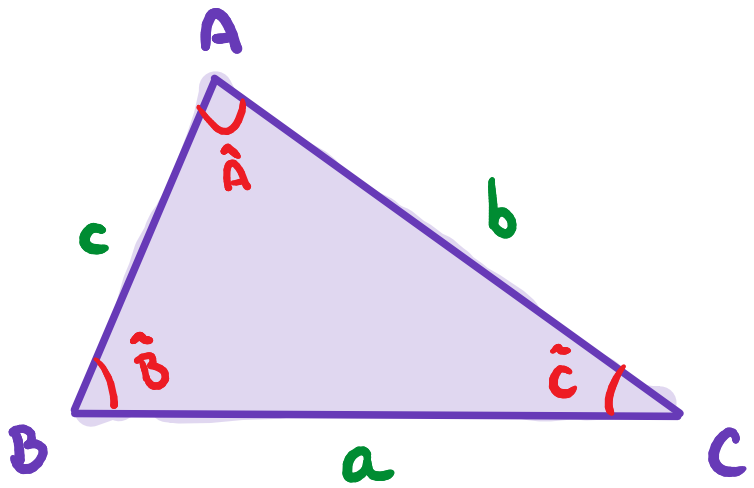
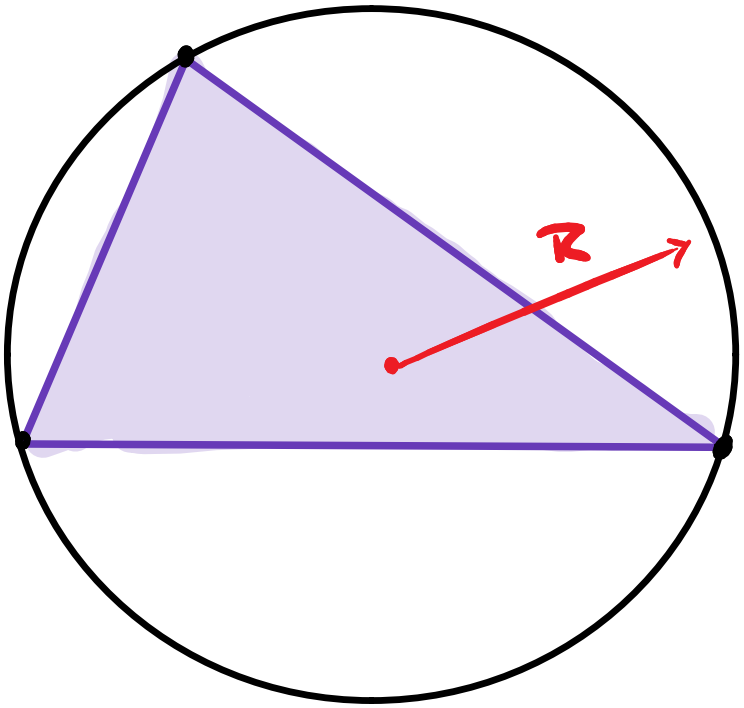
TRIGONOMETRIA EM TRIÂNGULOS

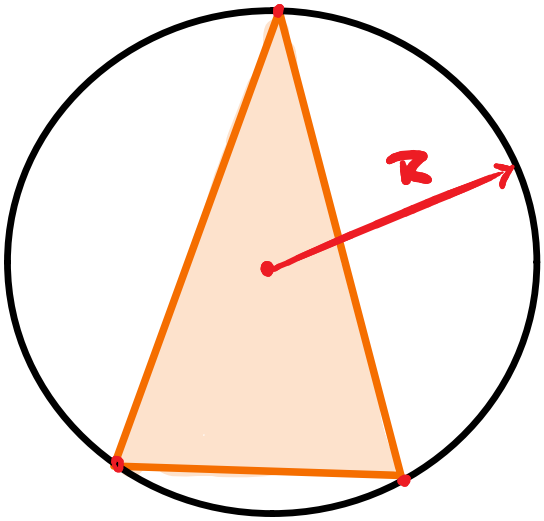
LEI DOS SENOS

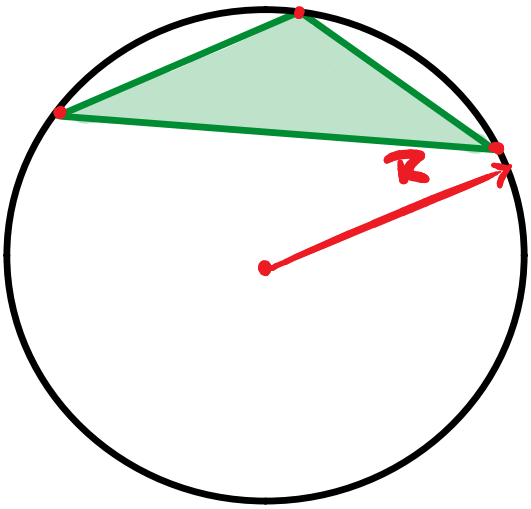


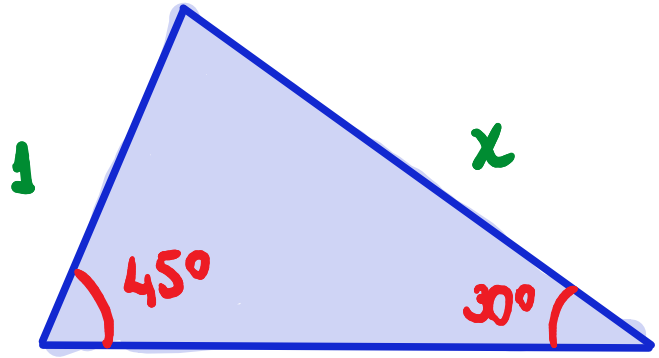
$$\frac{a}{\operatorname{sen}\hat{A}} = \frac{b}{\operatorname{sen}\hat{B}} = \frac{c}{\operatorname{sen}\hat{C}} = 2.R$$

R: RAIO DA CIRCUNFERÊNCIA CIRCUNSCRITA







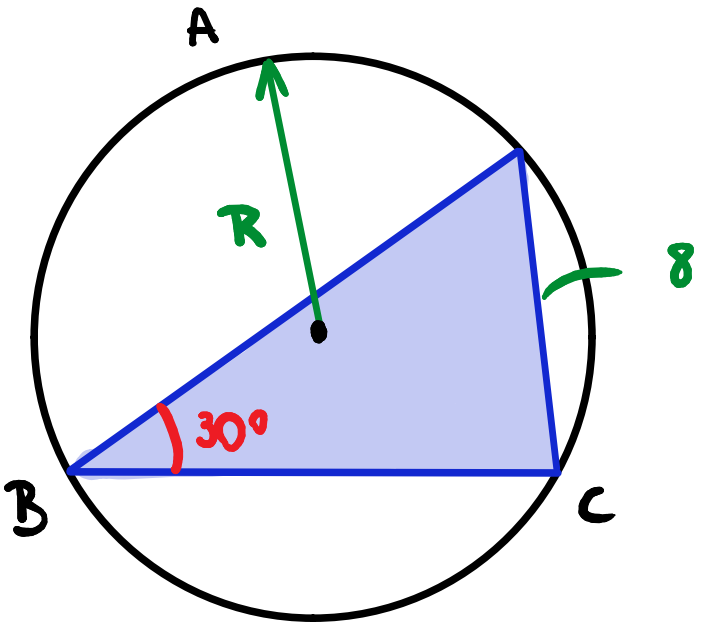


$$\frac{x}{\operatorname{sen} 45^\circ} = \frac{1}{\operatorname{sen} 30^\circ}$$

$$x \cdot \operatorname{sen} 30^\circ = 1 \cdot \operatorname{sen} 45^\circ$$

$$x \cdot \frac{1}{2} = 1 \cdot \frac{\sqrt{2}}{2}$$

$$x = \sqrt{2}$$



$$\frac{\overset{4}{\cancel{8}}}{\text{Sen } 30^\circ} = \cancel{2}.R$$

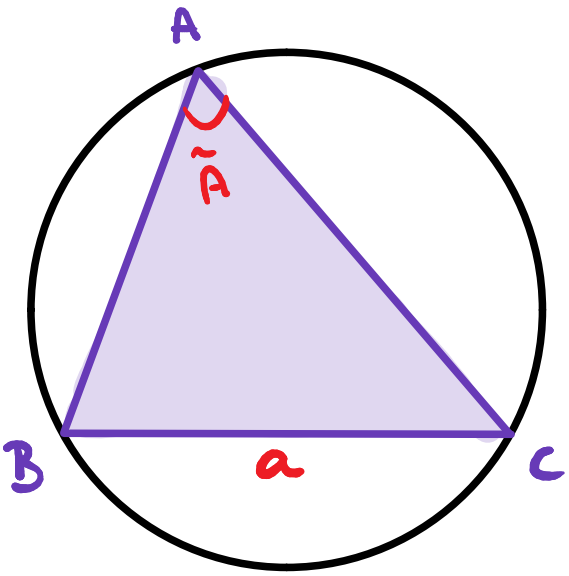
$$\frac{4}{1/2} = R$$

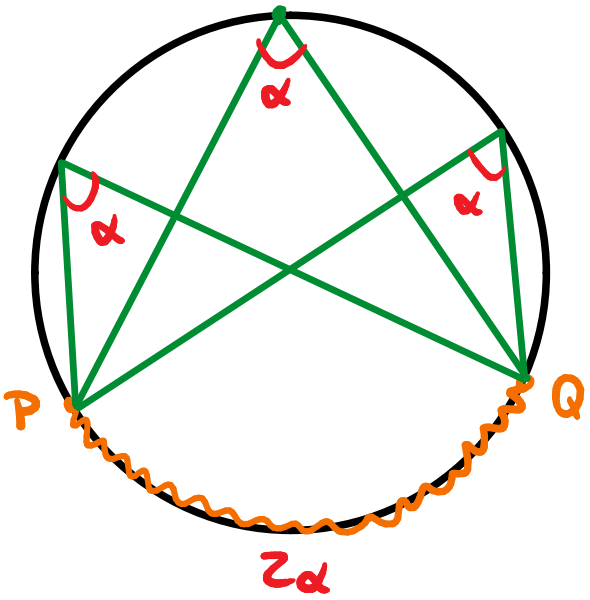
$$R = 4.\frac{2}{1}$$

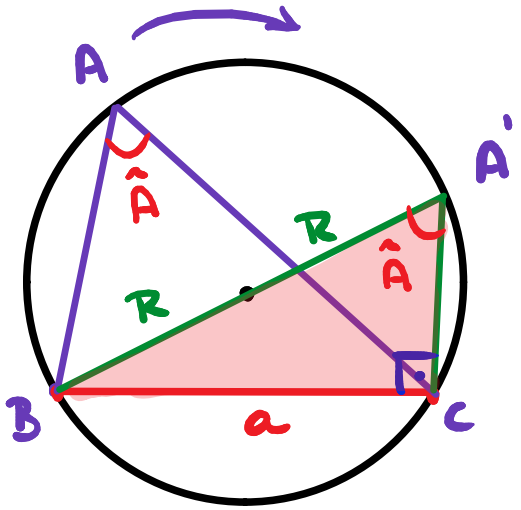
$$R = 8$$

LEI DOS SENOS

DEMONSTRAÇÃO







$$\operatorname{sen} \hat{A} = \frac{a}{2R}$$

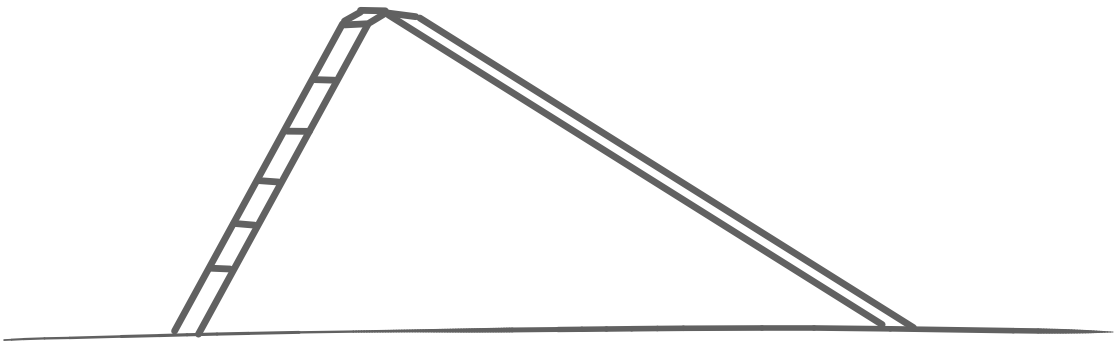
$$\frac{a}{\operatorname{sen} \hat{A}} = 2R$$

