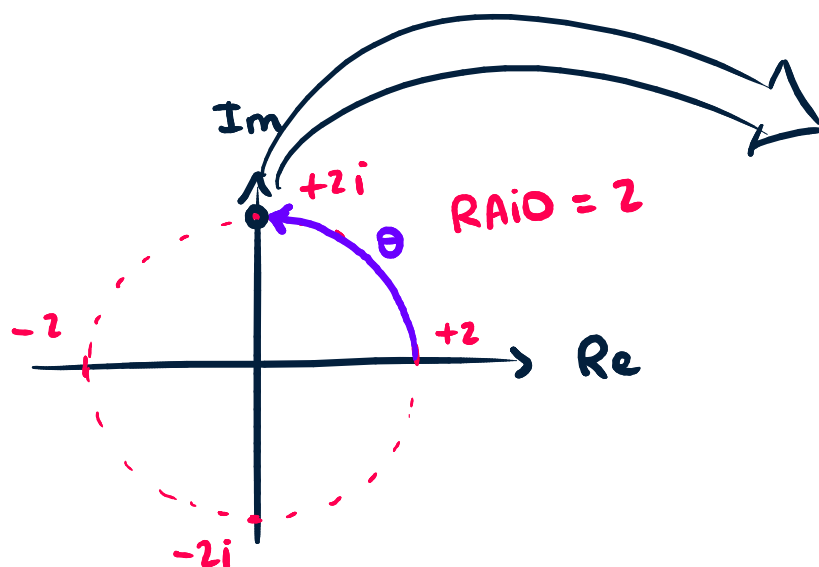
EXEMPLO 04

Escreva na forma algébrica

$$z = 2 (\cos 90^\circ + i \sin 90^\circ)$$

R:

i)



$$z = \overbrace{a}^{\text{zero}} + \overbrace{b}^2 \cdot i$$

$$z = 2 \cdot i$$

$$z = 2 (\cos 90^\circ + i \sin 90^\circ)$$

$$\text{ii) } z = 2 (\cancel{\cos 90^\circ} + i \overbrace{\sin 90^\circ}^1)$$

$$= 2 (0 + i)$$

$$= 2 \cdot i$$

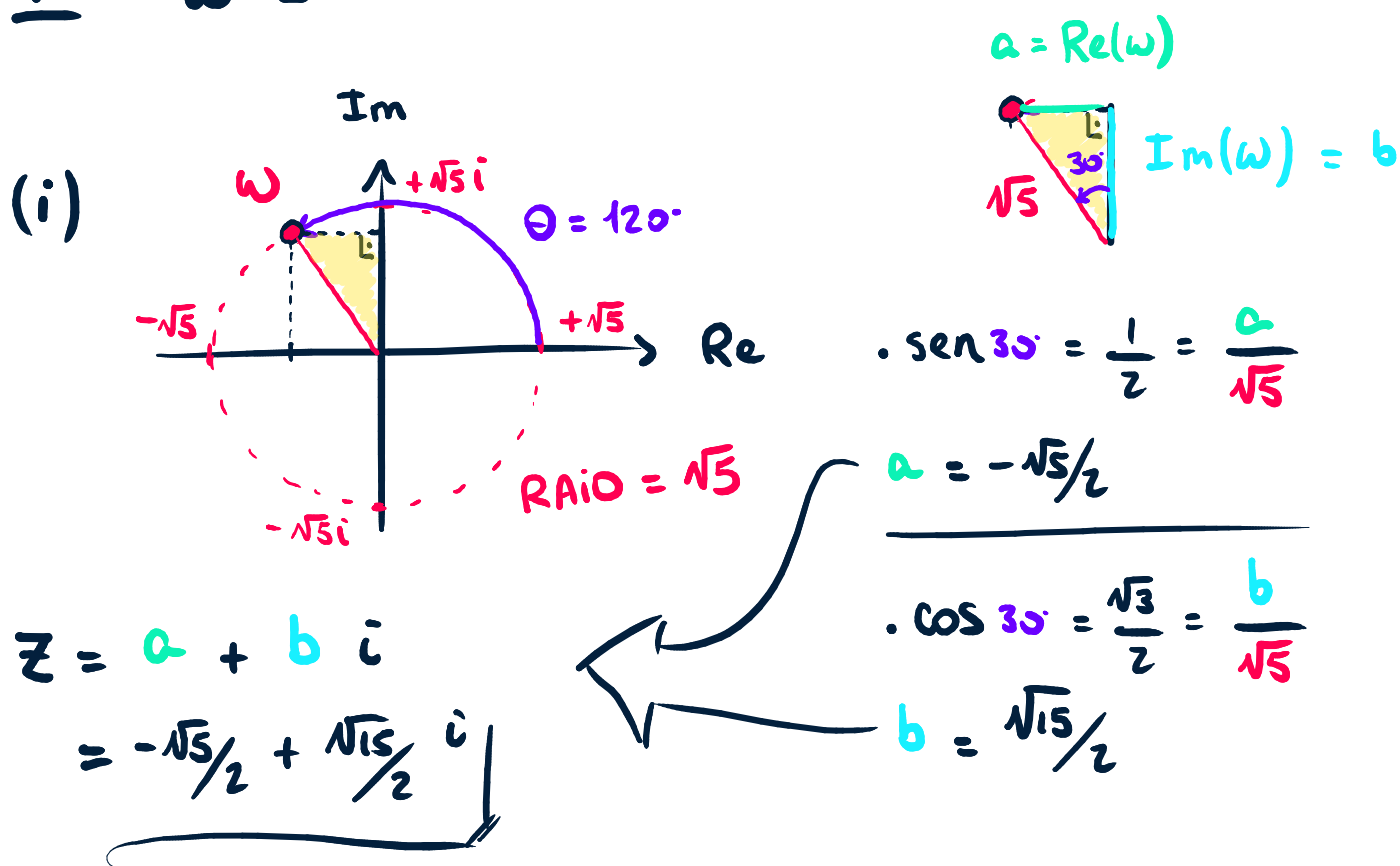


EXEMPLO 05

Escreva na forma algébrica

$$w = \sqrt{5} (\cos 120^\circ + i \sin 120^\circ)$$

R: $w = \sqrt{5} (\cos 120^\circ + i \sin 120^\circ)$



(ii) $w = \sqrt{5} (\underbrace{\cos 120^\circ}_{-\cos 60^\circ} + i \underbrace{\sin 120^\circ}_{\sin 60^\circ})$

$$= \sqrt{5} \left(-\frac{1}{2} + i \cdot \frac{\sqrt{3}}{2} \right)$$

$$= -\frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2} \cdot i$$

PRODUTO DE N^{os} COMPLEXOS

Seja $z_1 = \rho_1 (\cos \theta_1 + i \sin \theta_1)$ e

$$z_2 = \rho_2 (\cos \theta_2 + i \sin \theta_2).$$

O produto $z_1 \cdot z_2$ é dado, na forma trigonométrica, por:

$$z_1 \cdot z_2 = \rho_1 \cdot \rho_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

Demonstração

$$z_1 \cdot z_2 = \rho_1 (\cos \theta_1 + i \sin \theta_1) \rho_2 (\cos \theta_2 + i \sin \theta_2)$$

$$= \rho_1 \cdot \rho_2 \cdot [\underbrace{\cos \theta_1 \cdot \cos \theta_2}_{\text{}} + \underbrace{\cos \theta_1 \cdot \sin \theta_2 \cdot i}_{\text{}} + \underbrace{\sin \theta_1 \cdot \cos \theta_2 \cdot i}_{\text{}} - \underbrace{\sin \theta_1 \cdot \sin \theta_2}_{\text{}}]$$

$$= \rho_1 \cdot \rho_2 \left[(\underbrace{\cos \theta_1 \cdot \cos \theta_2 - \sin \theta_1 \cdot \sin \theta_2}_{\text{}}) + (\sin \theta_1 \cdot \cos \theta_2 + \cos \theta_1 \cdot \sin \theta_2) \cdot i \right]$$



$$\begin{aligned}
 z_1 \cdot z_2 &= \rho_1 \cdot \rho_2 \left[\underbrace{(\cos \theta_1 \cdot \cos \theta_2 - \operatorname{sen} \theta_1 \cdot \operatorname{sen} \theta_2)}_{\cos(\theta_1 + \theta_2)} + \right. \\
 &\quad \left. + \underbrace{(\operatorname{sen} \theta_1 \cdot \cos \theta_2 + \cos \theta_1 \cdot \operatorname{sen} \theta_2)}_{\operatorname{sen}(\theta_1 + \theta_2)} \cdot i \right]
 \end{aligned}$$

$$\begin{aligned}
 z_1 \cdot z_2 &= \rho_1 \cdot \rho_2 \left[\cos(\theta_1 + \theta_2) + i \cdot \operatorname{sen}(\theta_1 + \theta_2) \right] \\
 &= \rho \cdot \left[\cos \theta + i \cdot \operatorname{sen} \theta \right]
 \end{aligned}$$

De modo análogo:

$$\frac{z_1}{z_2} = \rho_1 / \rho_2 \cdot \left[\cos(\theta_1 - \theta_2) + i \cdot \operatorname{sen}(\theta_1 - \theta_2) \right]$$



EXEMPLO 01

Calcule $z \cdot w$ e z/w , sendo:

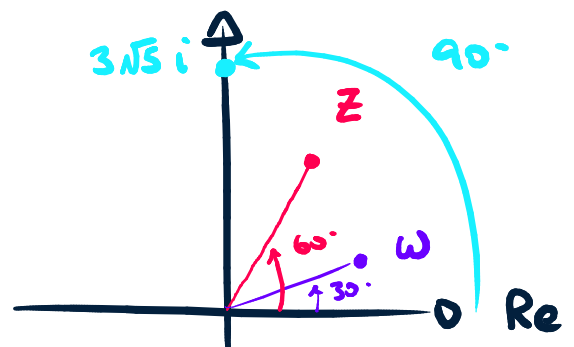
$$z = \sqrt{5} (\cos \pi/3 + i \operatorname{sen} \pi/3) \text{ e } w = 3 (\cos \pi/6 + i \operatorname{sen} \pi/6)$$

R:

$$\text{i) } z \cdot w = 3\sqrt{5} \cdot \left[\cos(\pi/3 + \pi/6) + i \operatorname{sen}(\pi/3 + \pi/6) \right]$$

$$= 3\sqrt{5} \left(\cancel{\cos \pi/2} + i \overbrace{\operatorname{sen} \pi/2}^1 \right)$$

$$= 3\sqrt{5} \cdot i$$



$$\text{ii) } z/w = \sqrt{5}/3 \left(\cos(\pi/3 - \pi/6) + i \operatorname{sen}(\pi/3 - \pi/6) \right)$$

$$= \sqrt{5}/3 \left(\cos \pi/6 + i \operatorname{sen} \pi/6 \right)$$

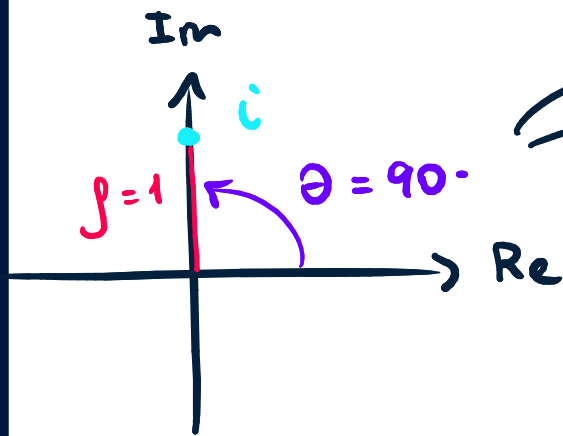


EXEMPLO 02

Calcule o produto de um número complexo qualquer pela unidade imaginária.

R:

$$\text{Seja } z = a + b \cdot i = \rho (\cos \theta + i \operatorname{sen} \theta)$$



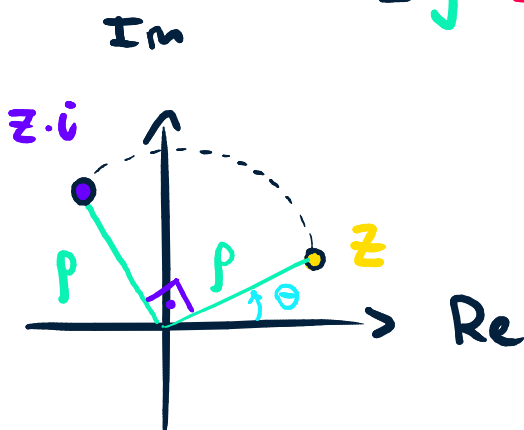
⇒ i na forma trigonométrica:

$$\begin{aligned} i &= \rho \cdot (\cos \theta + i \operatorname{sen} \theta) \\ &= 1 \cdot (\cos 90^\circ + i \operatorname{sen} 90^\circ) \end{aligned}$$

$$\hookrightarrow z \cdot i = (a + bi) \cdot i$$

$$= \rho (\cos \theta + i \operatorname{sen} \theta) \cdot 1 \cdot (\cos 90^\circ + i \operatorname{sen} 90^\circ)$$

$$= \rho \cdot 1 \cdot [\cos(\theta + 90^\circ) + i \operatorname{sen}(\theta + 90^\circ)]$$



$$z \cdot i = \rho [\cos(\theta + 90^\circ) + i \operatorname{sen}(\theta + 90^\circ)]$$



EXEMPLO 03

mostre que $\sin(a+b) = \sin a \cos b + \sin b \cos a$

e $\cos(a+b) = \cos a \cos b - \sin a \sin b$

R: Considere os complexos z_1 e z_2 :

$$z_1 = \underline{1} \cdot [\cos \alpha + i \sin \alpha] \text{ e } z_2 = \underline{1} \cdot [\cos \beta + i \sin \beta]$$

$$\text{Logo: } z_1 \cdot z_2 = \underline{1} \cdot \underline{1} \cdot [\underbrace{\cos(\alpha+\beta)}_{\text{Re}(z_1 \cdot z_2)} + i \underbrace{\sin(\alpha+\beta)}_{\text{Im}(z_1 \cdot z_2)}]$$

Mas, na brutalidade:

$$\begin{aligned} z_1 \cdot z_2 &= [\cos \alpha + i \sin \alpha] \cdot [\cos \beta + i \sin \beta] \\ &= \underline{\cos \alpha \cdot \cos \beta} + \underline{\sin \beta \cdot \cos \alpha \cdot i} + \underline{\sin \alpha \cos \beta \cdot i} + \\ &\quad - \underline{\sin \alpha \cdot \sin \beta} \end{aligned}$$

$$\cdot \underline{\text{Re}(z_1 \cdot z_2)} = \cos \alpha \cdot \cos \beta - \sin \alpha \sin \beta = \cos(\alpha + \beta)$$

$$\cdot \underline{\text{Im}(z_1 \cdot z_2)} = \sin \alpha \cos \beta + \sin \beta \cdot \cos \alpha = \sin(\alpha + \beta)$$

□



EXEMPLO 04 (ITA 1991)

Se $z = \cos t + i \sin t$, $0 < t < 2\pi$, mostre que $\frac{1+z}{1-\bar{z}}$ é igual a $i \cdot \tan(t/2)$

R:

$$\frac{1+z}{1-\bar{z}} = \frac{1 + \cos t + i \sin t}{(1 - \cos t) - i \sin t} \cdot \frac{[(1 - \cos t) + i \sin t]}{[(1 - \cos t) + i \sin t]}$$

$$= \frac{1 - \cancel{\cos t} + i \cancel{\sin t} + \cancel{\cos t} - \cancel{\cos^2 t} + i \cancel{\sin t} \cos t + \cancel{\sin t} - i \cancel{\sin^2 t} - \cancel{\sin t} \cos t}{2(1 - \cos t)}$$

$$= \frac{2 \cancel{\sin t}}{2(1 - \cos t)} \cdot i = \frac{\cancel{\sin t}}{(1 - \cos t)} \cdot i = \frac{2 \cancel{\sin(t/2)} \cancel{\cos(t/2)}}{2 \cancel{\cos(t/2)} \cancel{\cos(t/2)}} i$$

$$\cdot \sin t = 2 \sin(t/2) \cos(t/2)$$

$$\cdot \cos^2(t/2) = \frac{1 - \cos t}{2}$$

$$= \underline{\underline{\tan(t/2) \cdot i}}$$



EXEMPLO 05 (IME 1988)

Se $z \in \mathbb{C}$ mostre que $z + \bar{z}^{-1}$ é real apenas se $z \in \mathbb{R}$ ou $|z| = 1$

R: $z = a + bi$

$$w = z + \bar{z}^{-1} = a + bi + \frac{1}{(a+bi)} \cdot \frac{(a-bi)}{(a-bi)}$$

$$= (a+bi) \cdot \frac{(a^2+b^2)}{(a^2+b^2)} + \frac{a-bi}{a^2+b^2}$$

$$= \frac{a^3 + ab^2 + a^2b \cdot i + b^3 \cdot i + a - bi}{(a^2 + b^2)}$$

$$\text{Im}(w) = 0 \Rightarrow (a^2b + b^3 - b) = 0$$

$$b(a^2 + b^2 - 1) = 0$$

i) $b = 0$

✓ $\downarrow$
 $z \in \mathbb{R}$

ii) $a^2 + b^2 - 1 = 0$

$$a^2 + b^2 = 1$$

$$|z|^2 = 1$$

✓ $\downarrow$
 $|z| = 1$



~~PRIMEIRA FÓRMULA DE DE MOIVRE~~

Seja $z \in \mathbb{C}$ dado por $z = \rho (\cos \theta + i \sin \theta)$, logo:

$$z^n = \rho^n [\cos(n\theta) + i \sin(n\theta)]$$

Demonstração:

$$z^n = \underbrace{\rho (\cos \theta + i \sin \theta)} \cdot \underbrace{\rho (\cos \theta + i \sin \theta)} \cdot \dots \cdot \underbrace{\rho (\cos \theta + i \sin \theta)}_n$$