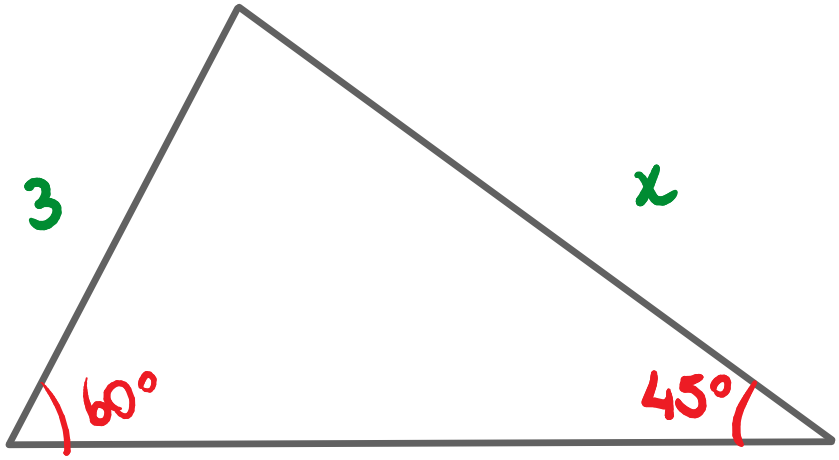
$$\frac{a}{\operatorname{sen} \hat{A}} = \frac{b}{\operatorname{sen} \hat{B}} = \frac{c}{\operatorname{sen} \hat{C}} = 2R$$

EXEMPLO

NO ESCORREDOR ABAIXO, A ESCADA FORMA 60° COM O SOLO E A RAMPA LISA PARA DESLIZAR FORMA 45° COM O SOLO.

SE A ESCADA TEM COMPRIMENTO DE 3m, QUAL O COMPRIMENTO DA RAMPA?





$$\frac{x}{\operatorname{sen} 60^\circ} = \frac{3}{\operatorname{sen} 45^\circ}$$

$$x \cdot \frac{\sqrt{2}}{2} = 3 \cdot \frac{\sqrt{3}}{2}$$

$$x = \frac{3 \cdot \sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = \frac{3\sqrt{6}}{2}$$

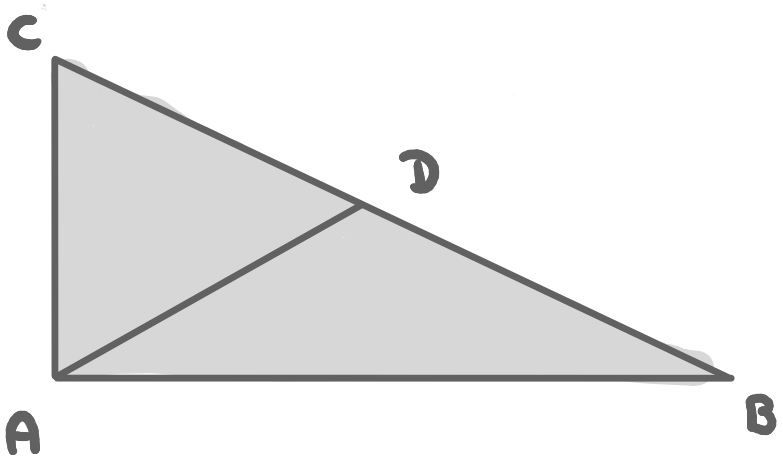
EXEMPLO

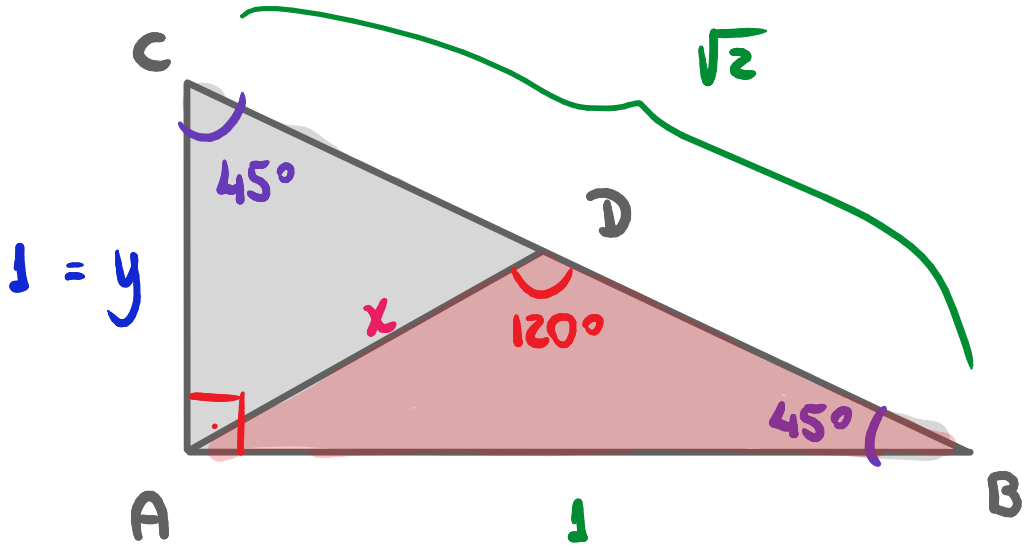
NO TRIÂNGULO ABC ABAIXO, TEM-SE:

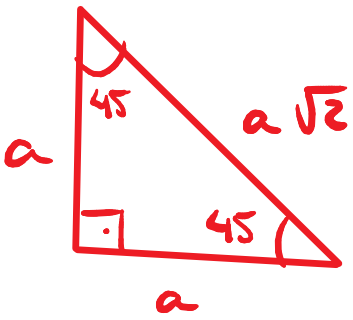
$B\hat{A}C = 90°$; $A\hat{D}B = 120°$

$AB = 1$; $BC = \sqrt{2}$

CALCULE O COMPRIMENTO DE AD.







$$y^2 + 1^2 = \sqrt{2}^2$$

$$y = 1$$

$$\frac{x}{\operatorname{sen} 45^\circ} = \frac{1}{\operatorname{sen} 120^\circ}$$

$$x \cdot \frac{\sqrt{3}}{2} = 1 \cdot \frac{\sqrt{2}}{2}$$

$$x = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{x = \frac{\sqrt{6}}{3}}$$

EXEMPLO

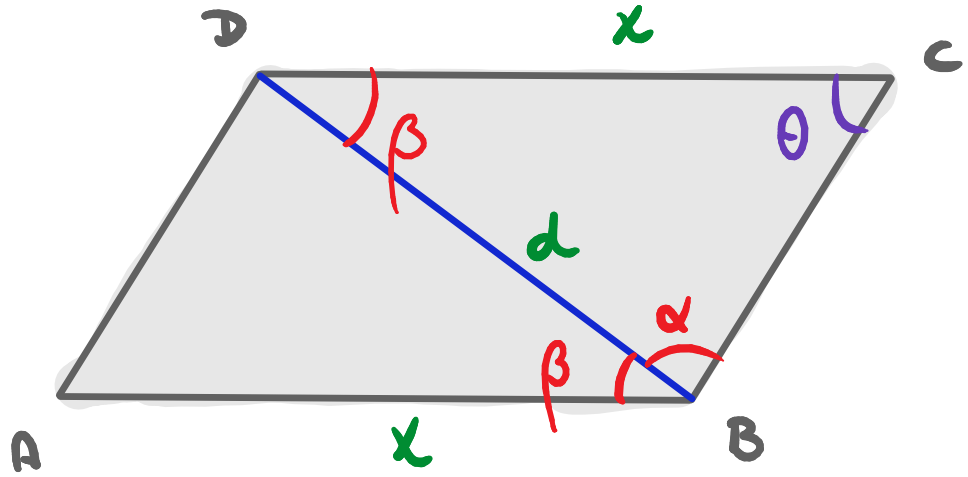
SEJA O PARALELOGRAMO ABCD COM:

ÂNGULO $A\hat{B}D = \beta$

ÂNGULO $C\hat{B}D = \alpha$

DIAGONAL BD = d

CALCULE O LADO AB EM FUNÇÃO DOS PARÂMETROS DADOS.



$$\frac{x}{\operatorname{sen}\alpha} = \frac{d}{\operatorname{sen}\theta} \rightarrow x = d \cdot \frac{\operatorname{sen}\alpha}{\operatorname{sen}\theta}$$

$$(\alpha + \beta) + (\theta) = 180^\circ$$

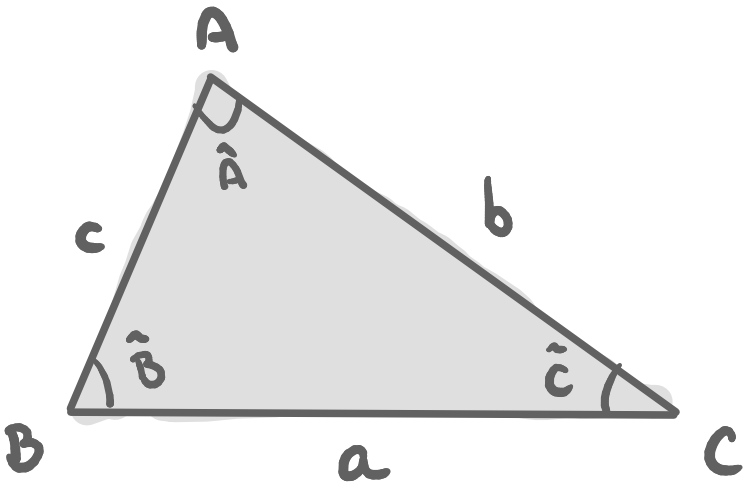
$$\operatorname{sen}\theta = \operatorname{sen}(\alpha + \beta)$$

$$\boxed{x = d \cdot \frac{\operatorname{sen}\alpha}{\operatorname{sen}(\alpha + \beta)}}$$

EXEMPLO

OS LADOS a, b E c ESTÃO, NESSA ORDEM, EM PROGRESSÃO ARITMÉTICA.