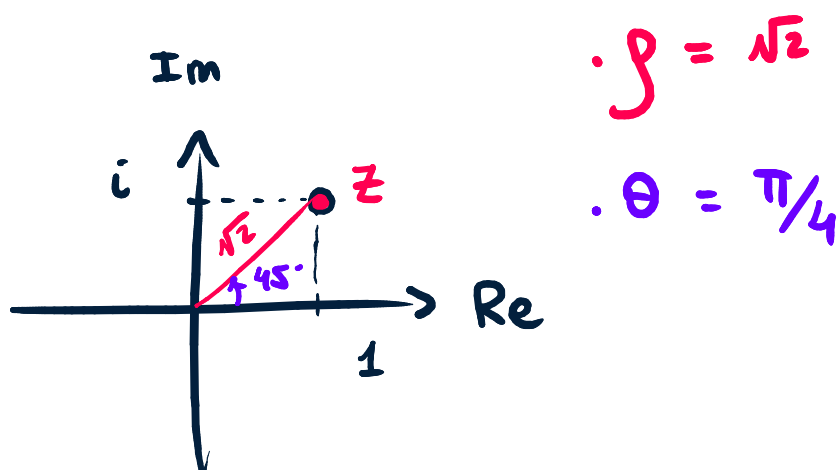
$$= \rho^n [\cos(n\theta) + i \sin(n\theta)] \quad (n \in \mathbb{R})$$



EXEMPLO 01

Seja $z = 1 - i$. Calcule $\bar{z}^{12}$

R: $\bar{z} = 1 + i = \sqrt{2} \cdot (\cos \pi/4 + i \sin \pi/4)$

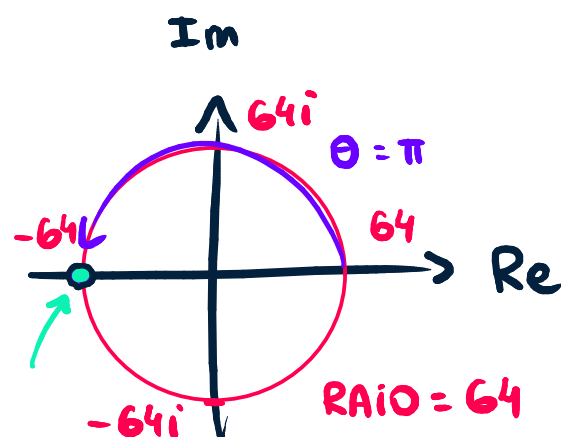


$$\bar{z}^{12} = \sqrt{2}^{12} \cdot [\cos(12 \cdot \pi/4) + i \sin(12 \cdot \pi/4)]$$

$$= 64 \cdot [\cos(\underbrace{3\pi}_{2\pi + \pi}) + i \sin(3\pi)]$$

$$= \underline{64} \cdot [\cos(\underline{\pi}) + i \sin(\underline{\pi})]$$

$$= -64$$

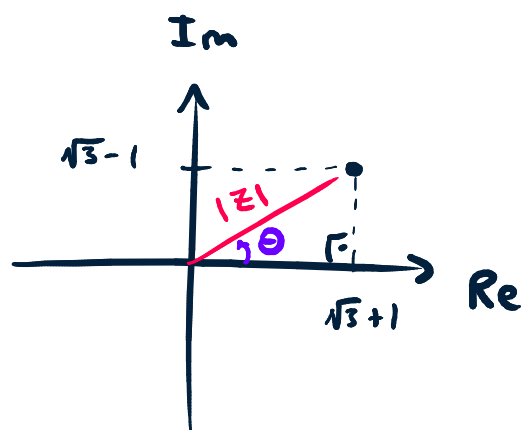


EXEMPLO 02

Seja $z = (1+i)(\sqrt{3}-i)$. Qual é o menor n , $n \in \mathbb{N}^*$, tal que $z^n \in \mathbb{R}$?

R: $z = (1+i)(\sqrt{3}-i) = \sqrt{3} - i + \sqrt{3}i + 1$
 $= (\sqrt{3}+1) + (\sqrt{3}-1) \cdot i$

i) $|z|^2 = (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2$
 $= 3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3} + 1$
 $= 8 \quad \therefore |z| = 2\sqrt{2}$



ii) $\text{sen } \theta = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$$\theta = \text{arc sen} \left(\frac{\sqrt{6}-\sqrt{2}}{4} \right) = 15^\circ$$

$$z = 2\sqrt{2} [\cos 15^\circ + i \text{sen } 15^\circ]$$

$$z^n = (2\sqrt{2})^n [\cos(n \cdot 15^\circ) + i \text{sen}(n \cdot 15^\circ)] \text{ , p/ } z^n \in \mathbb{R} :$$

$$\text{Im}(z^n) = 0 \quad \therefore (2\sqrt{2})^n \cdot \text{sen}(n \cdot 15^\circ) = 0 \quad \therefore \text{sen}(n \cdot 15^\circ) = 0$$

$$15^\circ \cdot n = 180^\circ K, \quad K \in \mathbb{Z} \quad | \quad n = 0, 12, 24, 36 \dots$$

$$n = \frac{180^\circ}{15^\circ} K = 12K$$



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R: $n = 12$

EXEMPLO 03 (AFA)

Seja $z \in \mathbb{C}$ tal que $\overline{|z| + z} = 2 - i$.

O número $(z - 3/4)^{1002}$ pertence a $\mathbb{R}_+$?

R : $z = a + bi$

$$\underbrace{\sqrt{a^2 + b^2}}_{\text{Re}} + \underbrace{a + bi}_{\text{Im}} = 2 - i$$

$$\underbrace{\sqrt{a^2 + b^2}}_{\text{Re}} + \underbrace{a}_{\text{Re}} - \underbrace{bi}_{\text{Im}} = \underbrace{2}_{\text{Re}} - \underbrace{1}_{\text{Im}} i$$

$$z = a + bi = \frac{3}{4} + i$$

$$z - \frac{3}{4} = i //$$

$$(z - \frac{3}{4})^{1002} = i^{1002} = -1 //$$

$$1002 = 250 \cdot \underline{\underline{4}} + 2$$

R : NÃO, $\notin \mathbb{R}_+$

i) $-b = -1$

$b = 1$

ii) $z = a + \sqrt{a^2 + b^2}$

$$2 - a = \sqrt{a^2 + 1}$$

$$4 - 4a + \cancel{a^2} = \cancel{a^2} + 1$$

$a = \frac{3}{4}$

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

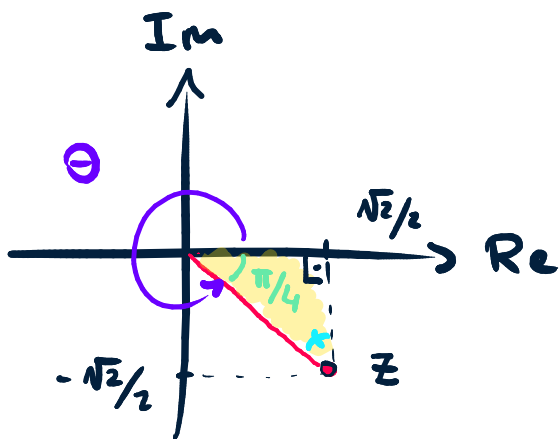
$$i^4 = 1$$



EXEMPLO 04 (ITA)

Mostre que $\left(\frac{\sqrt{2}}{1+i}\right)^{93} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

R : $z = \frac{\sqrt{2}}{(1+i)} \cdot \frac{(1-i)}{(1-i)} = \frac{\sqrt{2} - \sqrt{2}i}{1-i^2} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$



i) $\rho^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2$
 $= 2 \cdot \frac{2}{4} = 1$

$\rho = 1$

ii) $\Theta = 2\pi - \pi/4 \therefore \Theta = 7\pi/4$
 $= -\pi/4$

$z = 1 \cdot \left[\cos(-\pi/4) + i \sin(-\pi/4) \right]$

$z = 1^{93} \cdot \left[\cos(93 \cdot (-\pi/4)) + i \sin(93 \cdot (-\pi/4)) \right]$

$= 1 \cdot \left[\underbrace{\cos(-5\pi/4)}_{-\sqrt{2}/2} + i \underbrace{\sin(-5\pi/4)}_{\sqrt{2}/2} \right]$

$= -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

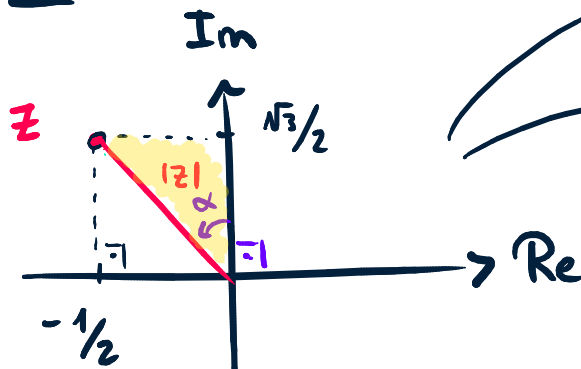


EXEMPLO 05 (ITA)

Seja $z = \frac{1}{2}(-1 + \sqrt{3}i)$.

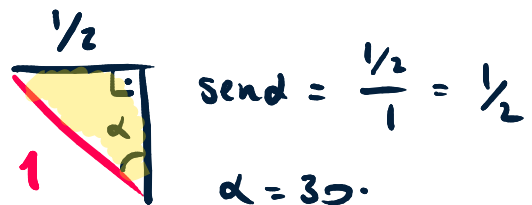
Mostre que $\sum_{n=1}^{89} z^n = -1$

R :



i) $\rho = 1$

ii) $\theta = 90^\circ + \alpha = 120^\circ = \frac{2\pi}{3}$



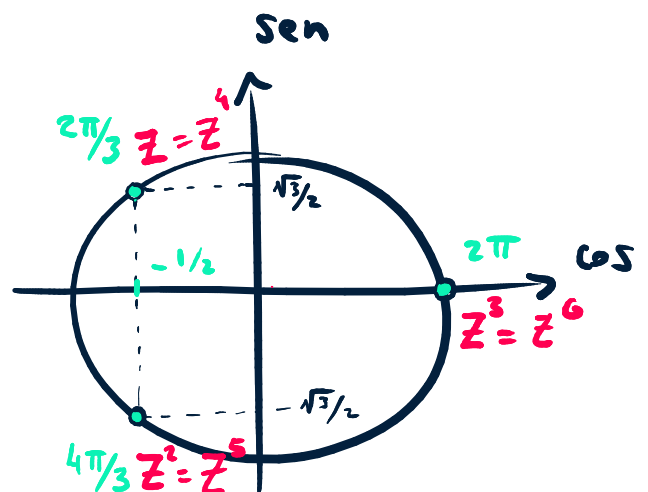
$z = 1 \cdot (\cos \frac{2\pi}{3} + i \text{sen} \frac{2\pi}{3})$

$$z^n = 1^n \cdot [\cos(n \cdot \frac{2\pi}{3}) + i \text{sen}(n \cdot \frac{2\pi}{3})]$$

$z^1 = 1^1 \cdot [\cos(\frac{1 \cdot 2\pi}{3}) + i \text{sen}(\frac{1 \cdot 2\pi}{3})]$

$z^2 = 1^2 \cdot [\cos(\frac{2 \cdot 2\pi}{3}) + i \text{sen}(\frac{2 \cdot 2\pi}{3})]$

$z^3 = 1^3 \cdot [\cos(\frac{3 \cdot 2\pi}{3}) + i \text{sen}(\frac{3 \cdot 2\pi}{3})]$



$$\begin{aligned} z^1 + z^2 + z^3 &= \cos(\frac{2\pi}{3}) + \cos(\frac{4\pi}{3}) + \cos(2\pi) + i \left[\text{sen}(\frac{2\pi}{3}) + \text{sen}(\frac{4\pi}{3}) + \text{sen}(2\pi) \right] \\ &= 0 \end{aligned}$$



$$\sum_{n=1}^{89} z^n =$$

$$= \underbrace{z^1 + z^2 + z^3}_{\text{zero}} + \underbrace{z^4 + z^5 + z^6}_{\text{zero}} + \dots + \underbrace{z^{85} + z^{86} + z^{87}}_{\text{zero}} + z^{88} + z^{89}$$

$$\sum_{n=1}^{89} z^n = z^{88} + z^{89} = z + z^2$$

$$z = 1 \cdot \left[\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right) \right]$$

$$+ z^2 = 1 \cdot \left[\cos\left(\frac{4\pi}{3}\right) + i \sin\left(\frac{4\pi}{3}\right) \right]$$

$$z + z^2 = \cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{4\pi}{3}\right) + \left[\sin\left(\frac{2\pi}{3}\right) + \sin\left(\frac{4\pi}{3}\right) \right] i$$

$$= -\frac{1}{2} - \frac{1}{2} + \underbrace{\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right)}_{\text{zero}} i$$

$$= -1$$

$$\sum_{n=1}^{89} z^n = -1$$

□



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SEGUNDA FÓRMULA DE DE MOIVRE

Seja $z \in \mathbb{C}$ dado por $z = \rho (\cos \theta + i \operatorname{sen} \theta)$, logo:

$$\sqrt[n]{z} = \sqrt[n]{\rho} \left[\cos \left(\frac{\theta + 2k\pi}{n} \right) + i \operatorname{sen} \left(\frac{\theta + 2k\pi}{n} \right) \right]$$

$$k \in \mathbb{N} \mid k < n$$

Demonstração:

$$\sqrt[n]{z} = \omega \iff \omega^n = z$$

$$\text{Se } \omega = \rho_\omega [\cos \theta_\omega + i \operatorname{sen} \theta_\omega]$$

$$\text{e } z = \rho [\cos \theta + i \operatorname{sen} \theta]$$

$$\omega^n = z$$

$$\rho_\omega^n [\cos(n \cdot \theta_\omega) + i \operatorname{sen}(n \cdot \theta_\omega)] = \rho [\cos \theta + i \operatorname{sen} \theta]$$

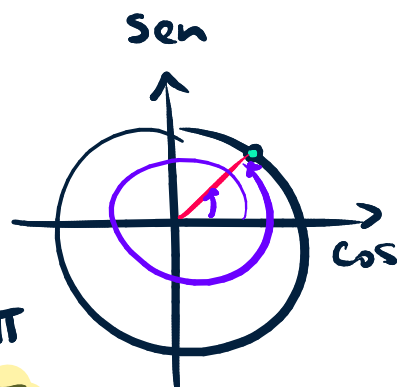
$$\text{i) } \rho_\omega^n = \rho \quad \therefore \rho_\omega = \sqrt[n]{\rho}$$

$$\text{ii) } \cos(n \cdot \theta_\omega) = \cos \theta \Rightarrow n \cdot \theta_\omega = \theta + 2k\pi$$

$$\operatorname{sen}(n \cdot \theta_\omega) = \operatorname{sen} \theta$$



$$\theta_\omega = \frac{\theta + 2k\pi}{n}$$



EXEMPLO 01

Resolva a equação

a) $z^3 - 1 = 0$

b) $z^3 - 8 = 0$

c) $z^3 - 27 = 0$



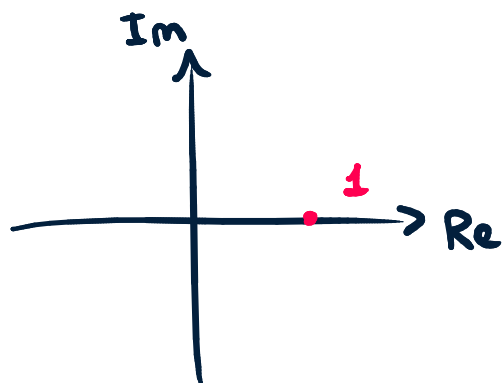
R:

$$a) z^3 = 1 \quad \therefore z = \sqrt[3]{1}$$

$$\boxed{z}$$

$$z = \rho [\cos \theta + i \sin \theta] \rightarrow \sqrt[3]{z} = \sqrt[3]{\rho} \left[\cos \left(\frac{\theta + 2k\pi}{3} \right) + i \sin \left(\frac{\theta + 2k\pi}{3} \right) \right]$$

$$1 = 1 \cdot [\cos 0 + i \sin 0] \rightarrow \sqrt[3]{1} = 1 \left[\cos \left(\frac{0 + 2k\pi}{3} \right) + i \sin \left(\frac{0 + 2k\pi}{3} \right) \right]$$



$$i) k=0: \boxed{z_0 = 1}$$

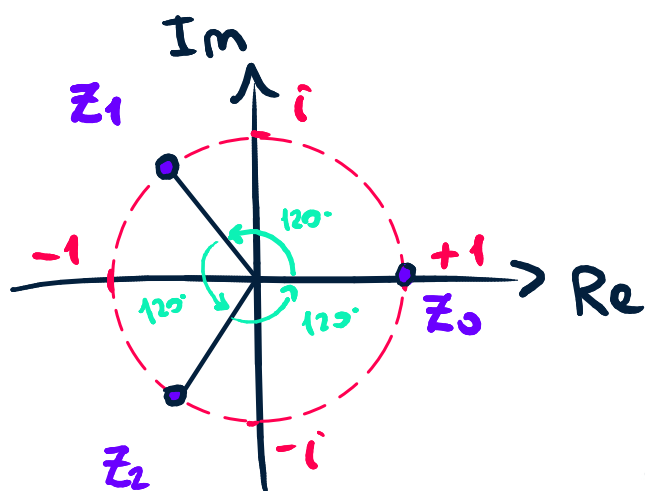
$$ii) k=1: z_1 = \cos(2\pi/3) + i \sin(2\pi/3)$$