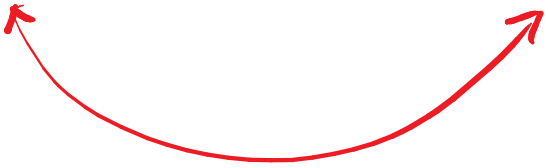
CALCULE O VALOR DE: $\dfrac{\cos \frac{A - C}{2}}{\cos \frac{A + C}{2}}$



$$(a, b, c) = (b - r, \ b, \ b + r)$$

$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} = \frac{c}{\operatorname{sen} C}$$

$$\frac{b - r}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} = \frac{b + r}{\operatorname{sen} C} = \frac{2b}{\operatorname{sen} A + \operatorname{sen} C}$$



$$\frac{\cancel{b}}{\operatorname{sen} B} = \frac{2\cancel{b}}{\operatorname{sen} A + \operatorname{sen} C}$$

$$\operatorname{sen} A + \operatorname{sen} C = 2 . \operatorname{sen} B$$

$$(A + C) + (B) = 180^\circ$$

$$\operatorname{sen} B = \operatorname{sen}(A + C)$$

$$\operatorname{sen} A + \operatorname{sen} C = 2 \cdot \operatorname{sen}(A + C)$$

$$2 \cdot \operatorname{sen}\left(\frac{A+C}{2}\right) \cdot \cos\left(\frac{A-C}{2}\right) = 2 \cdot 2 \cdot \operatorname{sen}\left(\frac{A+C}{2}\right) \cdot \cos\left(\frac{A+C}{2}\right)$$

$$\operatorname{sen} 2\alpha = 2 \cdot \operatorname{sen}\alpha \cdot \cos\alpha$$

$$\operatorname{sen}(A+C) = 2 \cdot \operatorname{sen}\left(\frac{A+C}{2}\right) \cdot \cos\left(\frac{A+C}{2}\right)$$

$$2 \cdot \operatorname{sen}\left(\frac{A+C}{2}\right) \cdot \cos\left(\frac{A-C}{2}\right) = 2 \cdot 2 \cdot \operatorname{sen}\left(\frac{A+C}{2}\right) \cdot \cos\left(\frac{A+C}{2}\right)$$

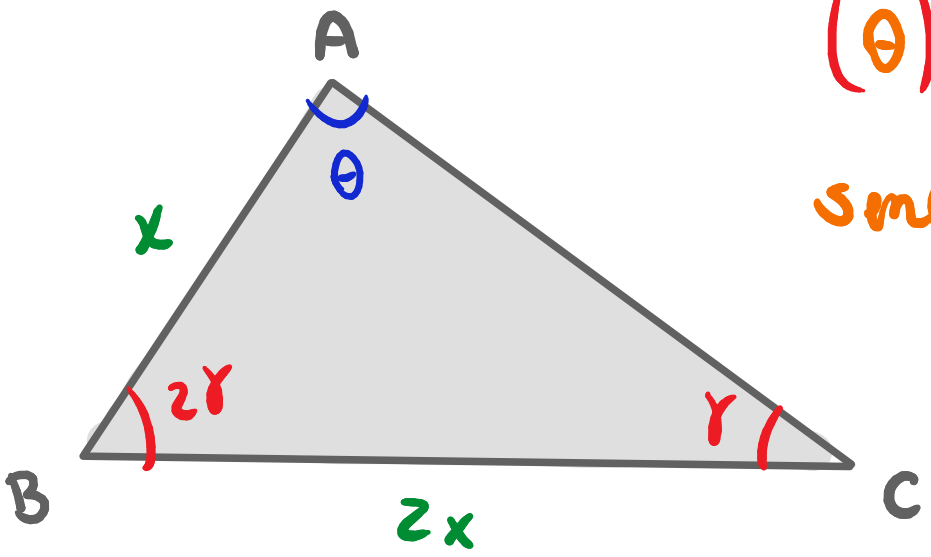
$$\cos\frac{A-C}{2} = 2 \cdot \cos\frac{A+C}{2}$$

$$\frac{\cos\frac{A-C}{2}}{\cos\frac{A+C}{2}} = 2$$

EXEMPLO

NO TRIÂNGULO ABC, $BC = 2.AB$ E $\hat{B} = 2.\hat{C}$

CALCULE O ÂNGULO $\hat{C}$.



$(\theta) + (3\gamma) = 180°$

$\operatorname{sen}\theta = \operatorname{sen} 3\gamma$

$$\frac{x}{\operatorname{sen}\gamma} = \frac{2x}{\operatorname{sen}\theta} \rightarrow \operatorname{sen}\theta = 2.\operatorname{sen}\gamma$$

$$\operatorname{sen} 3\gamma = 2.\operatorname{sen}\gamma$$

$$3\operatorname{sen}\gamma - 4\operatorname{sen}^3\gamma = 2\operatorname{sen}\gamma$$

$$\operatorname{sen}\gamma = 4\operatorname{sen}^2\gamma$$

$$4\operatorname{sen}^2\gamma = 1$$

$$\operatorname{sen}^2\gamma = \frac{1}{4}$$

$$\operatorname{sen}\gamma = \pm\frac{1}{2}$$

$$\operatorname{sen}\gamma = \frac{1}{2}$$

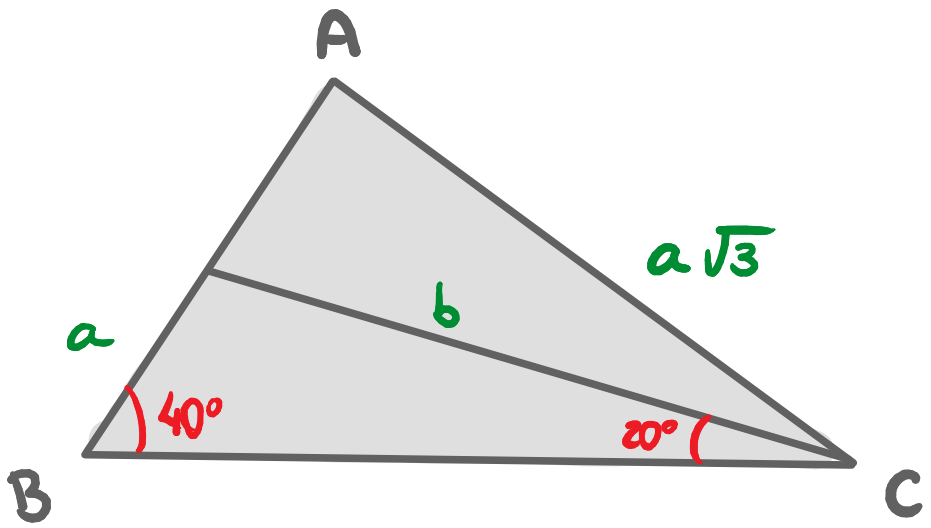
$\gamma = 30°$ ou ~~$\gamma = 150°$~~

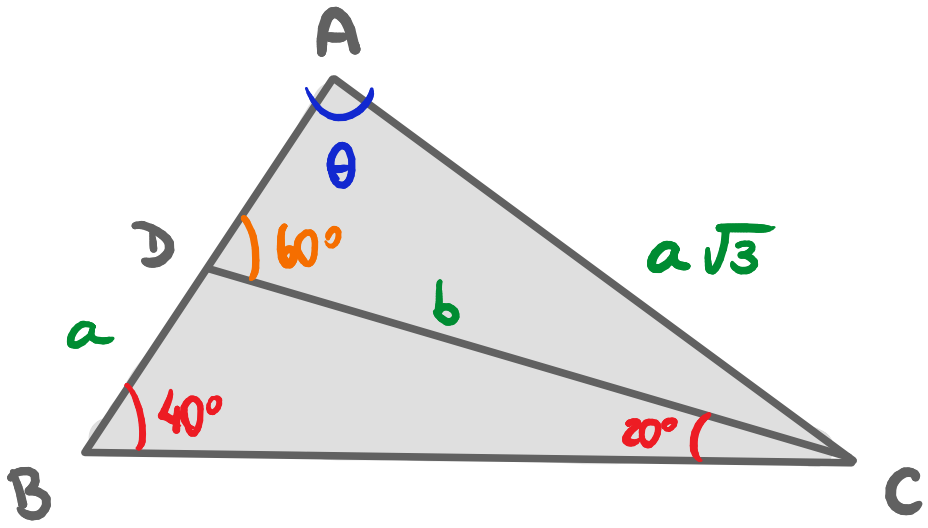
$3\gamma < 180°$

$$\gamma = 30°$$

EXEMPLO

CALCULE O VALOR DO ÂNGULO Â DO TRIÂNGULO.





$$\Delta BCD: \frac{b}{\operatorname{sen} 40^\circ} = \frac{a}{\operatorname{sen} 20^\circ}$$

$$\frac{b}{a} = \frac{\operatorname{sen} 40^\circ}{\operatorname{sen} 20^\circ}$$

$$\Delta ACD: \frac{b}{\operatorname{sen}\theta} = \frac{a\sqrt{3}}{\operatorname{sen} 60^\circ}$$

$$\frac{b}{a} = \frac{\sqrt{3}.\operatorname{sen}\theta}{\operatorname{sen} 60^\circ}$$

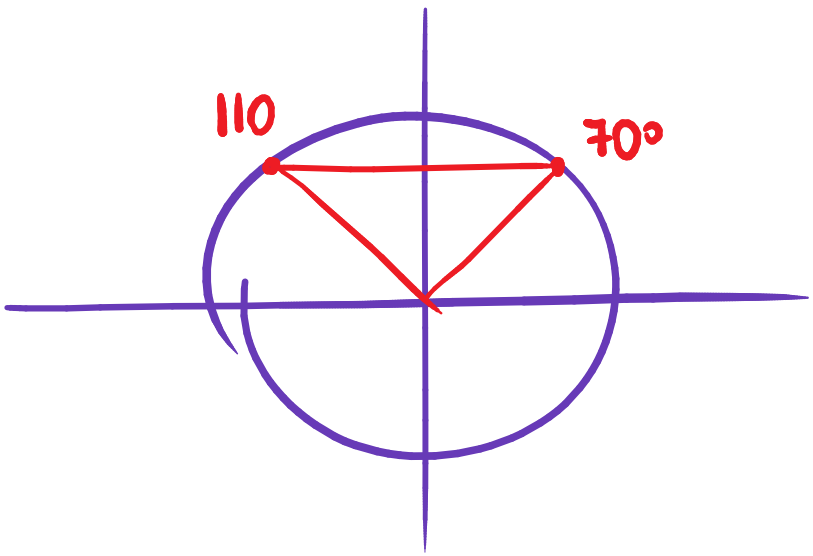
$$\frac{\sqrt{3}.\operatorname{sen}\theta}{\operatorname{sen}60^\circ} = \frac{\operatorname{sen}40^\circ}{\operatorname{sen}20^\circ}$$

$$\operatorname{sen}\theta \cdot \frac{\sqrt{3}}{\sqrt{3}/2} = \frac{2.\operatorname{sen}20^\circ.\cos 20^\circ}{\operatorname{sen}20^\circ}$$

$$\operatorname{sen}\theta \cdot \sqrt{3} \cdot \frac{2}{\sqrt{3}} = 2\cos 20^\circ$$

$$\operatorname{sen}\theta = \operatorname{sen}70^\circ$$

$$\theta = 70^\circ \quad \text{ou} \quad \theta = 110^\circ$$



EXEMPLO

NO TRIÂNGULO ABAIXO TEM-SE:

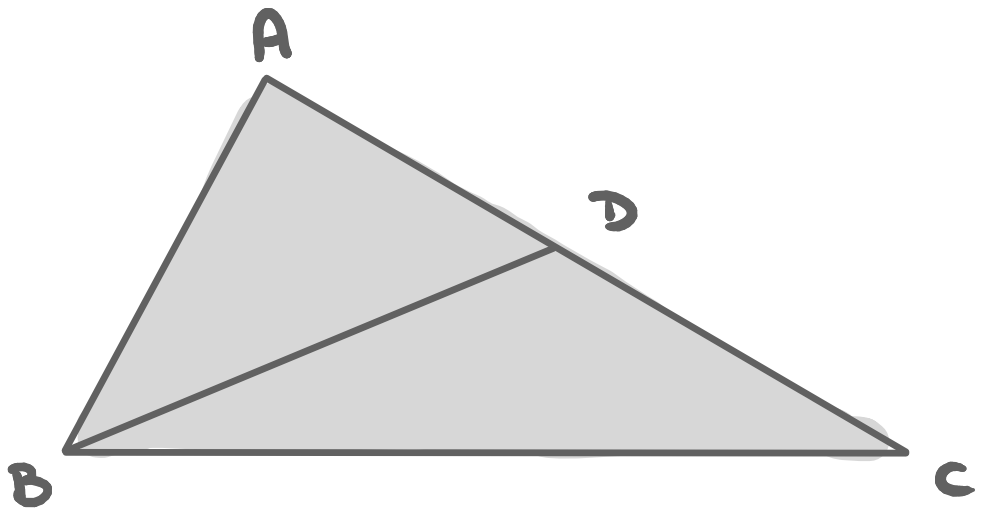
$AB = CD$

$D\hat{B}C = x$

$B\hat{C}D = 2x$

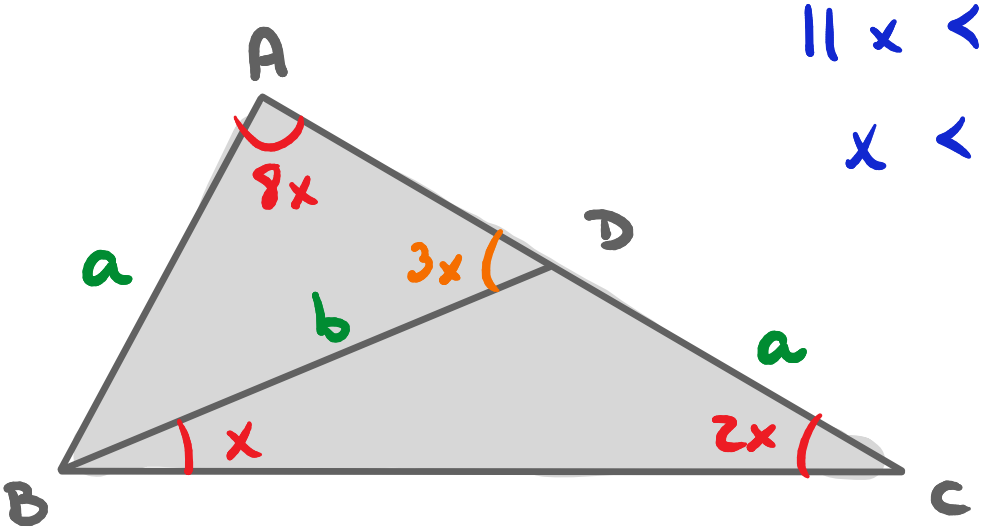
$B\hat{A}C = 8x$

DETERMINE O VALOR DO ÂNGULO x.