$$\boxed{z_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i}$$

$$iii) k=2: z_2 = \cos(4\pi/3) + i \sin(4\pi/3)$$

$$\boxed{z_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i}$$

$$iv) k=3: z_3 = z_0$$



$$b) z^3 = 8 \quad \therefore z = \sqrt[3]{8}$$

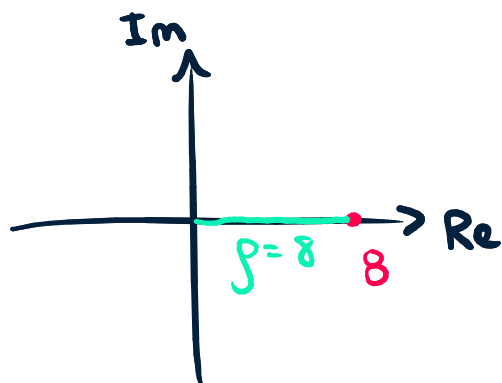
$$\boxed{z}$$

$$\boxed{\sqrt[3]{z}}$$

$$z = \rho [\cos \theta + i \sin \theta] \rightarrow \sqrt[3]{z} = \sqrt[3]{\rho} \left[\cos\left(\frac{\theta + 2k\pi}{3}\right) + i \sin\left(\frac{\theta + 2k\pi}{3}\right) \right]$$

$$8 = 8 \cdot [\cos 0 + i \sin 0] \rightarrow \sqrt[3]{z} = \sqrt[3]{8} \left[\cos\left(\frac{0 + 2k\pi}{3}\right) + i \sin\left(\frac{0 + 2k\pi}{3}\right) \right]$$

$$= 2 \left[\cos\left(\frac{0 + 2k\pi}{3}\right) + i \sin\left(\frac{0 + 2k\pi}{3}\right) \right]$$



$$i) k=0 : z_1 = 2(1 + i \cdot 0)$$

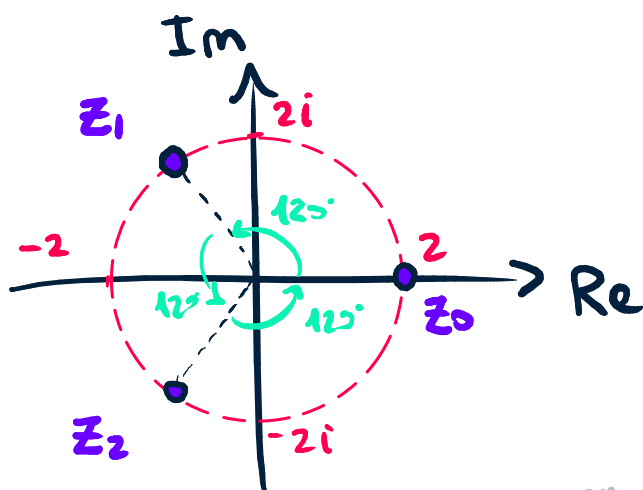
$$\boxed{z_1 = 2}$$

$$ii) k=1 : z_1 = 2 \cdot \left[\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right) \right] = 2 \left[-\frac{1}{2} + \frac{\sqrt{3}}{2} i \right]$$

$$\boxed{z_1 = -1 + \sqrt{3} i}$$

$$iii) k=2 : \boxed{z_2 = -1 - \sqrt{3} i}$$

$$k=3$$



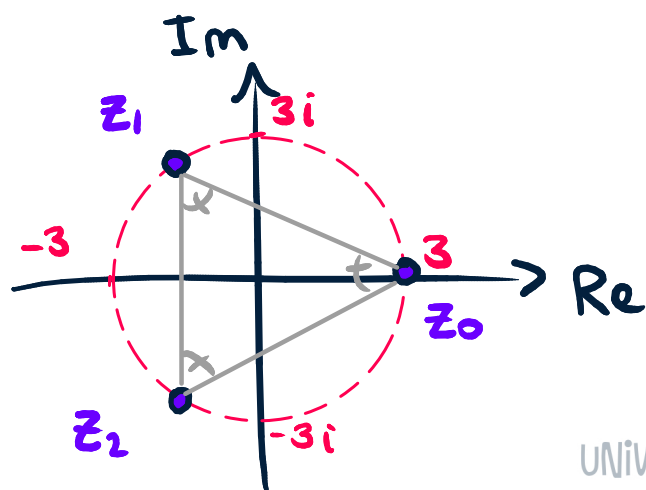
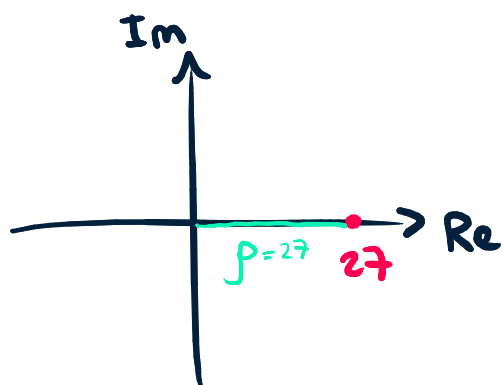
$$c) \quad z^3 = 27 \quad \therefore \quad z = \sqrt[3]{27}$$

$$\boxed{z}$$

$$\boxed{\sqrt[3]{z}}$$

$$z = \rho [\cos \theta + i \sin \theta] \rightarrow \sqrt[3]{z} = \sqrt[3]{\rho} \left[\cos \left(\frac{\theta + 2k\pi}{3} \right) + i \sin \left(\frac{\theta + 2k\pi}{3} \right) \right]$$

$$27 = 27 \cdot [\cos 0 + i \sin 0] \rightarrow = 3 \left[\cos \left(\frac{0 + 2k\pi}{3} \right) + i \sin \left(\frac{0 + 2k\pi}{3} \right) \right]$$



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EXEMPLO 02

Resolva a equação

a) $z^4 - 1 = 0$

b) $z^4 - 16 = 0$

c) $z^4 - 81 = 0$

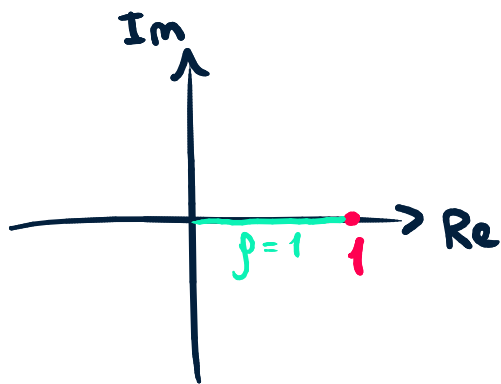


$$a) z^4 - 1 = 0 \quad \therefore z^4 = 1 \quad \therefore z = \sqrt[4]{1}$$

$$\boxed{\omega}$$

$$\omega = \rho [\cos \theta + i \sin \theta]$$

$$\omega = 1 \cdot [\cos 0 + i \sin 0]$$



$K < 4$

$$\boxed{\sqrt[4]{\omega}}$$

$$\xrightarrow{\quad} \sqrt[4]{\omega} = \sqrt[4]{\rho} \left[\cos\left(\frac{\theta + 2k\pi}{4}\right) + i \sin\left(\frac{\theta + 2k\pi}{4}\right) \right]$$

$$\xrightarrow{\quad} \sqrt[4]{\omega} = 1 \left[\cos\left(\frac{0 + 2k\pi}{4}\right) + i \sin\left(\frac{0 + 2k\pi}{4}\right) \right]$$