$11x < 180°$

$x < 16°, \ldots$



$$\Delta BCD: \quad \frac{b}{\operatorname{sen} 2x} = \frac{a}{\operatorname{sen} x}$$

$$\frac{b}{a} = \frac{\operatorname{sen} 2x}{\operatorname{sen} x}$$

$$\Delta ABD: \quad \frac{b}{\operatorname{sen} 8x} = \frac{a}{\operatorname{sen} 3x}$$

$$\frac{b}{a} = \frac{\operatorname{sen} 8x}{\operatorname{sen} 3x}$$

$$\frac{\sin 2x}{\sin x} = \frac{\sin 8x}{\sin 3x}$$

$$\sin 8x \cdot \sin x = \sin 3x \cdot \sin 2x$$

$$\sin 8x \cdot \sin x = \sin 3x \cdot 2 \cdot \sin x \cdot \cos x$$

$$\sin 8x = 2 \cdot \sin 3x \cos x$$

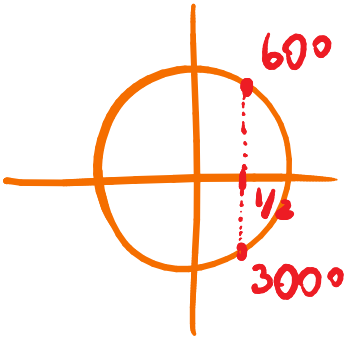
$$\sin 8x = \sin 4x + \sin 2x$$

$$\sin 8x - \sin 4x = \sin 2x$$

$$2 \cdot \cos 6x \cdot \sin 2x = \sin 2x$$

$$2 \cos 6x = 1$$

$$\cos 6x = \frac{1}{2}$$



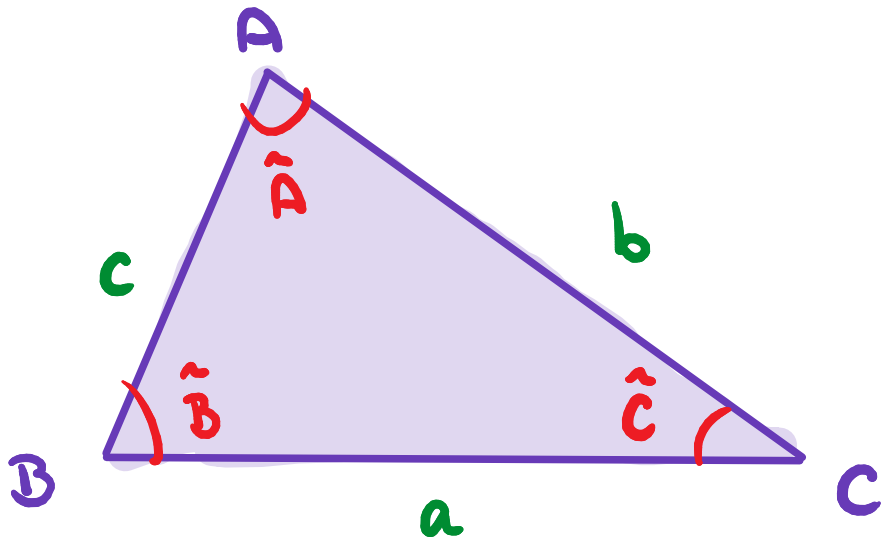
$$6x = 60°$$

$$\boxed{x = 10°}$$

~~$6x = 300°$~~

~~$x = 50°$~~

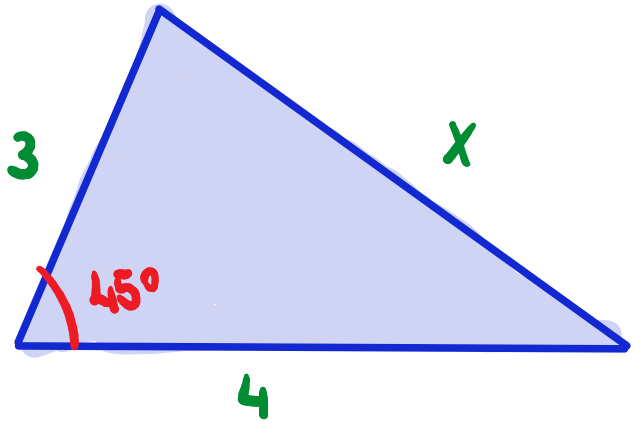
LEI DOS COSSENOS



$$a^2 = b^2 + c^2 - 2.b.c.\cos \hat{A}$$

$$b^2 = a^2 + c^2 - 2.a.c.\cos \hat{B}$$

$$c^2 = a^2 + b^2 - 2.a.b.\cos \hat{C}$$

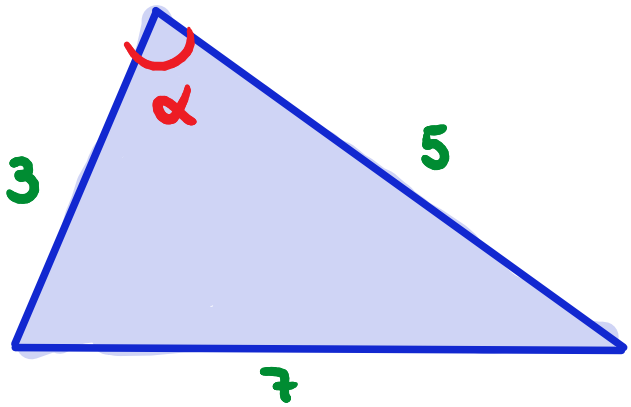


$$x^2 = 3^2 + 4^2 - 2.3.4.\cos 45°$$

$$x^2 = 9 + 16 - \cancel{2}.12.\frac{\sqrt{2}}{\cancel{2}}$$

$$x^2 = 25 - 12\sqrt{2}$$

$$\boxed{x = \sqrt{25 - 12\sqrt{2}}}$$

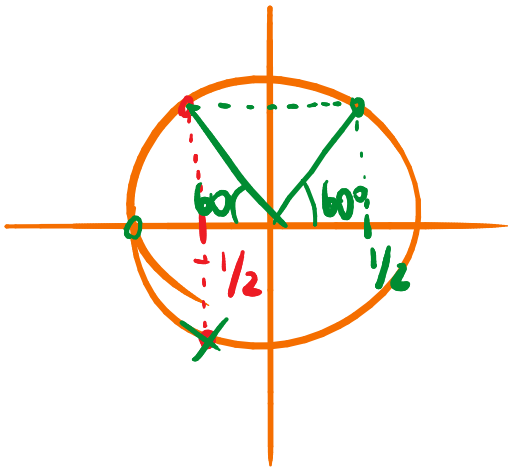


$$7^2 = 3^2 + 5^2 - 2.3.5.\cos\alpha$$

$$49 = 9 + 25 - 2.15.\cos\alpha$$

$$2.\cancel{15}^{1}.\cos\alpha = -\cancel{15}^{-1}$$

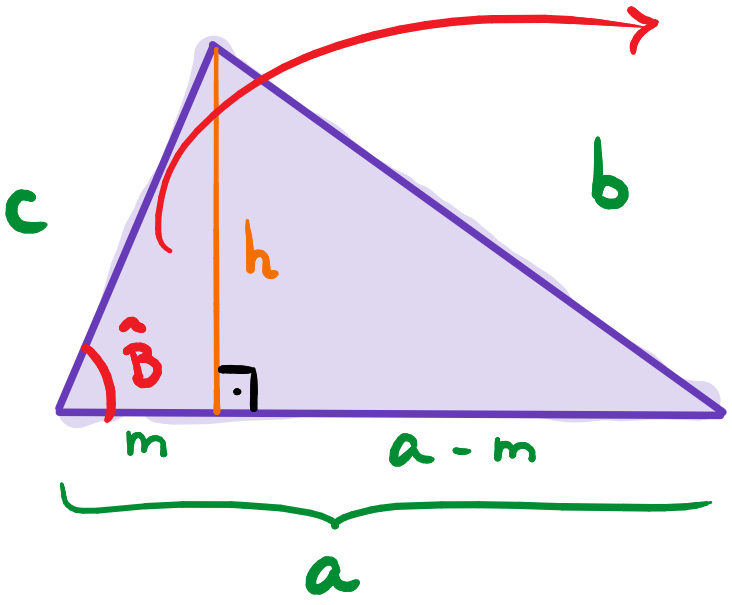
$$\cos\alpha = -\frac{1}{2}$$



$$\boxed{\alpha = 120°}$$

LEI DOS COSSENOS

DEMONSTRAÇÃO



$$\cos \hat{B} = \frac{m}{c}$$

$$m = c \cdot \cos \hat{B}$$

$$\begin{cases} h^2 = c^2 - m^2 \\ h^2 = b^2 - (a-m)^2 \end{cases} \rightarrow c^2 - m^2 = b^2 - (a-m)^2$$