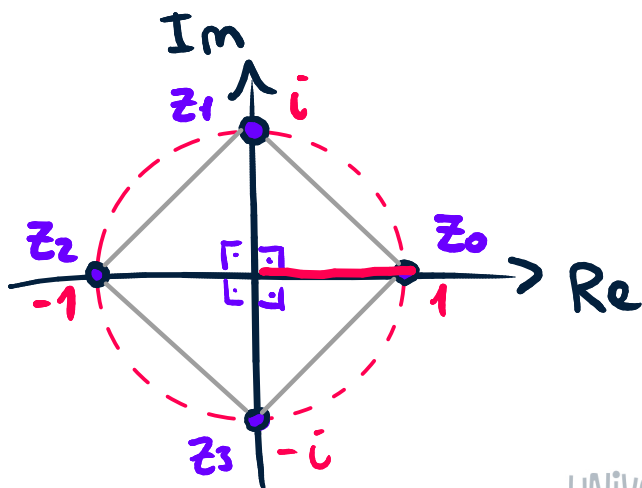
$$i) K=0 \therefore \boxed{z_0 = 1}$$

$$ii) K=1 \therefore \boxed{z_1 = i}$$

$$iii) K=2 \therefore \boxed{z_2 = -1}$$

$$iv) K=3 \therefore \boxed{z_3 = -i}$$

$$v) K=4 \therefore \cancel{z_4 = z_0}$$



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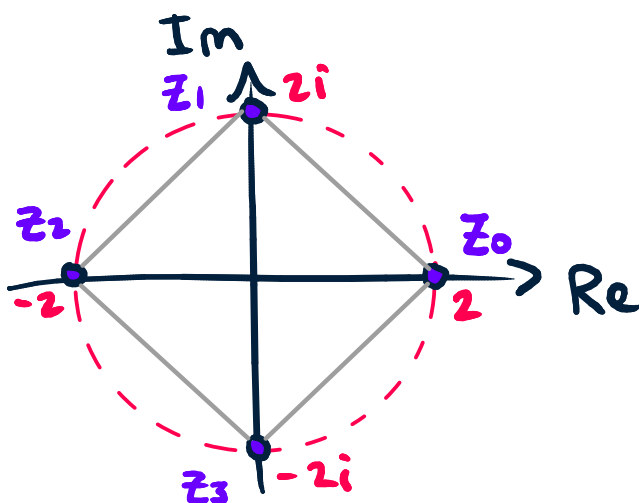
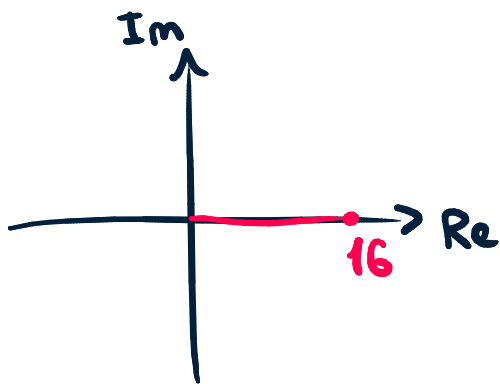
$$b) z^4 - 16 = 0 \quad \therefore z^4 = 16 \quad \therefore z = \sqrt[4]{16}$$

$$\boxed{\omega}$$

$$\boxed{\sqrt[4]{\omega}}$$

$$\omega = \rho [\cos \theta + i \sin \theta] \rightarrow \sqrt[4]{z} = \sqrt[4]{\rho} \left[\cos \left(\frac{\theta + 2k\pi}{4} \right) + i \sin \left(\frac{\theta + 2k\pi}{4} \right) \right]$$

$$\omega = 16 \cdot [\cos 0^\circ + i \sin 0^\circ] \rightarrow =$$



$$\boxed{z_0 = 2}$$

$$\boxed{z_1 = 2i}$$

$$\boxed{z_2 = -2}$$

$$\boxed{z_3 = -2i}$$



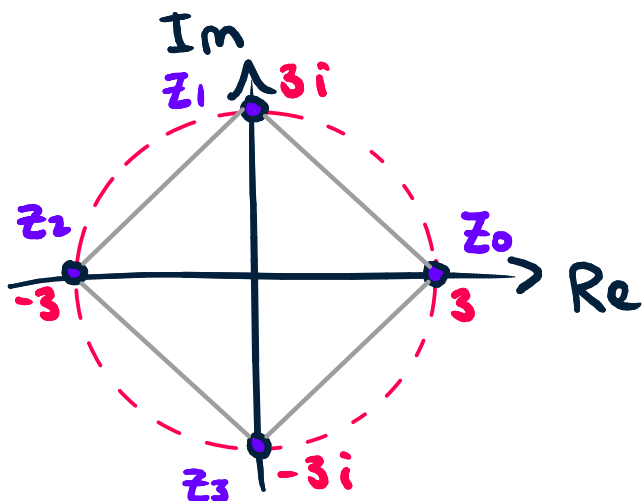
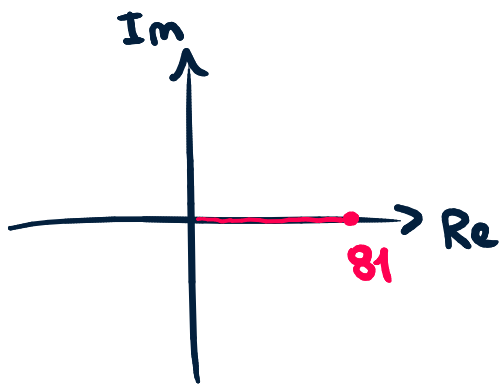
$$c) z^4 - 81 = 0 \quad \therefore z^4 = 81 \quad \therefore z = \sqrt[4]{81}$$

$$\boxed{\omega}$$

$$\boxed{\sqrt[4]{\omega}}$$

$$\omega = \rho [\cos \theta + i \sin \theta] \rightarrow \sqrt[4]{z} = \sqrt[4]{\rho} \left[\cos \left(\frac{\theta + 2k\pi}{4} \right) + i \sin \left(\frac{\theta + 2k\pi}{4} \right) \right]$$

$$\omega = 81 \cdot [\cos 0 + i \sin 0] \rightarrow =$$



$$\boxed{z_0 = 3}$$

$$\boxed{z_1 = 3i}$$

$$\boxed{z_2 = -3}$$

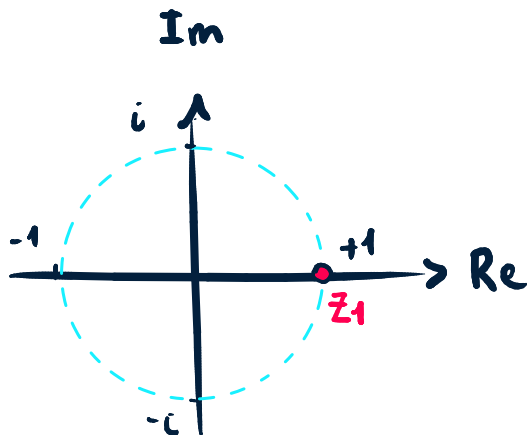
$$\boxed{z_3 = -3i}$$



INTERPRETAÇÃO GEOMÉTRICA

(i) Solução de $z^n - 1 = 0$

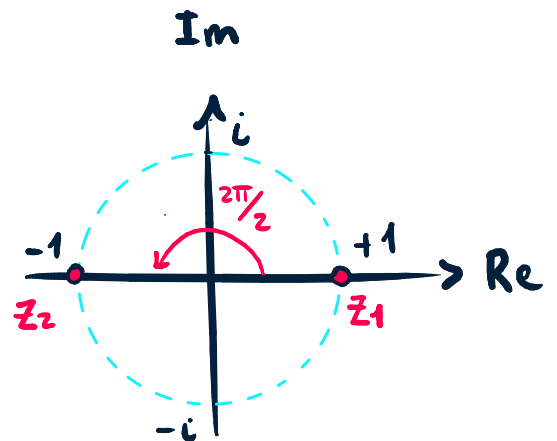
$n = 1$



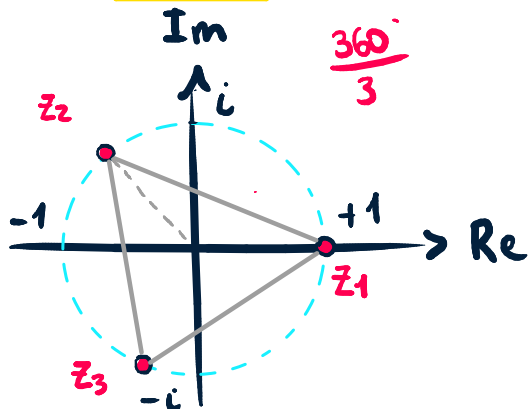
$$(z^n)^{1/n} = (1)^{1/n}$$

$$z = \sqrt[n]{1}$$

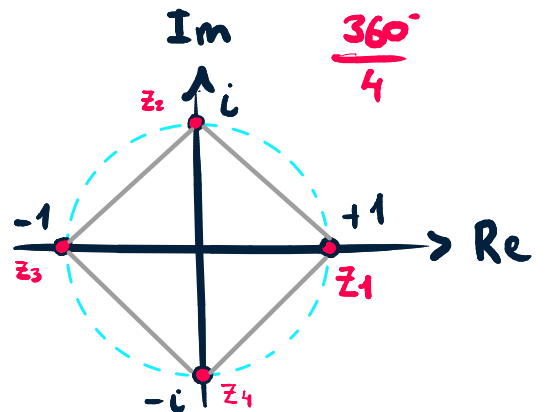
$n = 2$



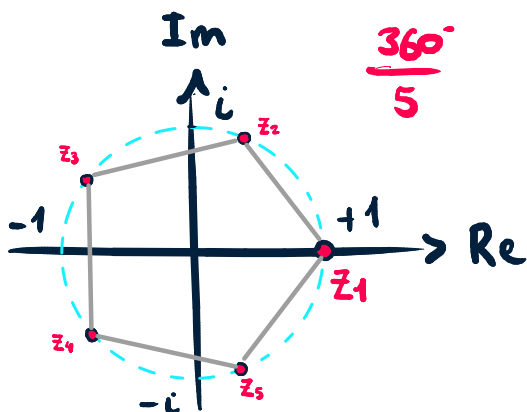
$n = 3$



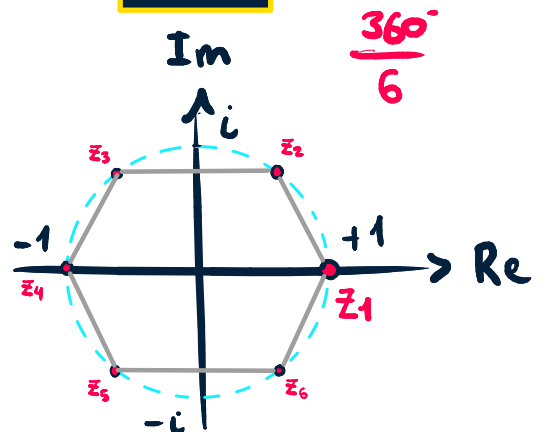
$n = 4$



$n = 5$

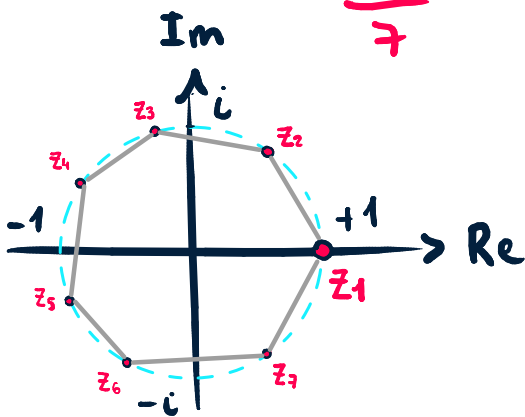


$n = 6$



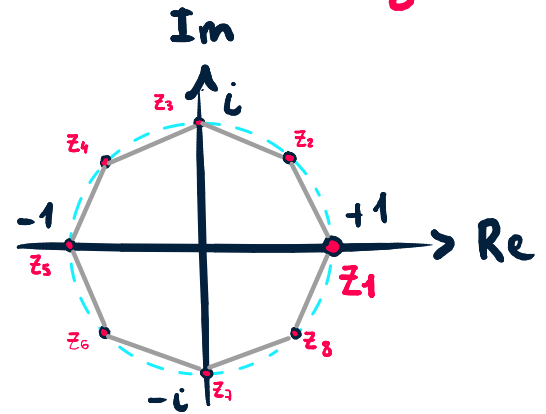
$$n = 7$$

$$\frac{360^\circ}{7}$$



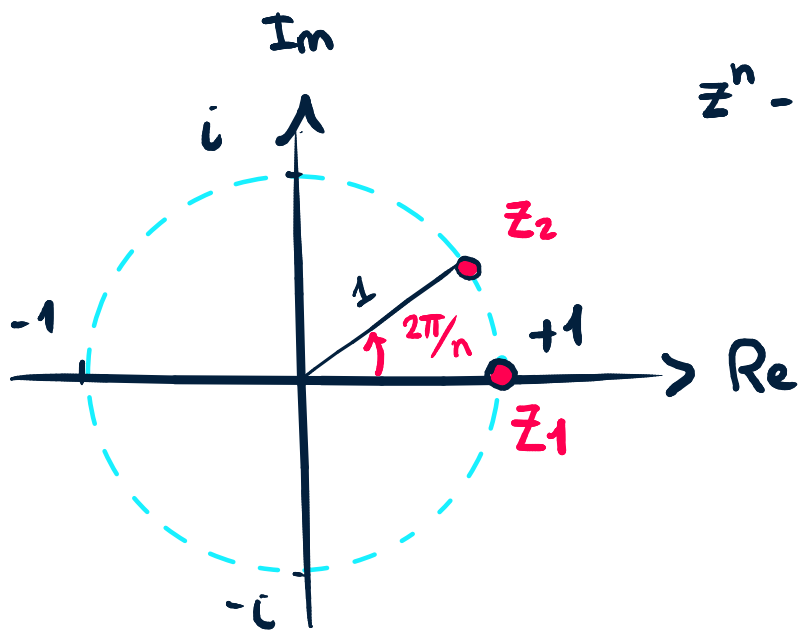
$$n = 8$$

$$\frac{360^\circ}{8}$$



n qualquer :

$$z^n - 1 = 0$$



As raízes

serão vertices de um poligono regular de n lados inscrito numa circunferência de raio 1.