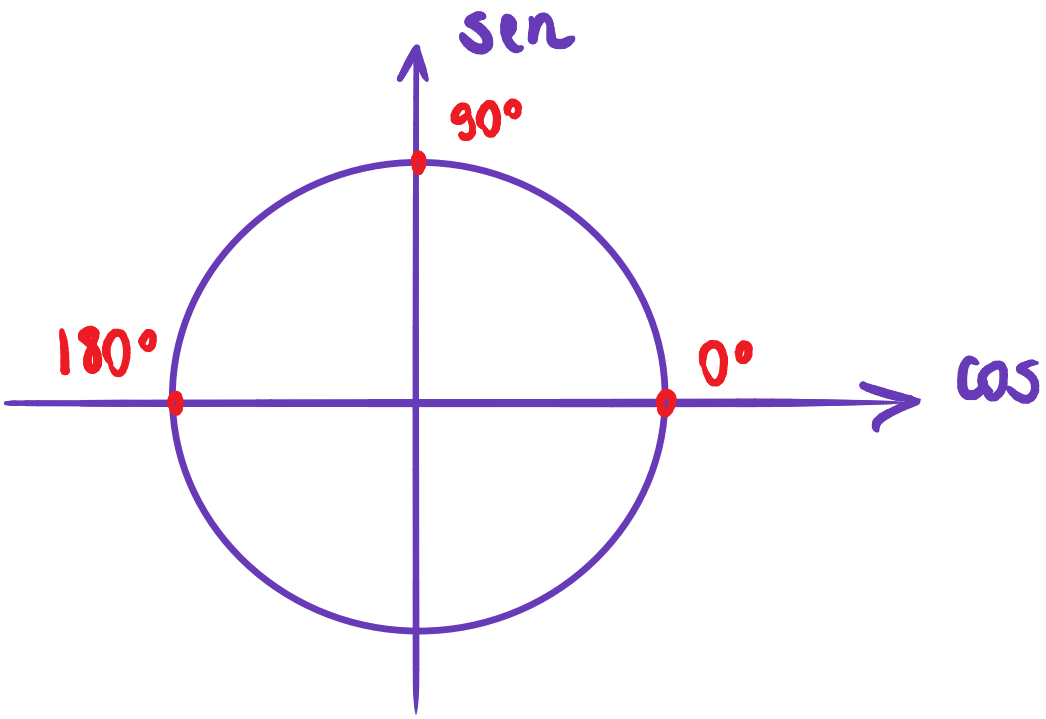
$$c^2 - m^2 = b^2 - a^2 + 2am - m^2$$

$$b^2 = a^2 + c^2 - 2am$$

$$\boxed{b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \hat{B}}$$

NATUREZA DO TRIÂNGULO

CICLO TRIGONOMÉTRICO

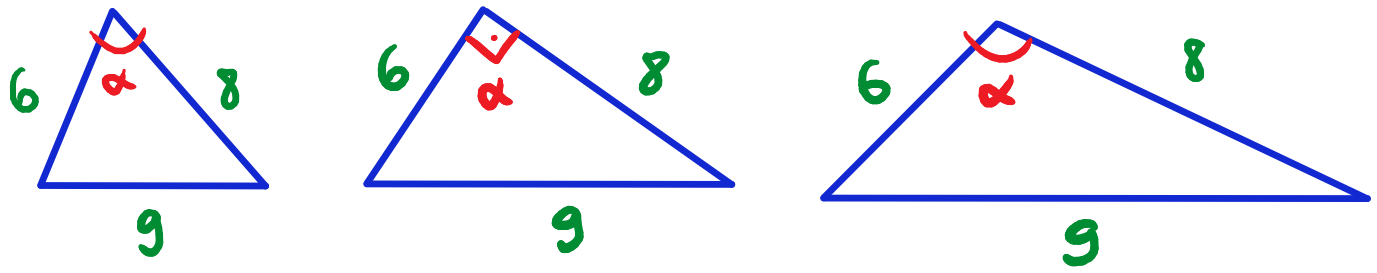


$0° < x < 90° \rightarrow \cos(x) > 0$

$x = 90° \rightarrow \cos(x) = 0$

$90° < x < 180° \rightarrow \cos(x) < 0$

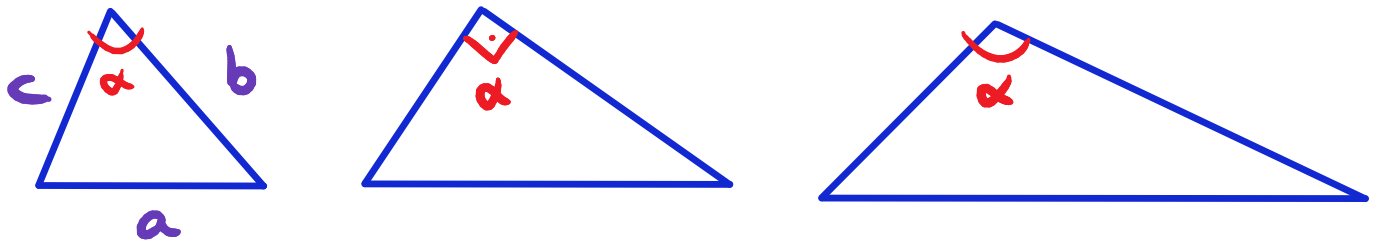
LADOS 6, 8, 9



$$9^2 = 6^2 + 8^2 - 2.6.8.\cos\alpha$$

$$\cos\alpha = \frac{-9^2 + 6^2 + 8^2}{2.6.8}$$

$$\cos\alpha = \frac{19}{96} > 0 \qquad \alpha < 90°$$



$$a^2 = b^2 + c^2 - 2.b.c.\cos\alpha$$

$$2.b.c.\cos\alpha = b^2 + c^2 - a^2$$

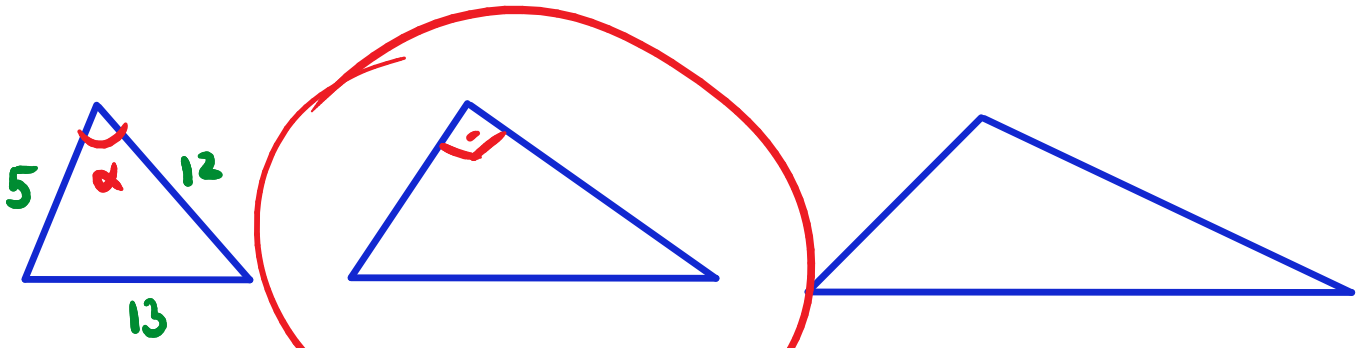
$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 + c^2 - a^2 > 0 \rightarrow \alpha < 90°$$

$$b^2 + c^2 - a^2 = 0 \rightarrow \alpha = 90°$$

$$b^2 + c^2 - a^2 < 0 \rightarrow 90° < \alpha < 180°$$

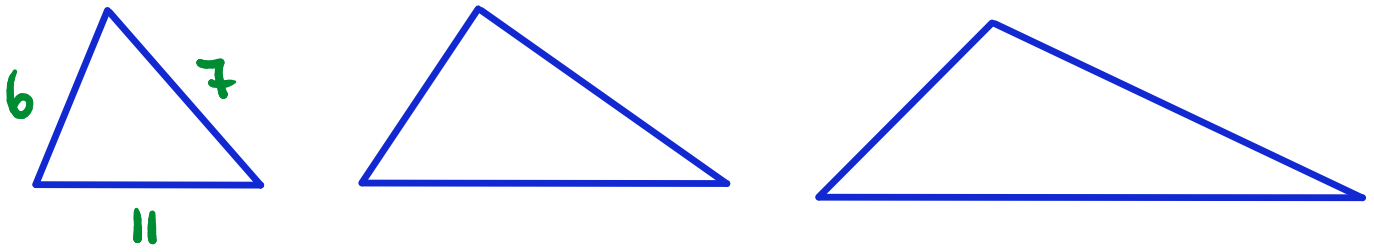
LADOS 5, 12, 13



$5^2 + 12^2 - 13^2 = 0 \rightarrow \alpha = 90°$

TR. RET.

LADOS 6, 7, 11



$$6^2 + 7^2 - 11^2 = 36 + 49 - 121$$

$$= 85 - 121$$

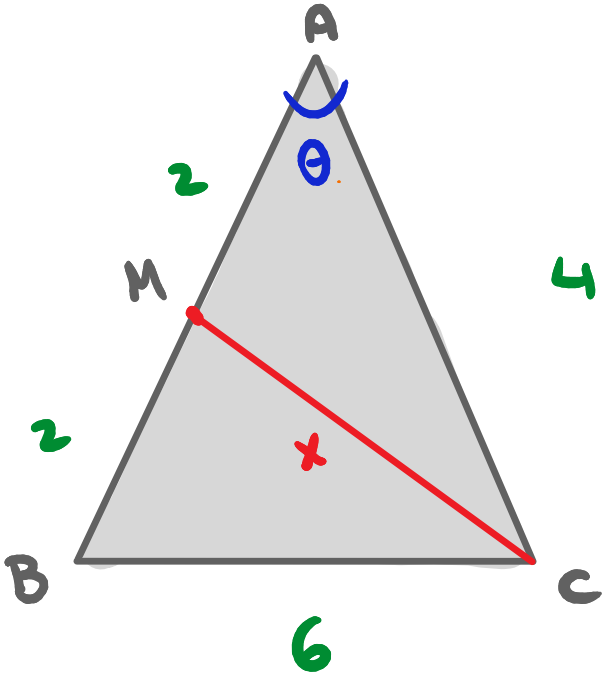
$$= -36 < 0$$

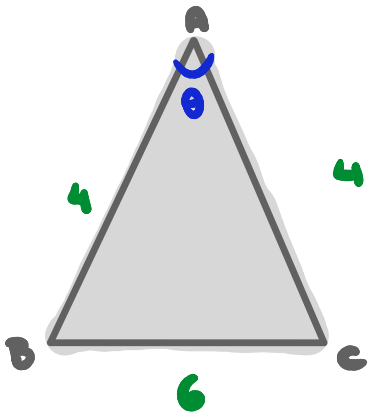
$$90° < \alpha < 180°$$

TR. OBTUSÂNGULO

EXEMPLO

SEJA M O PONTO MÉDIO DO LADO AB DO TRIÂNGULO ABC. CALCULE O COMPRIMENTO MC, SABENDO QUE AB = AC = 4 E BC = 6.





$$6^2 = 4^2 + 4^2 - 2.4.4.\cos\theta$$

$$2.\cancel{16}^{4}.\cos\theta = -\cancel{4}^{-1}$$

$$\cos\theta = \frac{-1}{8}$$

ΔAMC : $x^2 = 2^2 + 4^2 - 2.2.4.\cos\theta$

$$x^2 = 4 + 16 - 2.\cancel{2}.\cancel{4}\left(\frac{-1}{\cancel{8}}\right)$$

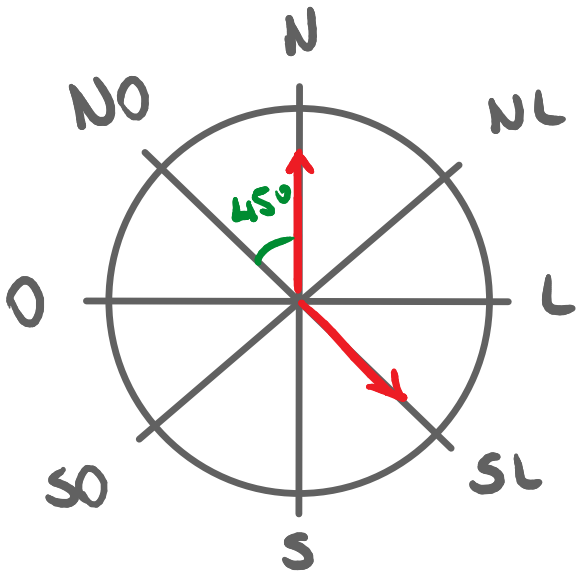
$$x^2 = 20 + 2$$

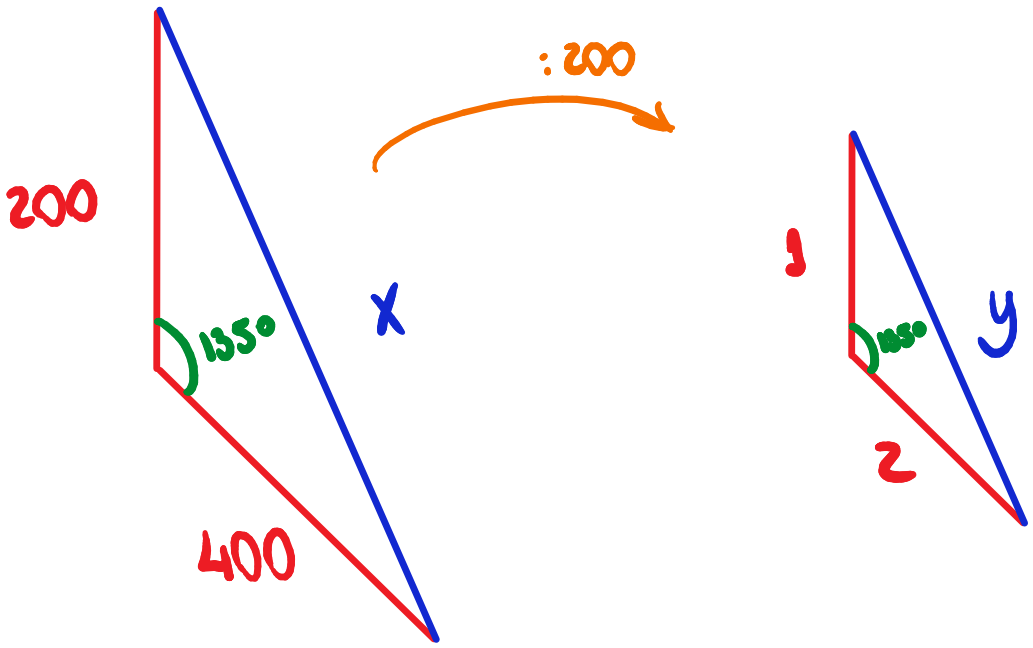
$$x = \sqrt{22}$$

EXEMPLO

OS BARCOS A E B PARTEM DE UM MESMO PONTO. O BARCO A VIAJA SENTIDO NORTE, COM VELOCIDADE 100Km/h, E O BARCO B VIAJA NA DIREÇÃO SUDESTE, COM VELOCIDADE 200Km/h.

QUAL A DISTÂNCIA ENTRE OS BARCOS 2 HORAS DEPOIS DA PARTIDA?





$$y^2 = 1^2 + 2^2 - 2 \cdot 1 \cdot 2 \cdot \cos 135°$$

$$y^2 = 5 - 2 \cdot 2 \cdot \left(-\frac{\sqrt{2}}{2}\right)$$

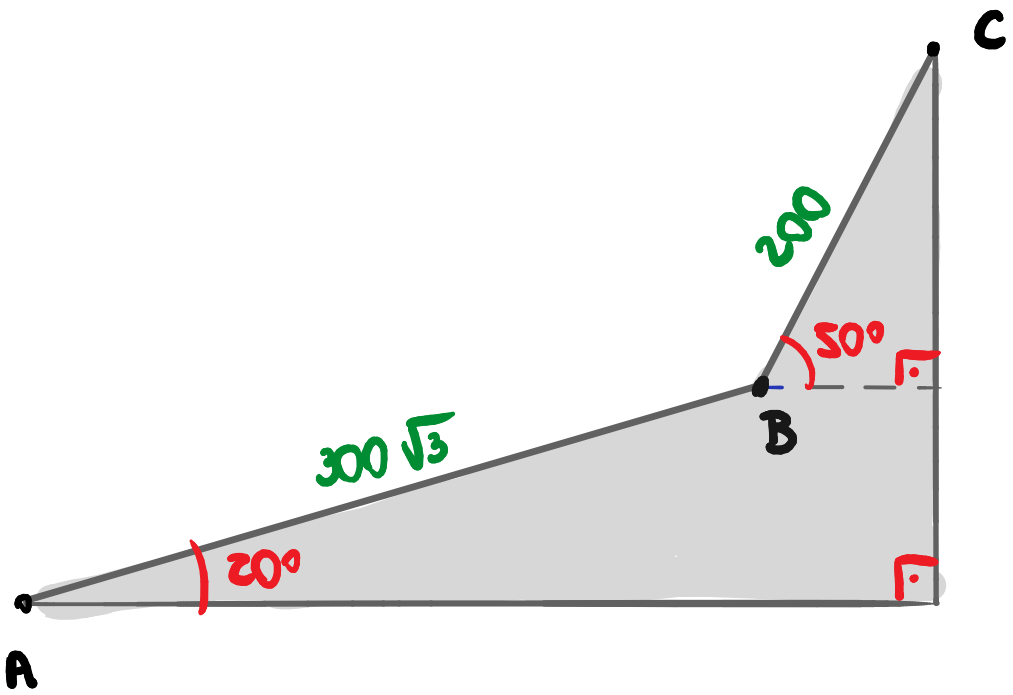
$$y^2 = 5 + 2\sqrt{2}$$

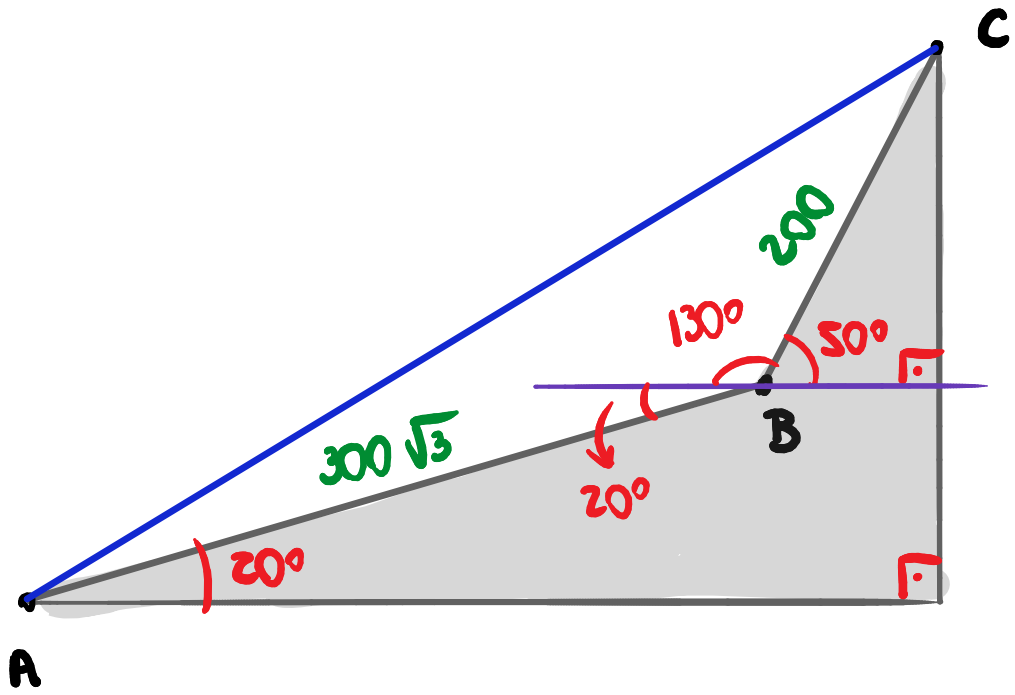
$$y = \sqrt{5 + 2\sqrt{2}} \rightarrow x = 200\sqrt{5 + 2\sqrt{2}}$$

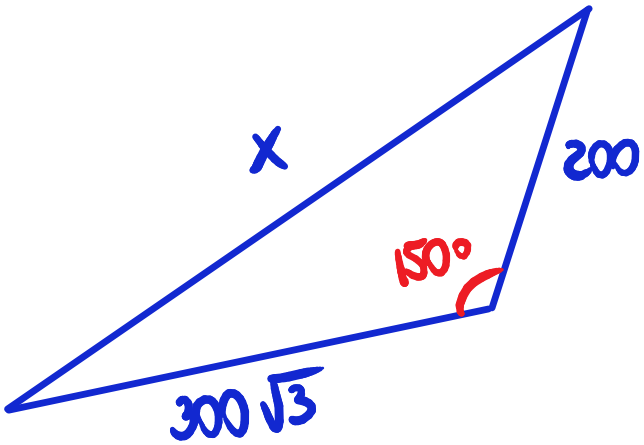
EXEMPLO

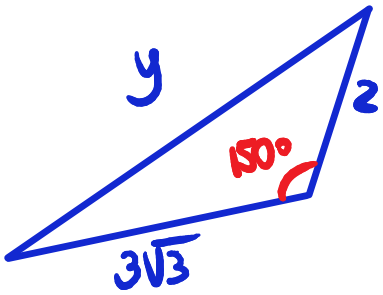
PARA DE DESLOCAR DO PONTO A AO PONTO C, ATRAVÉS DE UM TELEFÉRICO, PASSA-SE PELO PONTO B, SEGUNDO O ESQUEMA ABAIXO.

QUAL SERIA A DISTÂNCIA PERCORRIDA SE FOSSE POSSÍVEL SE DESLOCAR DO PONTO A DIRETAMENTE ATÉ O PONTO C?









$$y^2 = 2^2 + (3\sqrt{3})^2 - 2 \cdot 2 \cdot 3\sqrt{3} \cdot \cos 150°$$

$$y^2 = 4 + 27 - 2 \cdot 2 \cdot 3 \cdot \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)$$

$$y^2 = 31 + 18$$

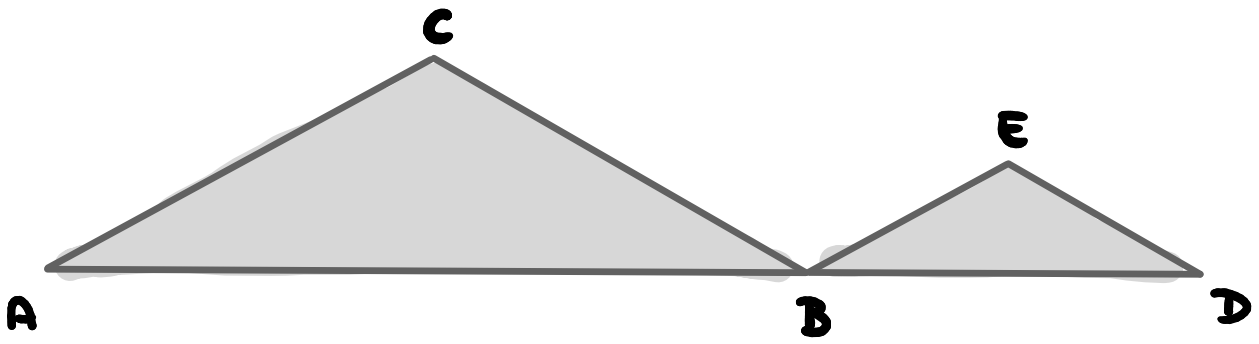
$$y^2 = 49$$

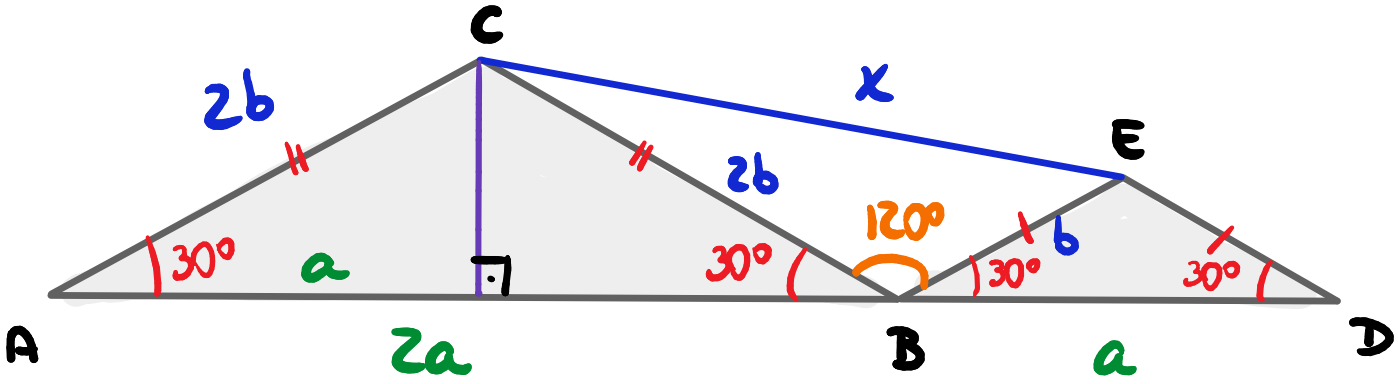
$$y = 7 \longrightarrow x = 700$$

EXEMPLO

NA FIGURA ABAIXO, ABC E BDE SÃO TRIÂNGULOS ISÓCELES SEMELHANTES DE BASES 2a E a, RESPECTIVAMENTE.