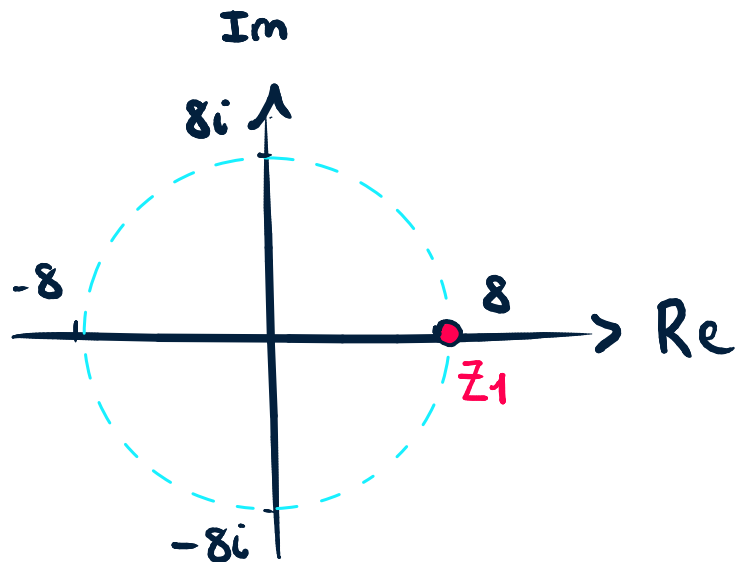
INTERPRETAÇÃO GEOMÉTRICA

(ii) Solução de $z^n - k = 0$, $k \in \mathbb{R}_+$

$$z^1 - 8 = 0$$

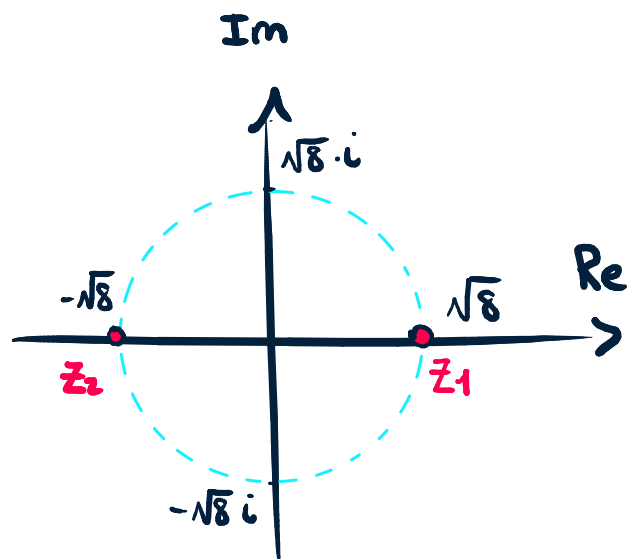
$$n = 1$$

$$(k = 8)$$



$$n = 2$$

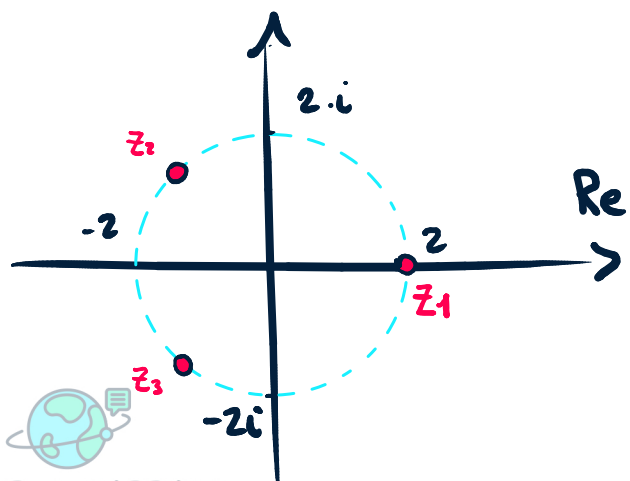
$$(k = 8)$$



$$z^3 - 8 = 0$$

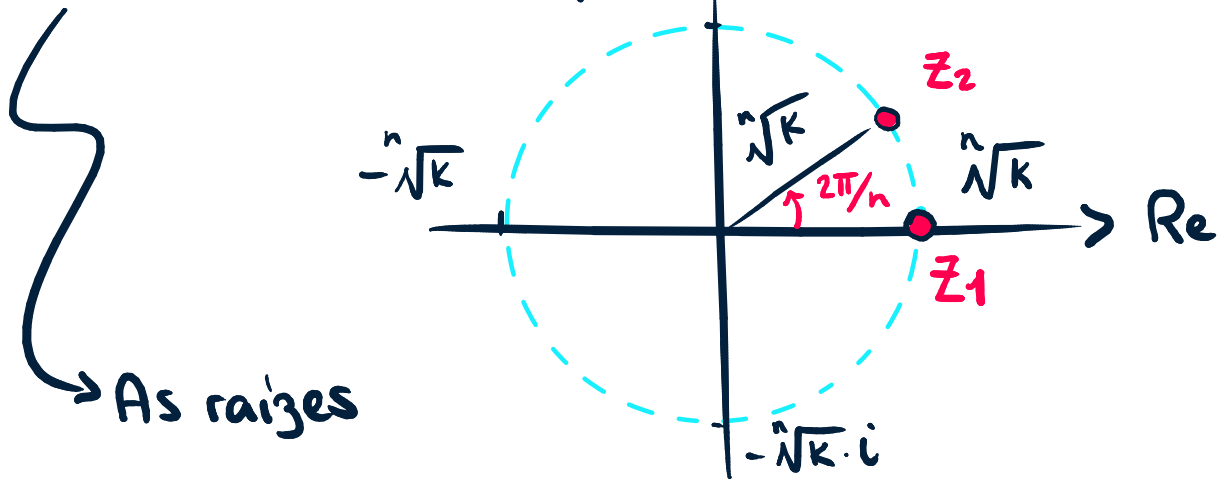
$$n = 3$$

$$(k = 8)$$



$$z^n - K = 0$$

n qualquer :



As raízes

serão vertices de um poligono regular de n lados inscrito numa circunferência de raio $\sqrt[n]{K}$



EXEMPLO 03

Calcule $\sqrt[3]{i}$

R:

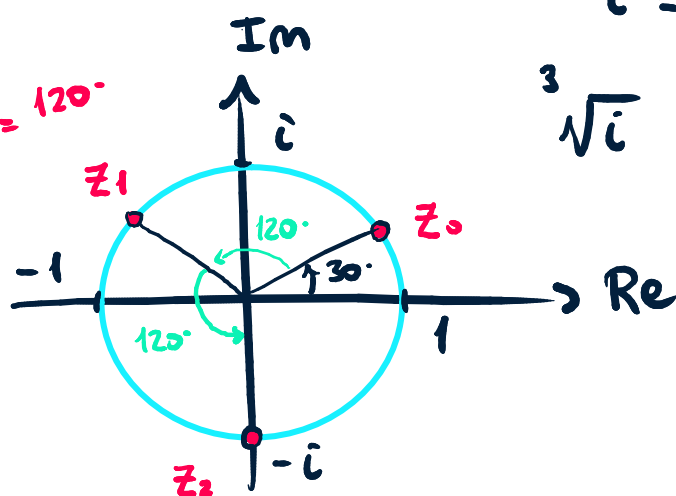
$$i = 1 \cdot [\cos 90^\circ + i \cdot \sin 90^\circ]$$

$$\sqrt[3]{i} = 1 \left\{ \cos \left[\frac{90 + 2K\pi}{3} \right] + \right.$$

$$\left. + i \sin \left[\frac{90 + 2K\pi}{3} \right] \right\}$$

$K = 0$

$$Z_0 = 1 \cdot [\cos 30^\circ + i \sin 30^\circ]$$



$$(i) \quad Z_0 = 1 \cdot [\cos 30^\circ + i \sin 30^\circ] = \frac{\sqrt{3}}{2} + \frac{i}{2}$$

$$(ii) \quad Z_1 = 1 \cdot [\cos 150^\circ + i \sin 150^\circ] = -\frac{\sqrt{3}}{2} + \frac{i}{2}$$

$$(ii) \quad Z_2 = 1 \cdot [\underbrace{\cos 270^\circ}_0 + i \underbrace{\sin 270^\circ}_{-1}] = -i$$



EXEMPLO 04

Resolva a equação $z^5 = \bar{z}$

R: $z^5 = \bar{z}$

$$|z^5| = |\bar{z}|$$

$$|z|^5 = |z|$$

i) $z = 0$

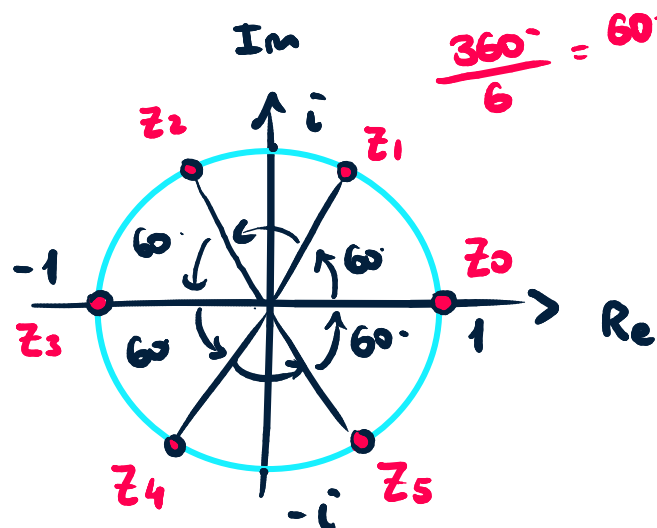
ii) $\frac{|z|^5}{|z|} = \frac{|z|}{|z|} \quad \therefore |z|^4 = 1 \quad \therefore |z| = 1$

iii) $z^5 = \bar{z} \quad \cdot z$

$$z^6 = z \cdot \bar{z}$$

$$z^6 = |z|^2$$

$$z^6 = 1$$



Sol.: $z = \left\{ 0, \overset{z_0}{1}, \overset{z_1}{\frac{1}{2} + i\frac{\sqrt{3}}{2}}, -\frac{1}{2} + i\frac{\sqrt{3}}{2}, -1, \right.$
 $\left. -\frac{1}{2} - i\frac{\sqrt{3}}{2}, +\frac{1}{2} - i\frac{\sqrt{3}}{2} \right\}$

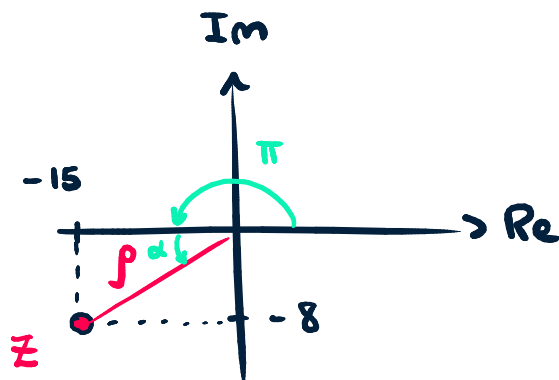
EXEMPLO 05 (IME)

Qual é a parte real do número $\sqrt{-15-8i}$?

R $\therefore Z = -15 - 8i$

$$\rho^2 = 15^2 + 8^2 = 289$$

$$\rho = 17$$



$$\cos \alpha = \frac{15}{17}, \quad \sin \alpha = \frac{8}{17}$$

$$Z = \rho \cdot [\cos \Theta + i \sin \Theta]$$

$$\Theta = \pi + \alpha$$

$$Z = 17 \cdot [\cos \Theta + i \sin \Theta]$$

$$\sin \Theta = -\frac{8}{17}$$

$$\cos \Theta = -\frac{15}{17}$$

$$W = \sqrt[2]{Z} = \sqrt[2]{17} \cdot \left[\cos\left(\frac{\Theta + 2K\pi}{2}\right) + i \sin\left(\frac{\Theta + 2K\pi}{2}\right) \right]$$

i) K=0 : $W_0 = \sqrt{17} \left[\cos(\Theta/2) + i \sin(\Theta/2) \right]$

$$\cos(\Theta/2) = -\sqrt{\frac{1 + \cos \Theta}{2}} = -\frac{1}{\sqrt{17}}$$

$$W_0 = \sqrt{17} \left[-\frac{1}{\sqrt{17}} + i \frac{4}{\sqrt{17}} \right]$$

$$\sin(\Theta/2) = +\sqrt{\frac{1 - \cos \Theta}{2}} = \frac{4}{\sqrt{17}}$$

$$W_0 = -1 + 4i$$

$$i) \underline{K=1} : \omega_1 = \sqrt{17} \left[\underbrace{\cos(\theta/2 + \pi)}_{-\cos(\theta/2)} + i \underbrace{\sin(\theta/2 + \pi)}_{-\sin(\theta/2)} \right]$$

$$\omega_1 = \sqrt{17} \left[\frac{1}{\sqrt{17}} + i \left(\frac{-4}{\sqrt{17}} \right) \right]$$

$$\omega_1 = 1 - 4i$$

$$\underline{R} : 1 \text{ ou } -1$$