SE O ÂNGULO BÂC = 30°, CALCULE CE.





$$x^2 = b^2 + (2b)^2 - 2 \cdot b \cdot 2b \cdot \cos 120°$$

$$x^2 = b^2 + 4b^2 - 2 \cdot 2 \cdot b^2 \cdot \left(\frac{-1}{2}\right)$$

$$x^2 = 7b^2 \longrightarrow \underline{x = b\sqrt{7}}$$

$$\cos 30° = \frac{a}{2b} \rightarrow \frac{\sqrt{3}}{2} = \frac{a}{2b}$$

$$b\sqrt{3} = a \rightarrow b = \frac{a}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$b = \frac{a\sqrt{3}}{3}$$

$$x = \frac{a\sqrt{3}}{3} \cdot \sqrt{7} \rightarrow \boxed{x = \frac{a\sqrt{21}}{3}}$$

EXEMPLO

OS LADOS a, b E c DE UM TRIÂNGULO SATISFAZEM A SEGUINTE RELAÇÃO:

$$(a + b + c)(a + b - c) = 3ab$$

CALCULE O ÂNGULO OPOSTO AO LADO c.

$$((a+b)+(c))((a+b)-(c)) = 3ab$$

$$(a+b)^2 - c^2 = 3ab$$

$$a^2 + 2ab + b^2 - c^2 = 3ab$$

$$\begin{cases} c^2 = a^2 + b^2 - ab \\ c^2 = a^2 + b^2 - 2.a.b.\cos C \end{cases}$$

$$a^2 + b^2 - ab = a^2 + b^2 - 2.a.b.\cos C$$

$$+ab = +2ab\cos\hat{C}$$

$$2\cos\hat{C} = 1$$

$$\cos\hat{C} = \frac{1}{2}$$

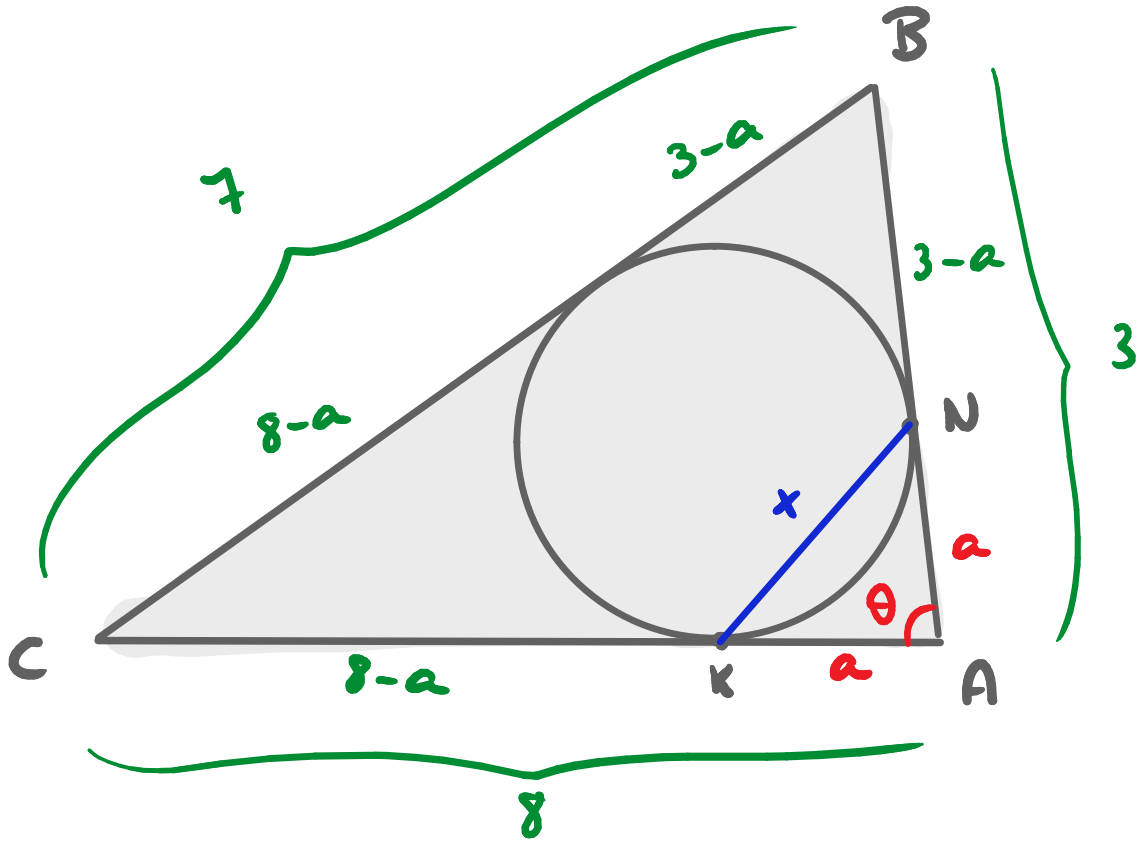
$$\hat{C} = 60^\circ$$

EXEMPLO

OS LADOS DO TRIÂNGULO ABC MEDEM:

$$AB = 3, \quad BC = 7, \quad AC = 8.$$

A CIRCUNFERÊNCIA INSCRITA NO TRIÂNGULO TANGENCIA AB EM N E AC EM K. CALCULE O COMPRIMENTO DO SEGMENTO KN.



$$8-a+3-a=7$$

$$-2a=-4$$

$$a=2$$

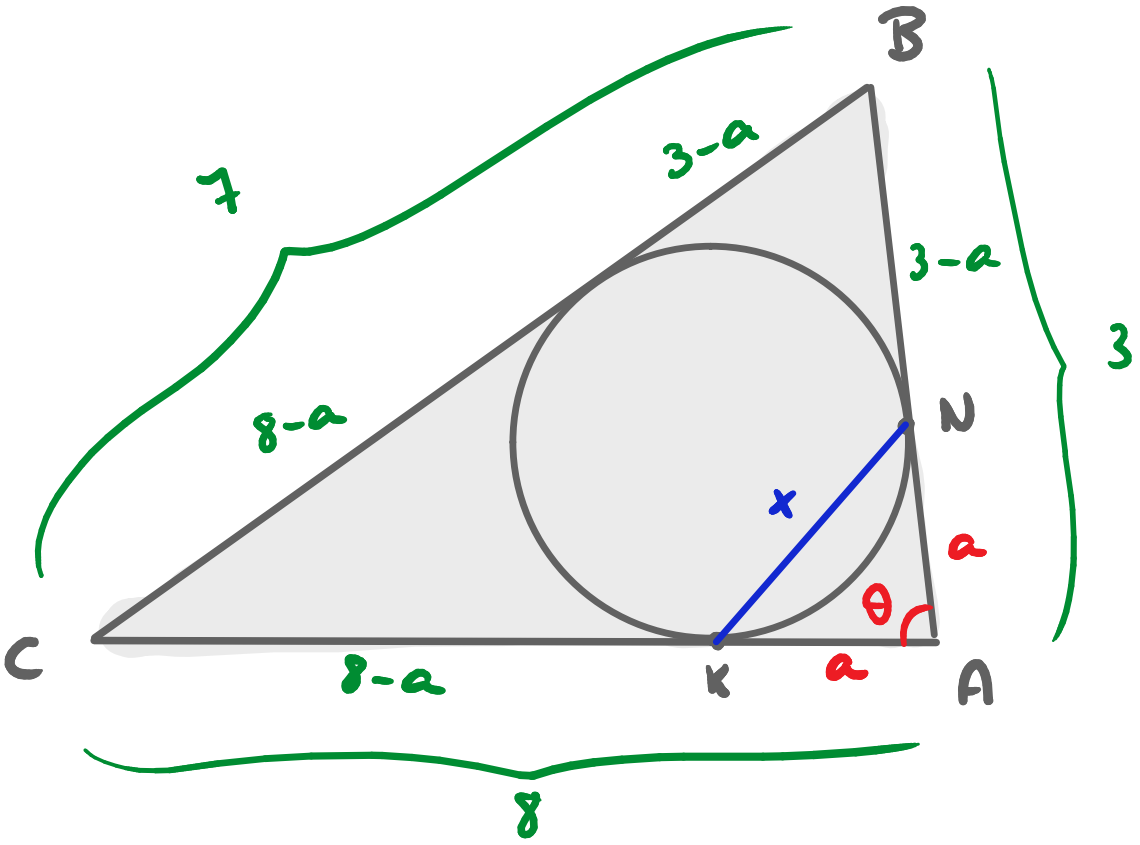
$$7^2=8^2+3^2-2.3.8.\cos\theta$$

$$49=64+9-2.3.8.\cos\theta$$

$$2.3.8\cos\theta=73-49$$

$$\cos\theta=\frac{24}{2.3.8}\longrightarrow\cos\theta=\frac{1}{2}$$

$$\theta=60^\circ$$



$$x^2 = a^2 + a^2 - 2 \cdot a \cdot a \cdot \cos\theta$$

$$x^2 = 2a^2 - 2a^2 \cdot \frac{1}{2}$$

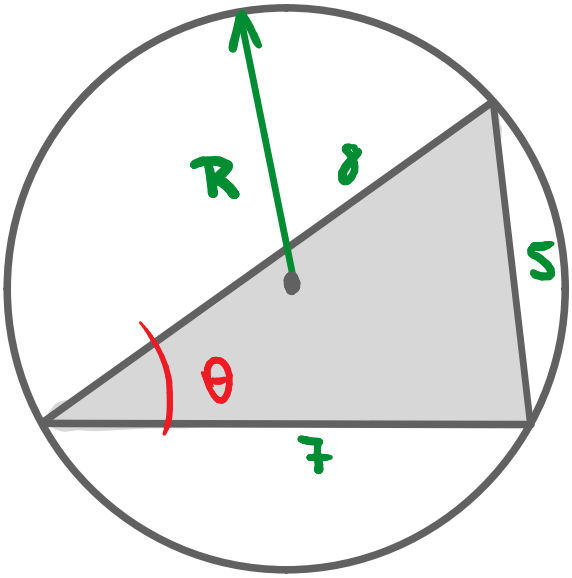
$$x^2 = a^2$$

$$x = a$$

$$x = 2$$

EXEMPLO

CALCULE O RAIO DO CÍRCULO CIRCUNSCRITO AO TRIÂNGULO DE LADOS 5, 7 E 8.



$$5^2 = 8^2 + 7^2 - 2.8.7.\cos\theta$$

$$2.8.7.\cos\theta = 64 + 49 - 25$$

$$2.8.7.\cos\theta = 88$$

$$\cos\theta = \frac{11}{14}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(\frac{11}{14}\right)^2 = 1$$

$$\sin^2\theta = \frac{196}{196} - \frac{121}{196}$$

$$\sin^2\theta = \frac{75}{196}$$

$$\sin\theta = \pm\sqrt{\frac{3.25}{196}}$$

$$\sin\theta = \frac{5\sqrt{3}}{14}$$

$$2R = \frac{5}{\sin\theta}$$

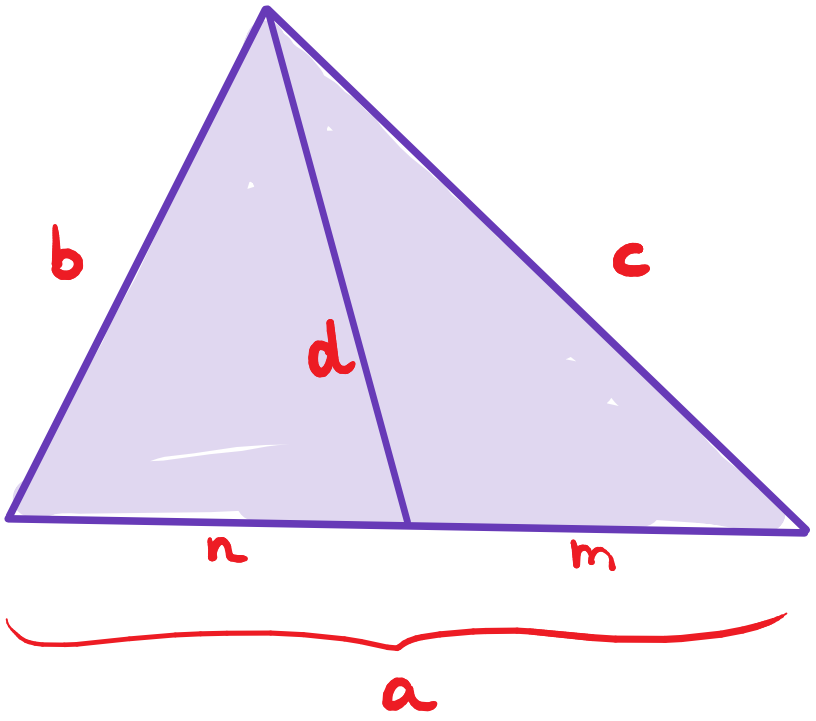
$$2R = \frac{5}{\frac{5\sqrt{3}}{14}}$$

$$2R = 5 \cdot \frac{14^{7}}{5\sqrt{3}}$$

$$R = \frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{R = \frac{7\sqrt{3}}{3}}$$

RELAÇÃO DE STEWART

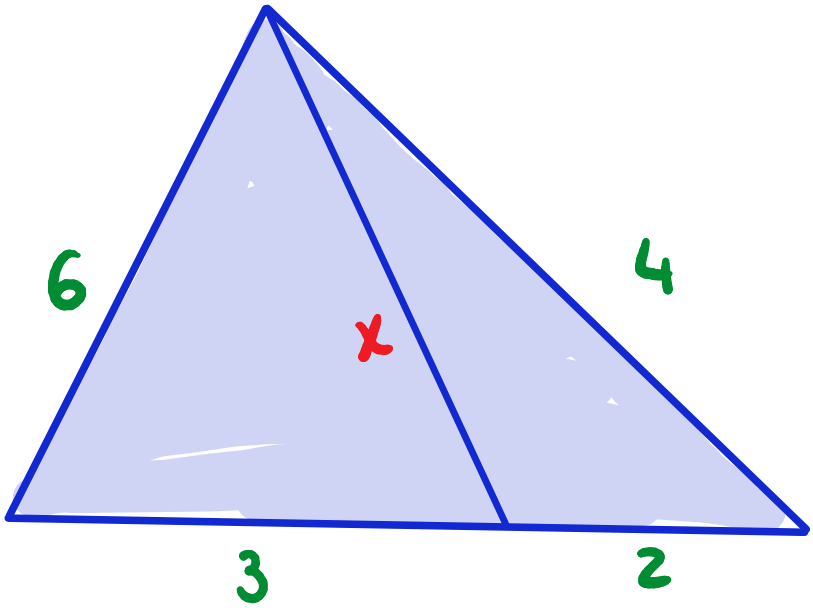


$$a(d^2 + mn) = b^2m + c^2n$$

man + dad = bmb + cnc

"A MAN AND HIS DAD

PUT A BOMB IN THE SINK"



$$3.5.2 + x.5.x = 6.2.6 + 4.3.4$$

$$5x^2 + 30 = 72 + 48$$

$$5x^2 = 90$$

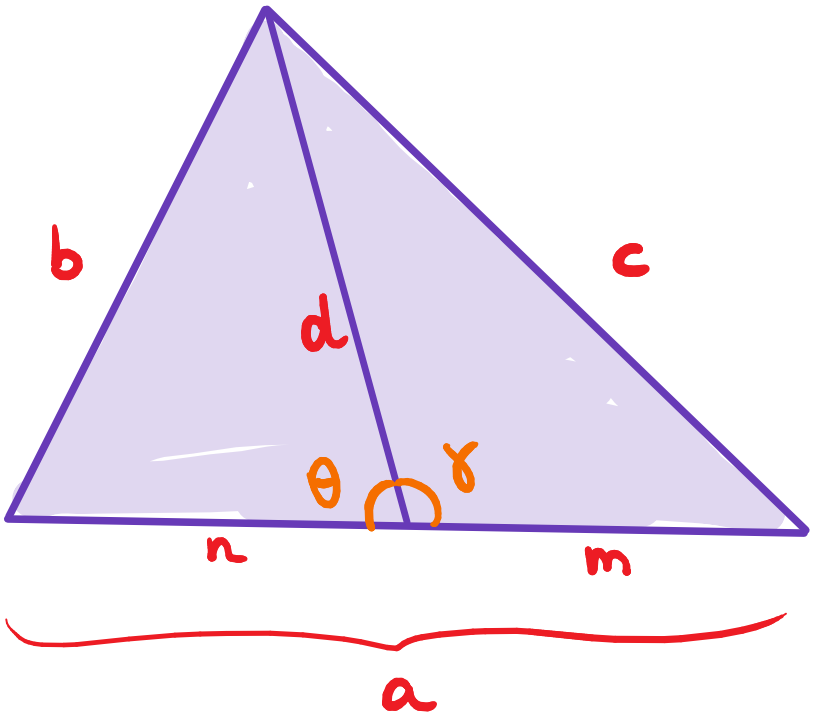
$$x^2 = 18$$

$$x = \sqrt{18}$$

$$x = 3\sqrt{2}$$

RELAÇÃO DE STEWART

DEMONSTRAÇÃO



$$\begin{cases} b^2 = d^2 + n^2 - 2.dn.\cos\theta \\ c^2 = d^2 + m^2 - 2dm.\cos\gamma \end{cases}$$

$$\theta + \gamma = 180° \rightarrow \cos\gamma = -\cos\theta$$

$$\begin{cases} b^2 = d^2 + n^2 - 2.dn.\cos\theta \\ c^2 = d^2 + m^2 - 2dm.(-\cos\theta) \end{cases}$$

$$\begin{cases} b^2 = d^2 + n^2 - 2.dn.\cos\theta \quad (m) \\ c^2 = d^2 + m^2 + 2dm.\cos\theta \quad (n) \end{cases}$$

$$\begin{cases} b^2 m = d^2 m + mn^2 - 2dmn\cos\theta \\ c^2 n = d^2 n + m^2 n + 2dmn\cos\theta \end{cases}$$

(+)

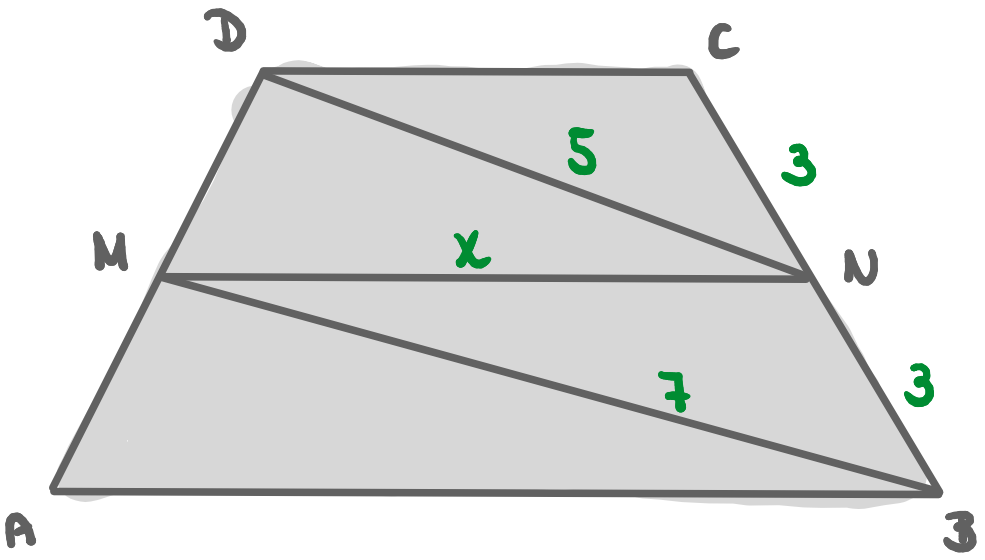
$$b^2 m + c^2 n = d^2 m + d^2 n + mn^2 + m^2 n$$

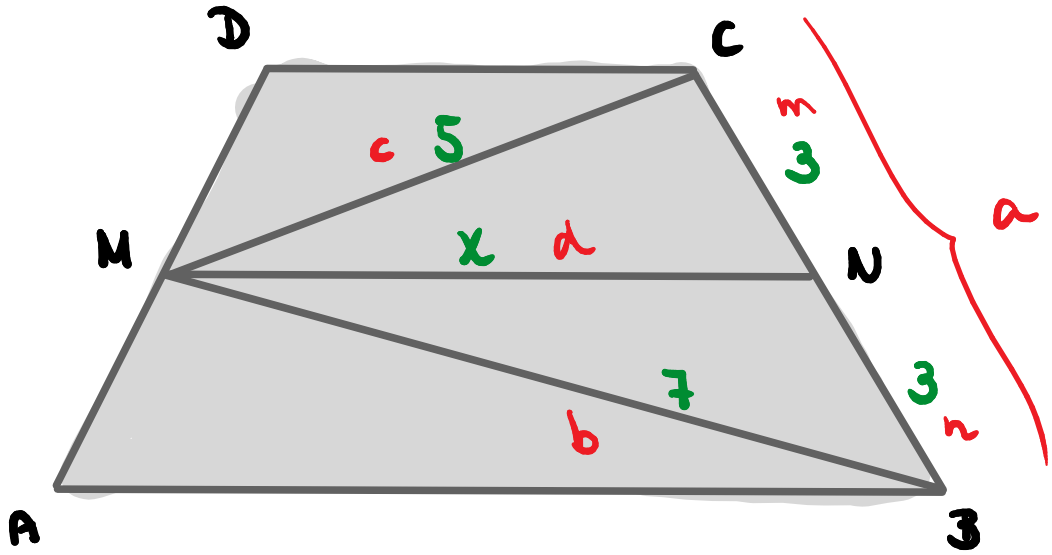
$$b^2 m + c^2 n = d^2 \underbrace{(m+n)}_{a} + mn\underbrace{(n+m)}_{a}$$

$$\boxed{man + d^2 a = b^2 m + c^2 n}$$

EXEMPLO

NO TRAPÉZIO ISÓCELES ABAIXO, CALCULE O COMPRIMENNTO DE MN SABENDO QUE ESSE SEGMENTO É PARALELO ÀS BASES DO TRAPÉZIO.





$$3.6.3 + x.6.x = 7.3.7 + 5.3.5$$

$$6x^2 = 3(49 + 25) - 54$$

$$2.3.x^2 = 3.74 - 3.18$$

$$x^2 = 37 - 9$$

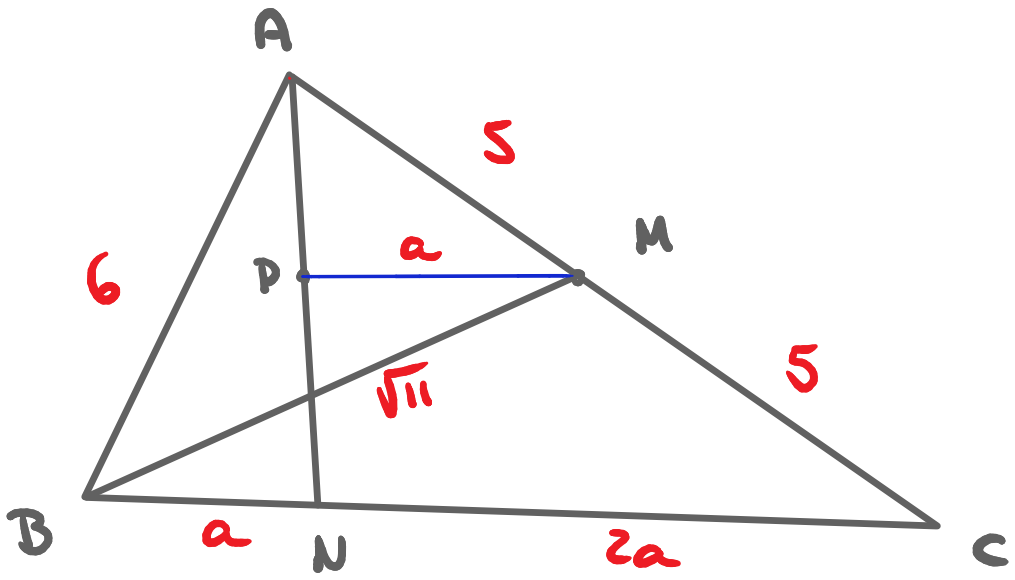
$$x^2 = 28$$

$$x = 2\sqrt{7}$$

EXEMPLO

SEJA UM TRIÂNGULO ABC COM MEDIANA BM E CEVIANA AN, TAL QUE CN = 2.BN.

CALCULE A MEDIDA DO SEGMENTO MP, ONDE P É PONTO MÉDIO DE AN, AC = 10, AB = 6 E BM = $\sqrt{11}$.



$$5.10.5 + \sqrt{11}.10.\sqrt{11} = 6.5.6 + 3a.5.3a$$

$$250 + 110 = 180 + 45a^2$$

$$45a^2 = 180 \quad (4)$$

$$a^2 = 4$$

$$a = 2$$